Gravitationally induced decoherence: from theoretical models to signatures in neutrino oscillations

Max Joseph Fahn

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Theory Seminar - Bologna, Jan 29, 2025

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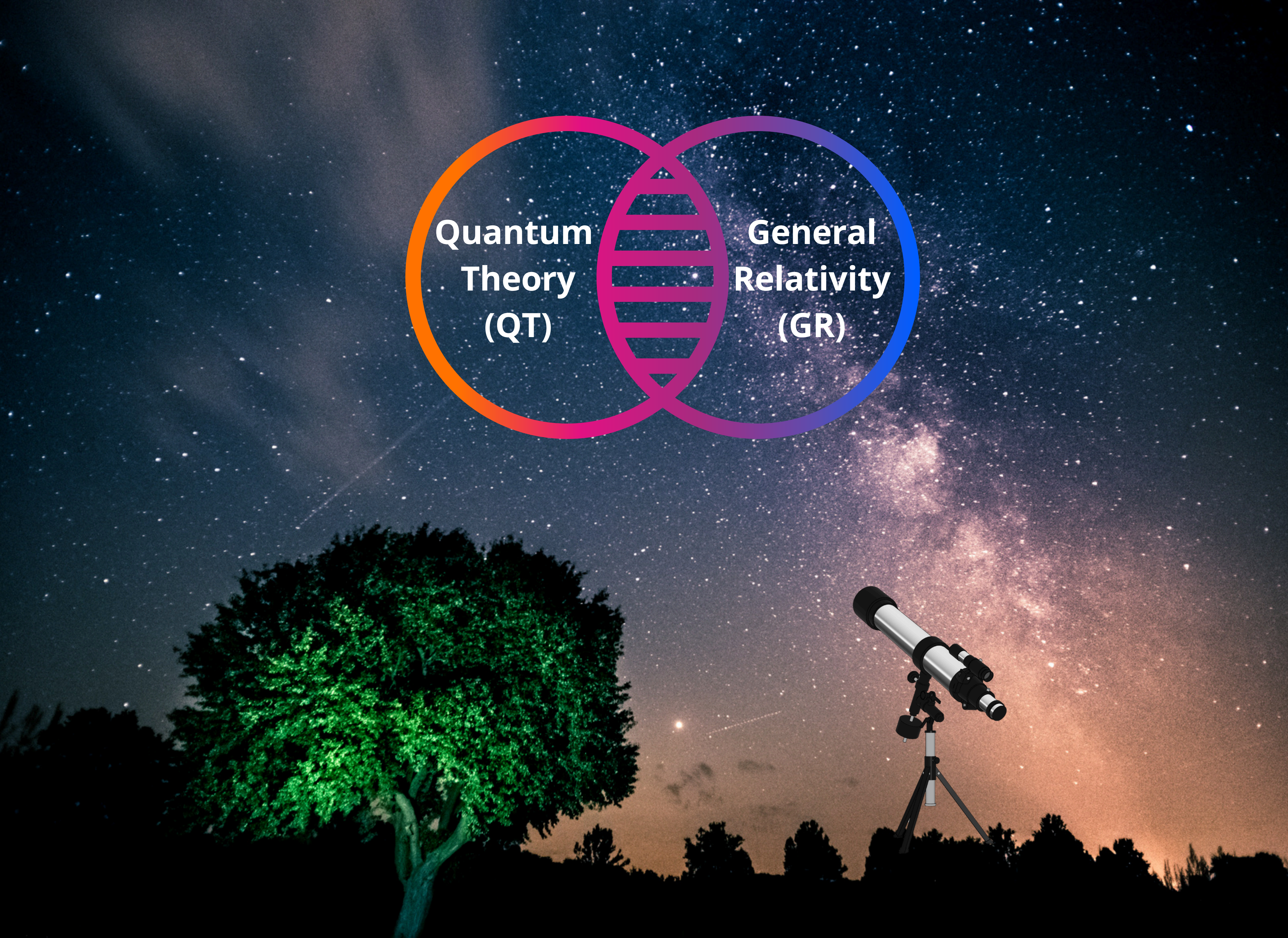

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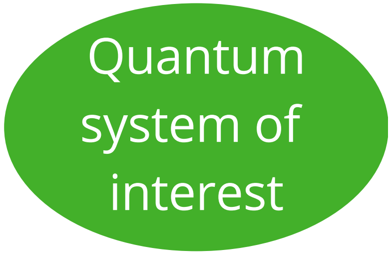
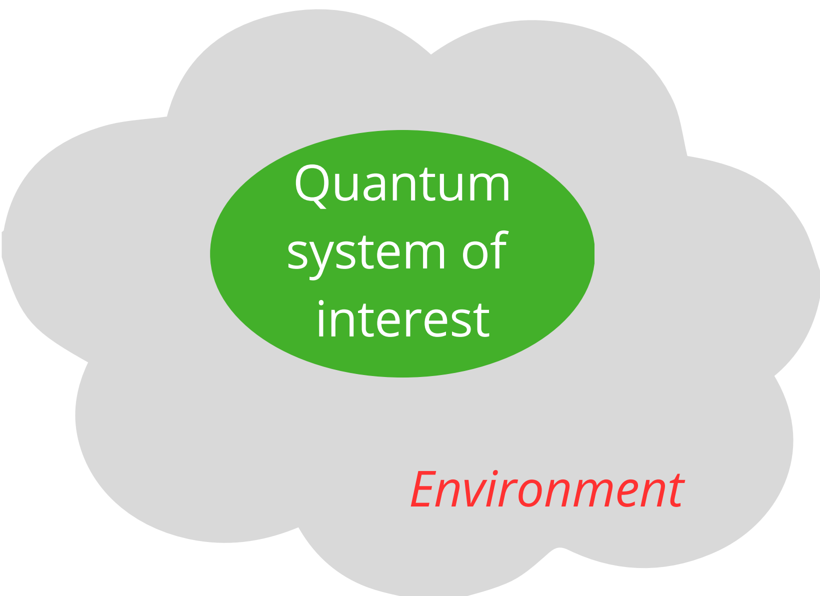
Isolated Quantum Systems



Liouville-von Neumann equation

$$\frac{\partial \rho_S(t)}{\partial t} = -i[H_S(t), \rho_S(t)]$$

Open Quantum Systems



Master equation

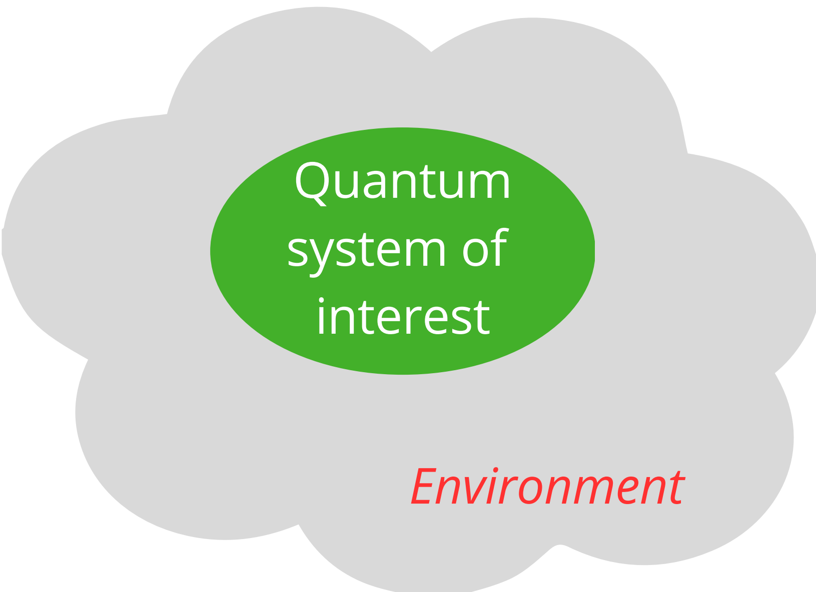
$$\frac{\partial \rho_S(t)}{\partial t} = -i[H_S(t) + H_{add}(t), \rho_S(t)] + \mathcal{D}[\rho_S(t)]$$

New processes encoded in red terms:

- Energy shifts / Renormalisation
- Dissipation
- Decoherence

Vanishing of off-diagonal elements of $\rho_S(t)$
→ no interference
→ "classicalisation"

Open Quantum Systems



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- Energy shifts / Renormalisation
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- Without gravity: Frequently used (e.g. Quantum Optics, QED, Solid State Physics, …)
- With (quantised) gravity as environment: Rather new

$\longleftrightarrow$ Need some Quantum Gravity Theory as basis

Overview of the talk

Part I: Field-theoretical master equation

"A gravitationally induced decoherence model using Ashtekar variables" (2022);
***MJF, K. Giesel, M. Kobler* (CQG)**

Part II: One-particle projection of the master equation

"Gravitationally induced decoherence of a scalar field: investigating the one-particle sector and its interplay with renormalisation" (2024);
***MJF, K. Giesel* (arXiv)**

Part III: Application to neutrino oscillations

"Understanding gravitationally induced decoherence parameters in neutrino oscillations using a microscopic quantum mechanical model" (2024);
***A. Domi, T. Eberl, MJF, K. Giesel, L. Hennig, U. Katz, R. Kemper, M. Kobler* (JCAP)**

Part I

Field-theoretical master equation

"A gravitationally induced decoherence model using Ashtekar variables" (2022);
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Master equations with gravity

[Anastopoulos, Hu 2013], [Blencowe 2013], [Oniga, Wang 2016], [Lagouvardos, Anastopoulos 2021], [Fahn, Giesel, Kobler 2022]

Classical system

$$S = S_{EH}[g_{ij}, P^{ij}] + S_{\Phi_i}[\Phi_i, \Pi^j, g_{ij}, P^{ij}]$$

Coupling of gravity and matter by GR

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Linearisation

Fock quantisation

Quantum system

$$H_T = H_S \otimes 1_E + 1_S \otimes H_E + H_I$$

$$\frac{\partial \rho_T(t)}{\partial t} = -i[H_T(t), \rho_T(t)]$$

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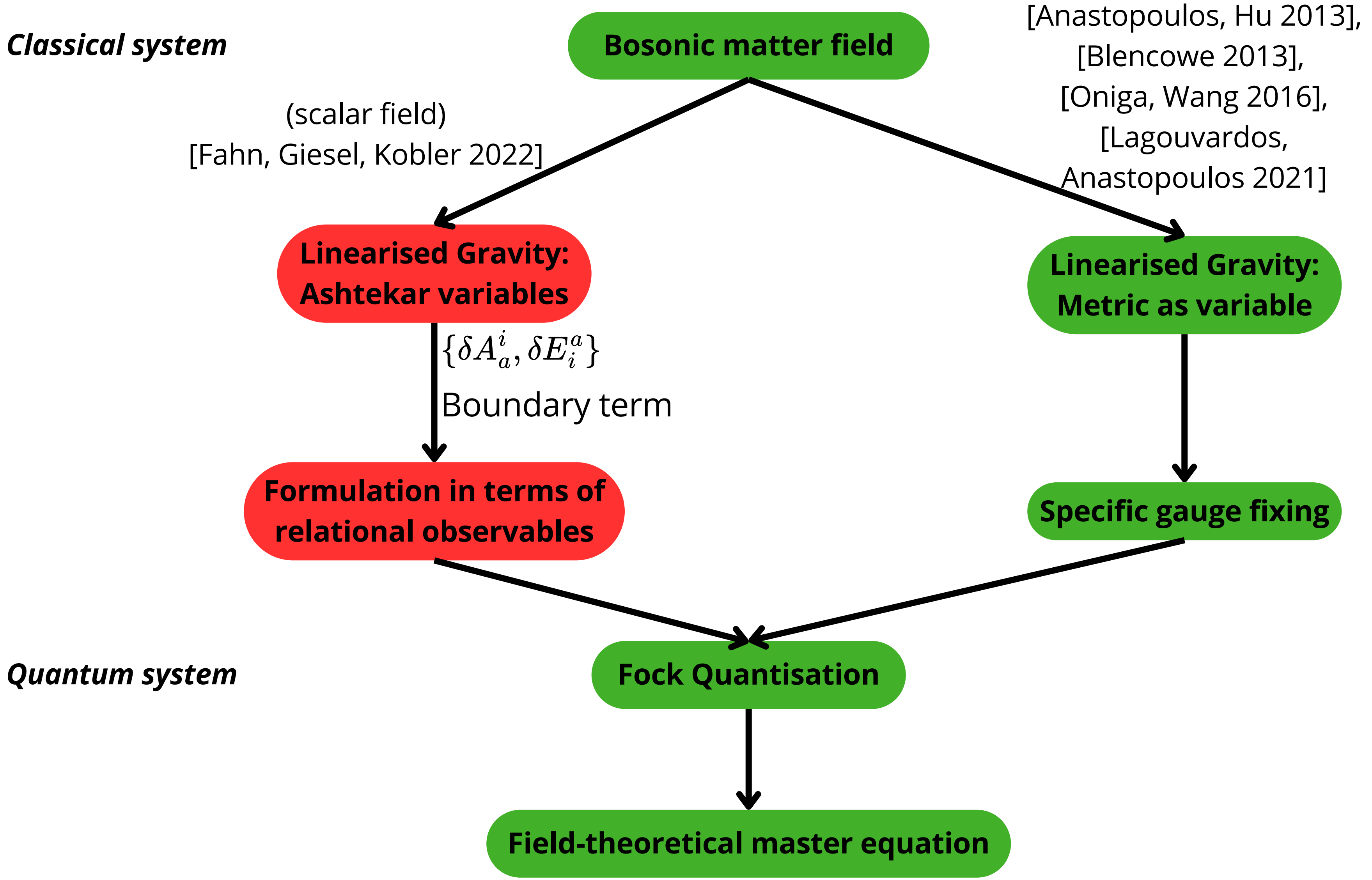
Trace out environment

$$\frac{\partial \rho_S(t)}{\partial t} = -i[H_S + H_{add}(t), \rho_S(t)] + \mathcal{D}[\rho_S(t)]$$

Procedure Part I



Relational formalism and observables

[Rovelli 1991], [Rovelli 1991], [Vytheeswaran 1994], [Rovelli 2002], [Dittrich 2006], [Dittrich, 2007]

- Method to deal with gauge freedom in physical systems and to formulate dynamics
- Basic idea: Describe evolution in terms of other objects; here: geometric clocks

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- Construct (weakly) gauge invariant observables up to second order in κ using the observable map

$$\mathcal{O}_{f,\{T\}}(\tau^I) = \left[\exp(\xi^I\{C_I,\cdot\})\cdot f\right]\Big|_{\xi^I:=T^I-\tau^I}$$

with first class constraints $\{C_I = \delta C_I + \delta^2 C_I\}$, clocks $\{T^I = \delta T^I\}$

- Construct geometrical clocks that fulfill $\{\delta T^I, \delta C_J\} = \delta^I_J$

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- Use dual map to make reference fields commute mutually, $\{\delta T^I, \delta T^J\} = 0$
- Canonical Transformation:

$$(\phi,\pi),\quad (\delta A^i_a, \delta E^a_i) \quad\longrightarrow\quad (\delta T^I, \delta C_J),\quad \underbrace{(\mathcal{O}_\phi, \mathcal{O}_\pi),(\mathcal{O}_{\delta A^i_a}, \mathcal{O}_{\delta E^a_i})}_{\text{Closed subalgebra}}$$

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↪ Separation of the phase space into physical and unphysical degrees of freedom

Field theoretical master equation

Fock quantisation of the physical phase space (Minkowski background):

$$\frac{\partial}{\partial t}\rho_T(t) = -i[H_T(t), \rho_T(t)]$$

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Projection operator technique
[Nakajima 1958], [Zwanzig 1960], [Prigogine 1962], [Shibata, Takahashi, Hashitsume 1977], [Chaturvedi, Shibata 1979]

$$\rho_T \mapsto \mathcal{P}(\rho_T) := tr_{\mathcal{E}}(\rho_T) \otimes \rho_{\mathcal{E}} =: \rho_S(t) \otimes \rho_{\mathcal{E}}$$

$$\rho_T \mapsto \mathcal{Q}(\rho_T) := \rho_T - \mathcal{P}\rho_T$$

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Gravitons thermally distributed

2nd order in κ

$$\frac{\partial}{\partial t}\rho_S(t) = -i[H_S + \kappa\, U(t) + \kappa\, H_{LS}(t), \rho_S(t)] + \mathcal{D}[\rho_S(t)]$$

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Challenges:

- Structure still very complicated → further approximations required for solution
- Connect to experiments → dynamics of a single scalar particle
- Extract physics → renormalisation

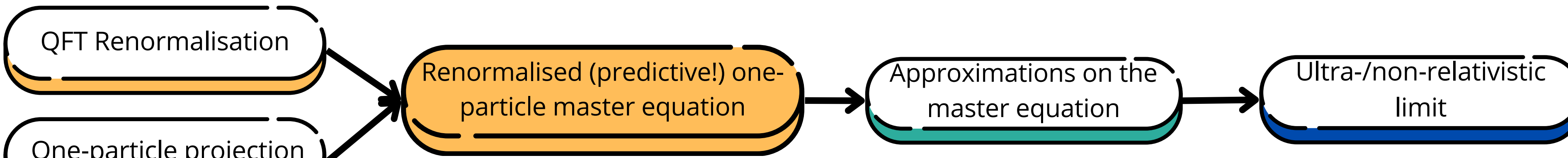
Part II

One-particle projection of the master equation

“Gravitationally induced decoherence of a scalar field: investigating the one-particle sector and its interplay with renormalisation” (2024);
***MJF, K. Giesel* (arXiv)**

Plan for Part II

$$\frac{\partial \rho_S(t)}{\partial t} = -i[H_S + H_{add}(t), \rho_S(t)] + \mathcal{D}[\rho_S(t)]$$



$$\rho_S(t) \rightarrow \int d^3p \int d^3q \; \rho(\vec{p}, \vec{q}, t) \; |\vec{p}\rangle\langle\vec{q}|$$

$$H_{add}(t) \rightarrow H'_{add}(t)$$

$$\mathcal{D}[\rho_S(t)] \rightarrow \mathcal{D}'[\rho_S(t)]$$

$$|\vec{p}|, |\vec{q}| \gg m$$

$$|\vec{p}|, |\vec{q}| \ll m$$

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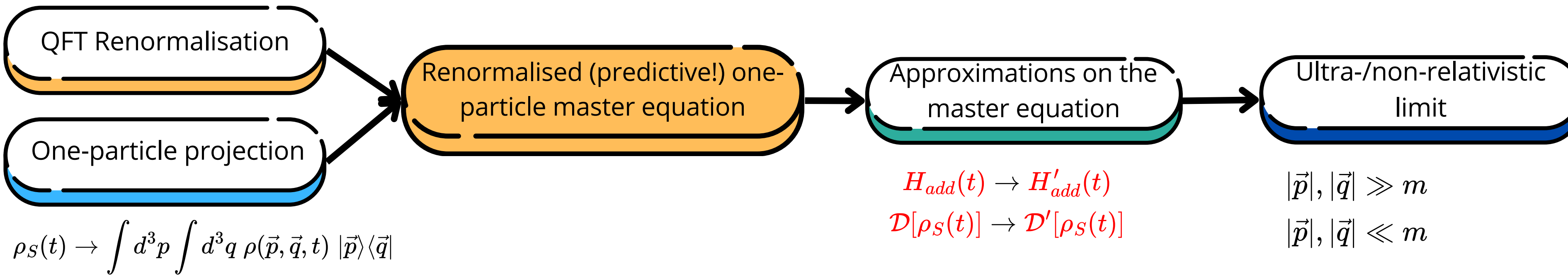

With QM/ Without Renormalisation: [Anastopoulos, Hu 2013], [Oniga, Wang 2016], [Lagouvardos, Anastopoulos 2021]

↪ Interpretation of divergences? Physical effect of the approximations?

QFT Renormalisation

- Some terms of the master equation exhibit logarithmic UV divergences
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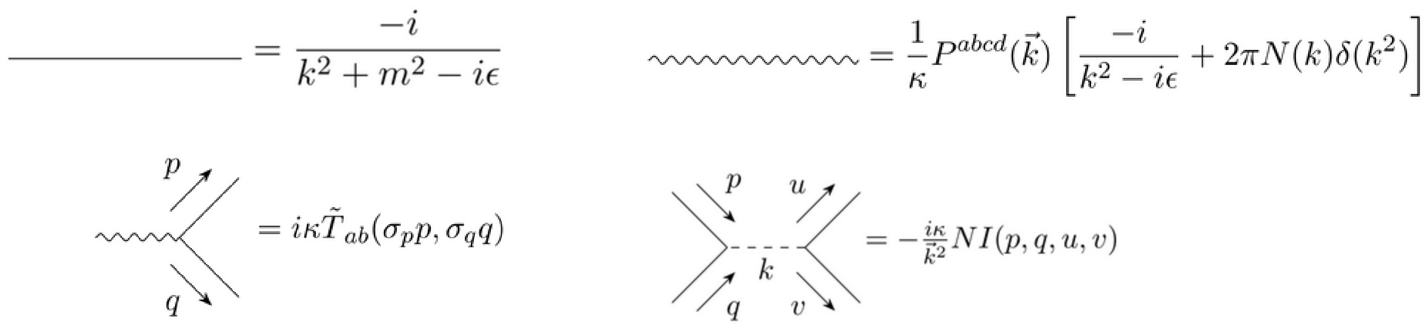
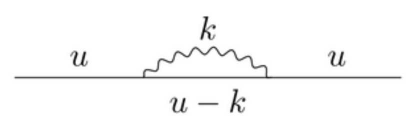


- UV divergence comes from vacuum self-energy: 

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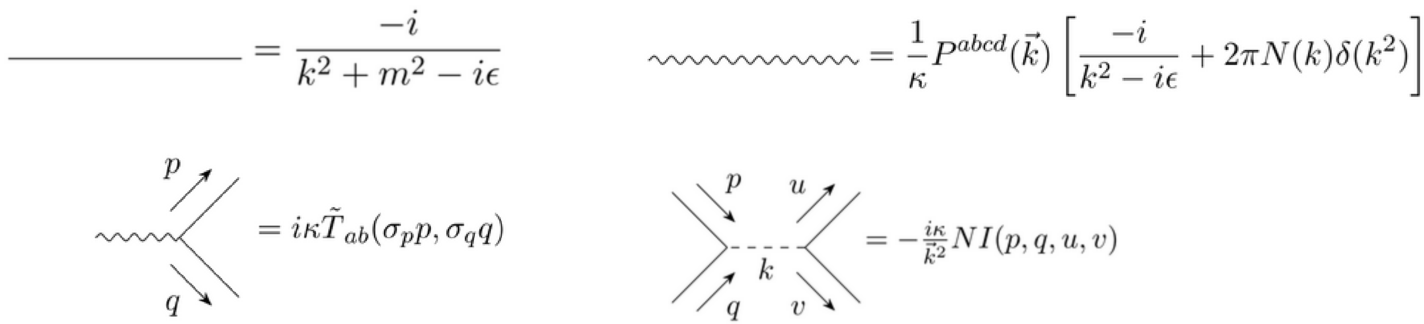

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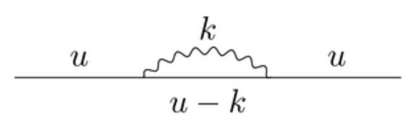

- Restore covariance (similar to QED in Coulomb gauge) [Tong QFT script], [Weinberg QFT book]
- Renormalise self-energy loop (dimensional regularisation and small artificial graviton mass to cure IR divergence; on-shell renormalisation scheme)
- Effect: Renormalisation removes all vacuum terms from the dissipator

(Further) Approximations

Typical aim: obtain a completely positive master equation

↪ further approximations:

Markov approximation

Idea: Environment forgets rapidly the influence of the system

↪ Justified for ultra-relativistic particles

Rotating wave approximation

Idea: “Coarse graining”: Detectors cannot measure arbitrarily fast oscillations

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↪ Dissipator has Lindblad form, no Lamb-shift left*:

$$\frac{\partial}{\partial t}\rho_1(t) = -i[H_S, \rho_1(t)] + \mathcal{D}[\rho_1(t)]$$

$$\mathcal{D}[\rho_1] = \kappa \sum_{r\in\{+,-\}} \int_{\mathbb{R}^3} \frac{d^3k}{(2\pi)^2} \delta(k) \frac{N(k)}{\Omega_k} \left(L_r(\vec{k})\rho_1 L_r(\vec{k})^\dagger - \frac{1}{2}\left\{\rho_1, L_r(\vec{k})^\dagger L_r(\vec{k})\right\} \right)$$

Specific limits

Non-relativistic limit:

$$\frac{\partial}{\partial t}\rho(\vec{p},\vec{q},t) = -i\rho(\vec{p},\vec{q},t)\left(\frac{\vec{p}^2}{2m} - \frac{\vec{q}^2}{2m}\right) - \frac{2\kappa k_B T}{3m^2}\left[\frac{1}{3}(\vec{p}^4 + \vec{q}^4) - \frac{1}{3}\vec{p}^2\vec{q}^2\left(1+\cos^2(\gamma)\right)\right]\rho(\vec{p},\vec{q},t)$$

Agrees approx. with [Anastopoulos, Hu 2013]

Ultra-relativistic limit:

$$\frac{\partial}{\partial t}\rho(\vec{p},\vec{q},t) = -i\rho(\vec{p},\vec{q},t)\left(|\vec{p}| - |\vec{q}|\right) - \frac{\kappa k_B T}{4\pi}\left[\frac{1}{3}(\vec{p}^2 + \vec{q}^2) + |\vec{p}||\vec{q}|\left(\frac{1}{3} - \cos^2(\gamma)\right)\right]\rho(\vec{p},\vec{q},t)$$

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As operator equation in 1D:

$$\frac{\partial}{\partial t}\hat{\rho}(t) = -i[\hat{H}_S, \hat{\rho}(t)] + \frac{\kappa k_B T}{6\pi}\left(\hat{H}_S\hat{\rho}(t)\hat{H}_S - \frac{1}{2}\left\{\hat{H}_S^2, \hat{\rho}(t)\right\}\right)$$

[Assumed approximately $\vec{p} \parallel \vec{q}$]

Solution in energy basis assuming discrete energy levels:

$$\rho_{ij}(t) = \rho_{ij}(0)e^{-i(E_i - E_j)t - \frac{\kappa}{12\pi\beta}(E_i - E_j)^2 t}$$

Part III

Application to neutrino oscillations

“Understanding gravitationally induced decoherence parameters in neutrino oscillations using a microscopic quantum mechanical model” (2024);
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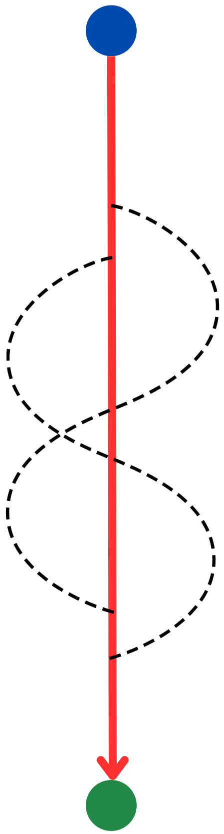
Model of interest



$$H_S = H_{vac}$$

Image credits: Scrudje (Earth), statu-nascendi from Getty Images (Background), via canva.com

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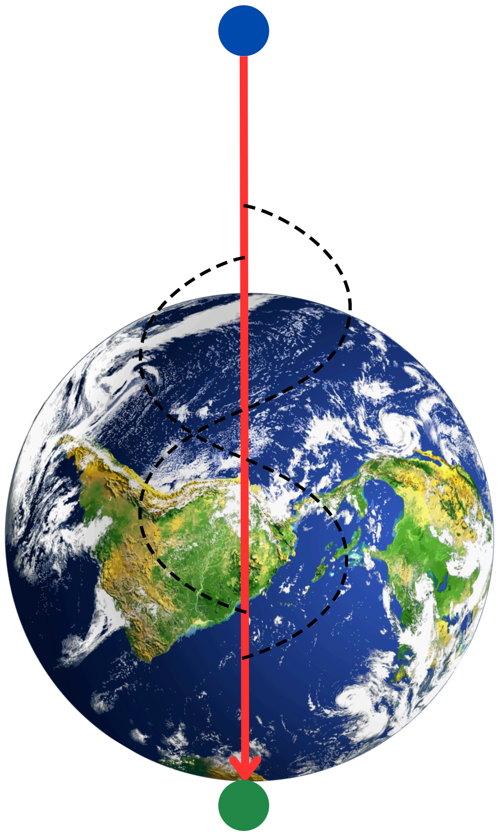

$$H_S = H_{vac} + H_{mat}$$

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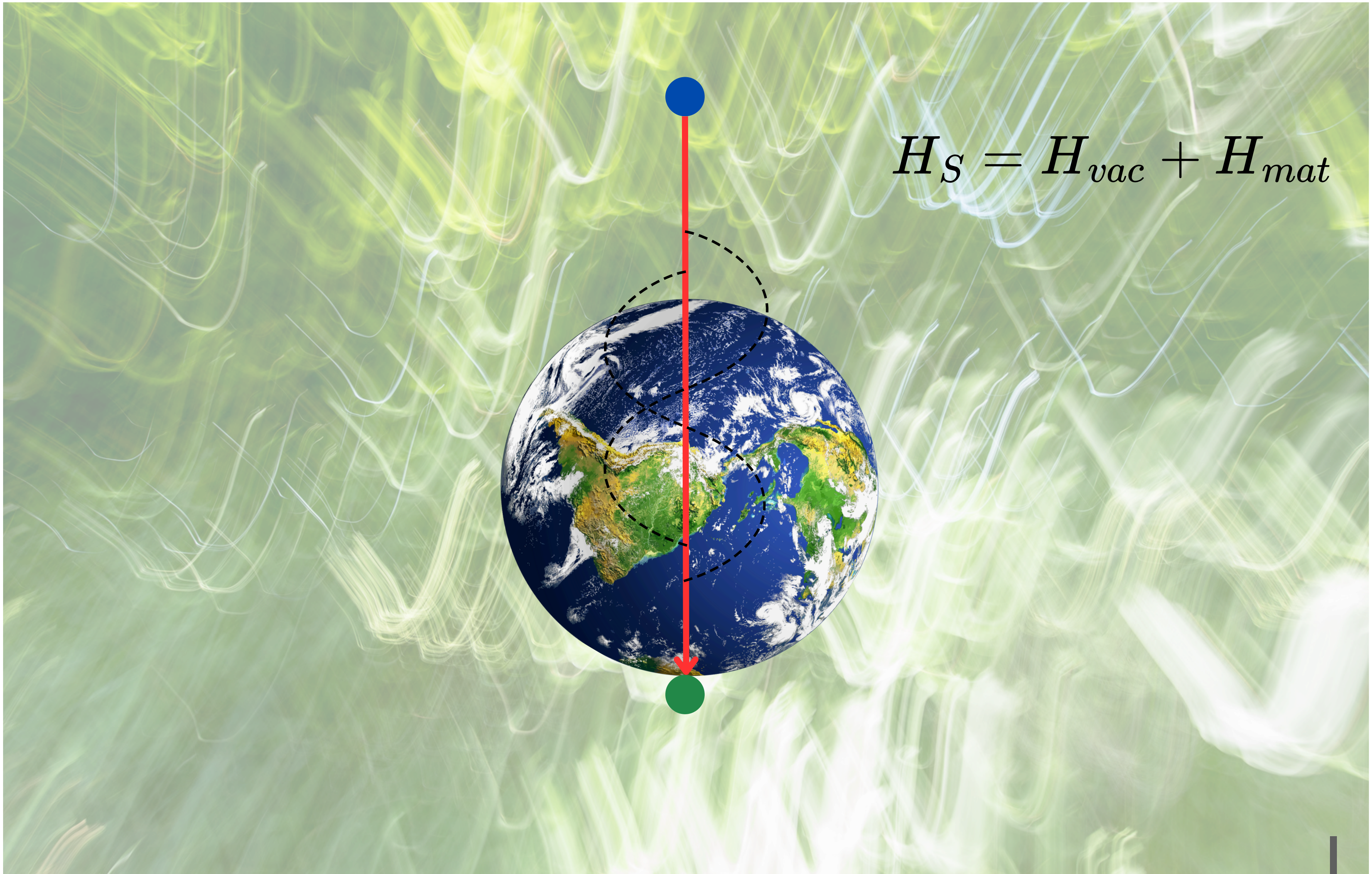

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Phenomenological models

Phenomenological quantum mechanical models

[Lisi, Marrone, Montanino, 2000], [Benatti, Floreanini 2000], [Gago, Santos, Teves , Zukanovich Funchal 2002], [Morgan, Winstanley, Brunner, Thompson 2006], [Guzzo, Holanda, Oliveira 2016], [Coelho, Mann 2017], [Coloma, Lopez-Pavon, Martinez-Soler, Nunokawa 2018], [Gomes, Gomes, Peres 2023], …

Typical procedure:

- Lindblad equation
- Solution (in effective mass basis):

$$\widetilde{\rho}_{ij}(t) = \widetilde{\rho}_{ij}(0) \cdot e^{-\frac{i}{\hbar}\left(\widetilde{H}_i - \widetilde{H}_j\right)t - \Gamma_{ij} t} \qquad \Gamma_{ij} = \gamma_{ij} \cdot \left(\frac{E}{E_0}\right)^n$$

- Gravitationally induced decoherence manifests as damping of the neutrino oscillations

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Based on Part I and Part II, can we come up with a microscopic model to resolve these unknown parameters?

Quantum mechanical toy model

Hamiltonian, inspired by [Xu, Blencowe 2022]:

$$\hat{H} = \underbrace{\left(\hat{H}_S^{(0)} + \hat{H}_S^{(C)}\right)}_{\hat{H}_S} \otimes 1_{\mathcal{E}} + 1_S \otimes \underbrace{\left[\frac{1}{2}\sum_{i=1}^{N}\left(\hat{p}_i^2 + \omega_i^2 \hat{q}_i^2\right)\right]}_{\hat{H}_{\mathcal{E}}} \underbrace{- \hat{H}_S \otimes \sum_{i=1}^{N} g_i \hat{q}_i}_{\hat{H}_{\text{int}}}$$

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Major steps:

- Small coupling constant, environment follows Gibbs state ← **Part II**
- Continuum limit of the environment ← **Part I**
- Markov approximation
- Renormalisation

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Solution of the master equation (in effective mass basis):

$$\widetilde{\rho}_{ij}(t) = \widetilde{\rho}_{ij}(0) \cdot e^{-\frac{i}{\hbar}\left(\widetilde{H}_i - \widetilde{H}_j\right)t - \frac{4\eta^2}{\hbar^3 \beta(T)}\left(\widetilde{H}_i - \widetilde{H}_j\right)^2 t}$$

(same form as one-particle master equation from Part II in ultra-relativistic limit)

Comparison to the phenomenological models

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Solution for phenomenological models:

$$\widetilde{\rho}_{ij}(t) = \widetilde{\rho}_{ij}(0) \cdot e^{-\frac{i}{\hbar}\left(\widetilde{H}_i - \widetilde{H}_j\right)t - \Gamma_{ij} t} \qquad \Gamma_{ij} = \gamma_{ij} \cdot \left(\frac{E}{E_0}\right)^n$$

Results:

- Expression for phenomenological parameters
- Energy dependence fixed
- Match to phenomenological models only in vacuum possible

Comparison to the phenomenological models

Solution of the master equation:

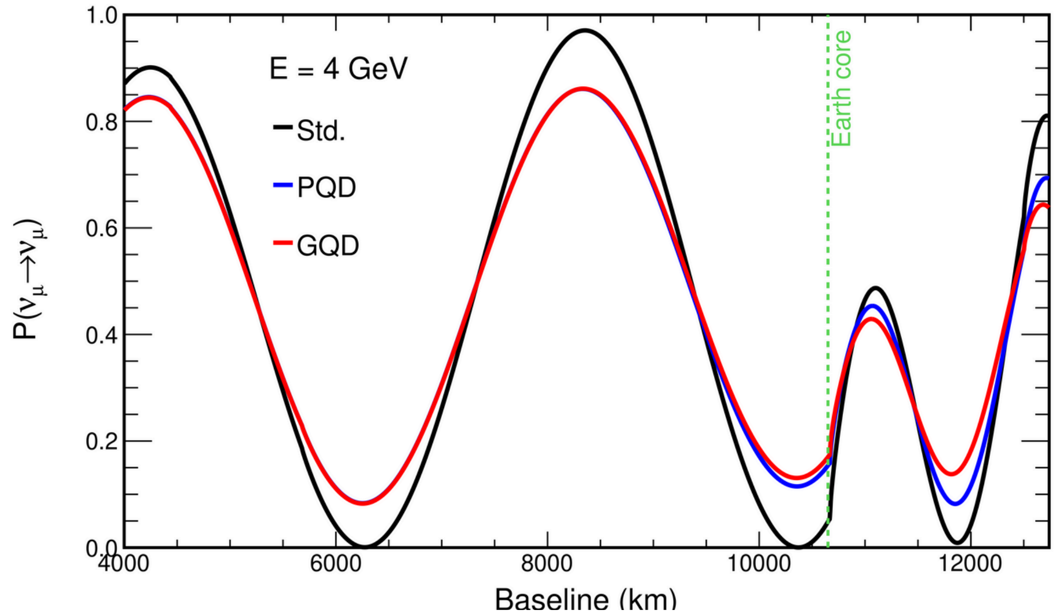
$$\widetilde{\rho}_{ij}(t) = \widetilde{\rho}_{ij}(0) \cdot e^{-\frac{i}{\hbar}\left(\widetilde{H}_i - \widetilde{H}_j\right)t - \frac{4\eta^2}{\hbar^3 \beta(T)}\left(\widetilde{H}_i - \widetilde{H}_j\right)^2 t}$$

Solution for phenomenological models:

$$\widetilde{\rho}_{ij}(t) = \widetilde{\rho}_{ij}(0) \cdot e^{-\frac{i}{\hbar}\left(\widetilde{H}_i - \widetilde{H}_j\right)t - \Gamma_{ij} t} \qquad \Gamma_{ij} = \gamma_{ij} \cdot \left(\frac{E}{E_0}\right)^n$$

Results:

- Expression for phenomenological parameters
- Energy dependence fixed
- Match to phenomenological models only in vacuum possible



(Plot from Paper 3, created by Dr. Alba Domi)

Deviations in the oscillation probabilities in matter for $\eta = 10^{-8} s, T = 0.9 K$

Summary

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(Quantised linearised) Gravity as environment in Open Quantum Systems

Main model: Scalar field coupled to linearised gravity in Ashtekar variables

Part I: Field-theoretical master equation

- Relational formalism:
  - Observables
  - Notion of dynamics
  - Physical time and length
- Projection operator technique

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- Markov and Rotating wave approximations and their physical effects
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Part III: Application to neutrino oscillations

- QM toy model:
  - Renormalisation and Markov approximation
  - Resolution of the decoherence parameters in the phenomenological models; match only in vacuum

Outlook I

- Similar analysis for different matter systems (matter gauge degrees, different quantisation and renormalisation, ...)
  [Fahn, Giesel, Kemper, soon]
- Different gravitational constituent/background (cosmological, black hole, ...)
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  [Giesel, Kobler 2022]

Outlook I

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- Analysis of the validity of Markov and Rotating Wave approximation

- Toy model → favourable energy ranges and dependencies for phenomenological models
- Modification of the phenomenological models in matter
- Sensitivity analyses

Thank you for your attention!

Image credits: Andras Klein from Getty Images (Nightsky) via canva.com