Decoding the Standard Model with Flavour Physics

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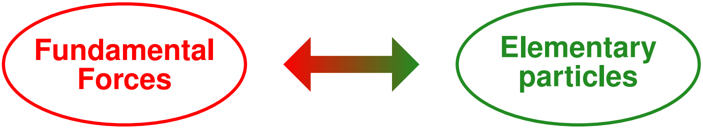
Outline:

1. The Standard Model and the Flavour Problem

2. The V_{cb} puzzle

3. Outlook and prospects on BSM

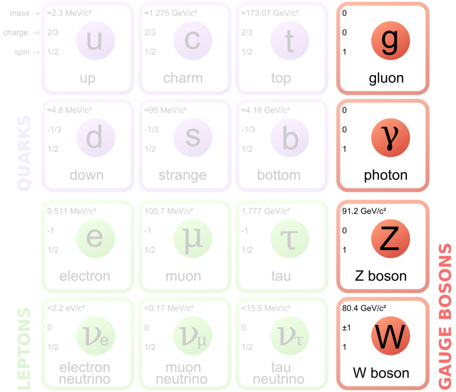
The Standard Model of Particle Physics



⇒ Describe the building blocks of the universe from a microscopic point of view

⇒ Easily characterised by symmetry laws

⇒ Forces and elementary particles are described by Quantum Fields, in the context of Quantum Field Theories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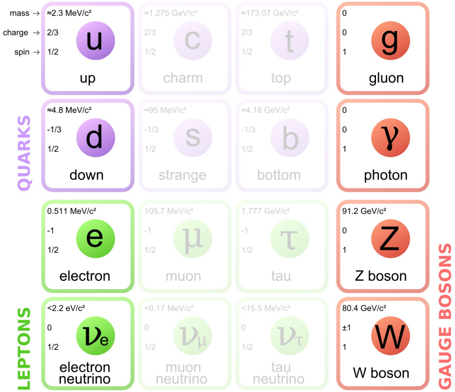
QUARKS
LEPTONS
GAUGE BOSONS



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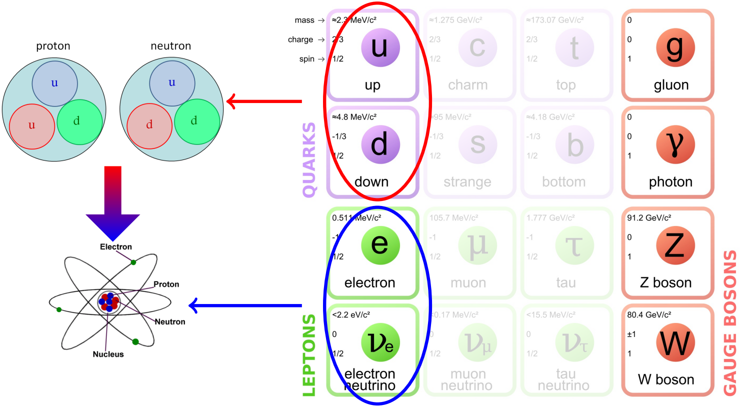
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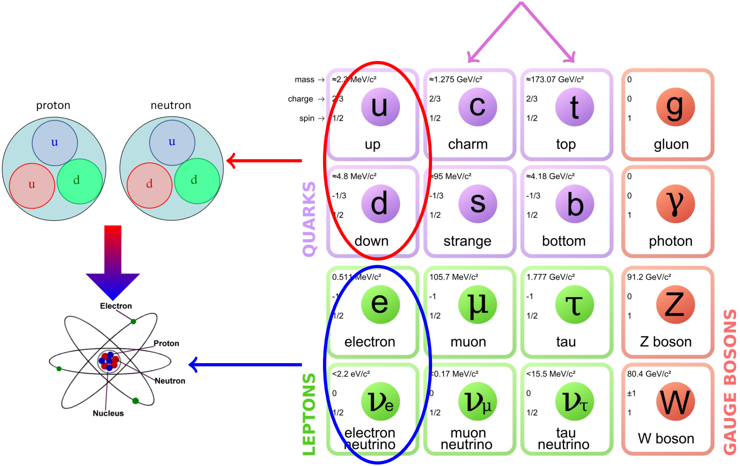
“copies” of the first generation with heavier masses



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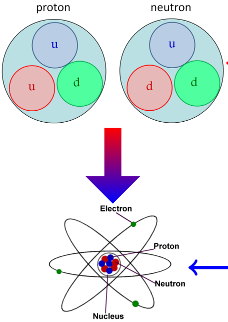

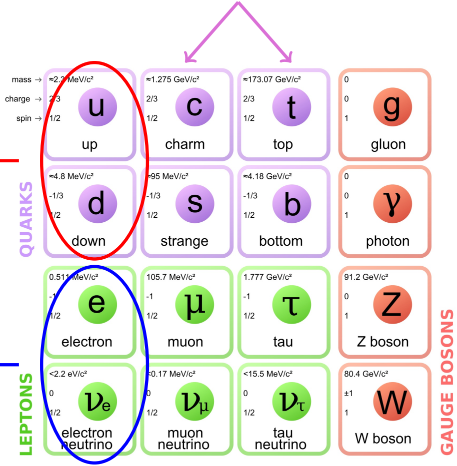

flavour: what is differentiating the three families

The flavour problem

$$\mathcal{L}_{\text{Yukawa}} \supset Y_u^{ij} \bar{Q}_L^i H u_R^j$$

$$Y_u \sim y_t \begin{pmatrix} & & 0.003 \\ & & 0.04 \\ & & 1 \end{pmatrix}^{(\star)}$$

- Yukawa couplings are not predicted in the SM
  - $\Rightarrow$ Extracted from experimental measurements
- No explanation for the hierarchical structure
  - $\Rightarrow$ Why the third generation seems so special?
- Why do we have exactly three families?

($\star$) in the down basis

Beyond the Standard Model

The SM is rather successful but...

Dark Matter

Hierarchy problem

Flavour Problem

Neutrino masses

Gravity

Beyond the Standard Model

The SM is rather successful but...

Dark Matter

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Can some beyond the SM theory accommodate all these problems?

Beyond the Standard Model

The SM is rather successful but...

Dark Matter **Hierarchy problem** **Flavour Problem**

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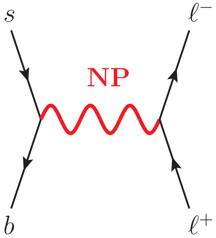
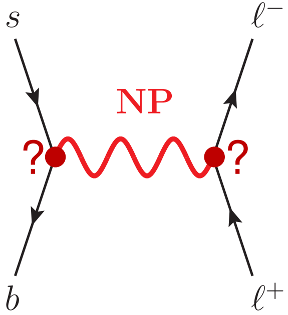


⇒ Looking for new physics in flavour-changing processes can give us hints on the structure of theories beyond the SM

⇒ It provides possible links to the flavour problem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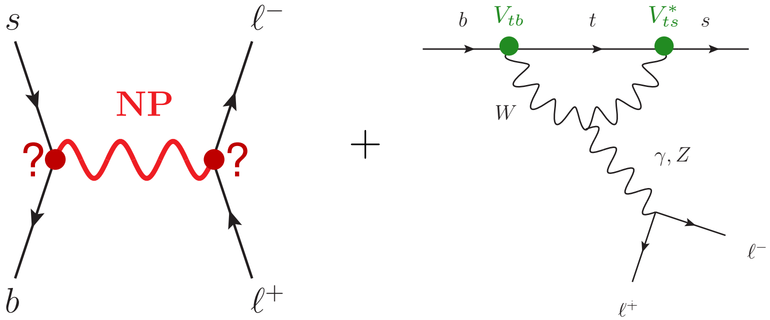
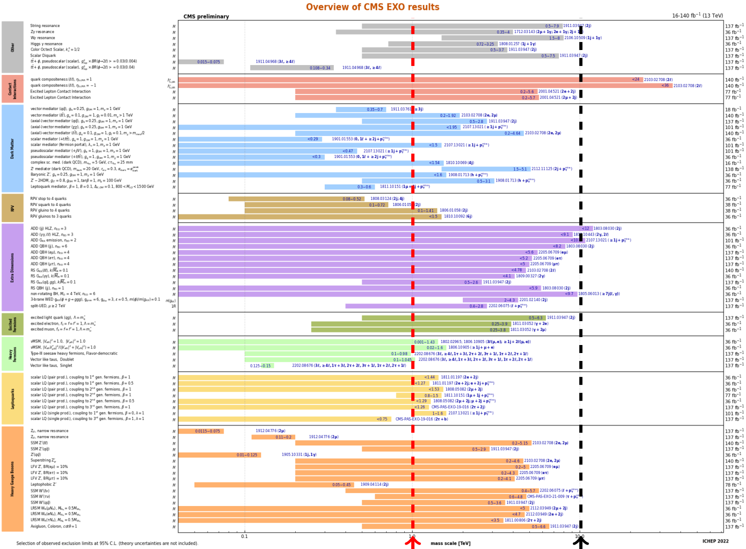


⇒ SM predictions for flavour-changing processes are parametrically small and therefore allow for testing heavy new physics

⇒ High control of theoretical and experimental precision is needed

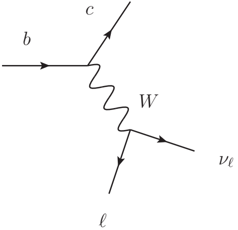
Status of high energy bounds

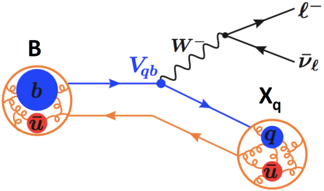


3rd generation

universal new physics

Partonic vs Hadronic



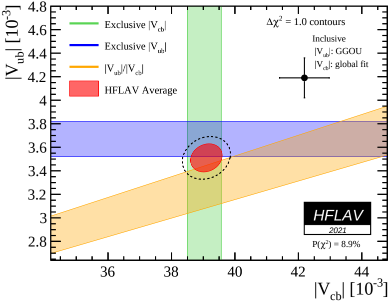


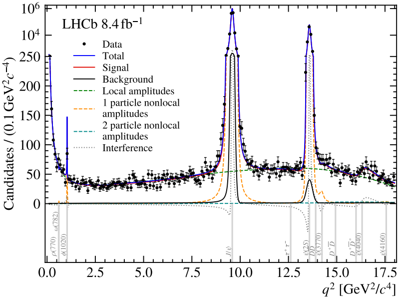
$\mu_{\text{partonic}} = m_b$

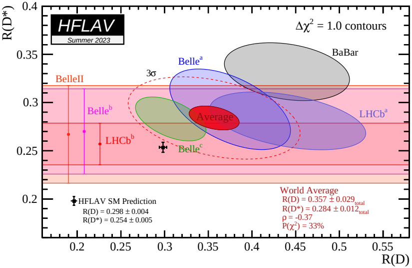
$\mu_{\text{hadronic}} = \Lambda_{\text{QCD}}$

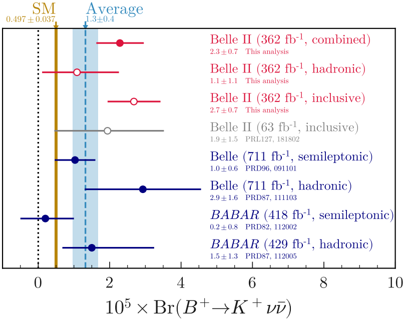
Fundamental challenge to match partonic and hadronic descriptions

Old and new puzzles in flavour physics

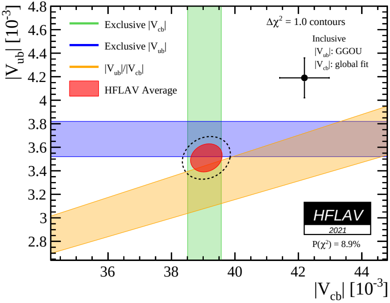


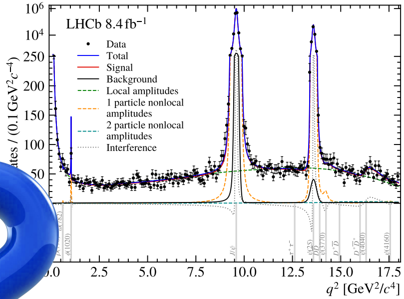


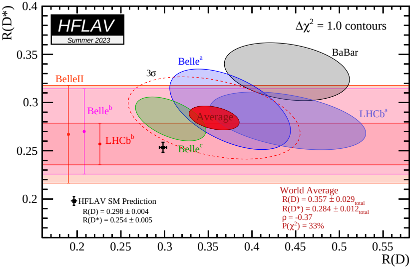


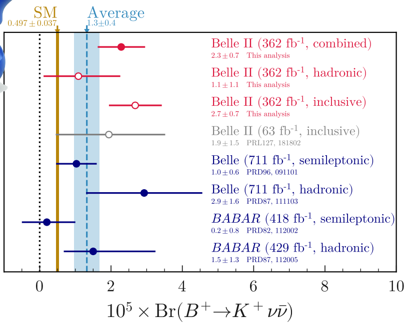


Old and new puzzles in flavour physics









The V_{cb} puzzle

The CKM matrix

Interaction basis

⇒ gauge interactions are diagonal

⇒ mass terms are not diagonal

$$-\mathcal{L}_{\rm Y} = Y_d^{ij}\bar{Q}_L^i H d_R^j + Y_u^{ij}\bar{Q}_L^i \tilde{H} u_R^j + \text{h.c.}$$

Non-diagonal Yukawa

Mass basis

⇒ Yukawa couplings are diagonal

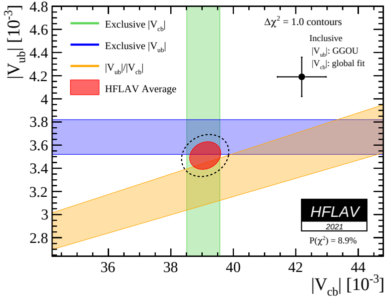
⇒ The CKM matrix is the remnant of the diagonalisation

$$\mathcal{L}_{cc} \propto \bar{u}_L^i \gamma^\mu d_L^j W_\mu^+ V_{ij}$$

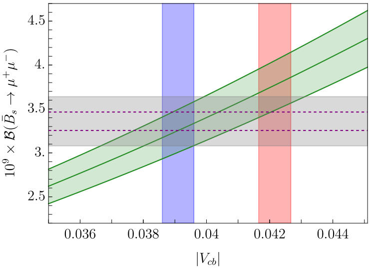
CKM matrix

The V_{cb} puzzle

- Inclusive determination: $B \to X_c \ell \bar{\nu}$
  - $\Rightarrow$ Stable against various datasets
- Exclusive decays: $B \to D^{(*)} \ell \bar{\nu}$, $\Lambda_b \to \Lambda_c \ell \bar{\nu}$
  - $\Rightarrow$ Lattice QCD results are in tension
  - $\Rightarrow$ Experimental measurement show various disagreements

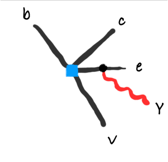


- $|V_{cb}|$ is a fundamental parameter to predict all flavour-changing observables



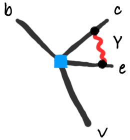
QED effects for inclusive V_{cb}

1. Collinear logs: captured by splitting functions



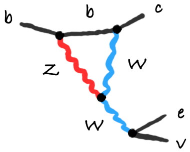
$$\sim \frac{\alpha_e}{\pi} \log^2\left(\frac{m_b^2}{m_e^2}\right)$$

2. Threshold effects or Coulomb terms



$$\sim \frac{2\pi\alpha_e}{3}$$

3. Wilson Coefficient



$$\sim \frac{\alpha_e}{\pi}\left[\log\left(\frac{M_Z^2}{\mu^2}\right) - \frac{11}{6}\right]$$

Branching ratio

[Bigi, MB, Gambino, Haisch, Piccione, '23]

- The total branching ratio is not affected by large logs due to KLN theorem
- The large corrections are from the Wilson Coefficient and the threshold effects

$$\frac{\Gamma}{\Gamma^{(0)}g(\rho)} = 1 + \frac{\alpha}{\pi}\left[\ln\left(\frac{M_Z^2}{m_b^2}\right) - \frac{11}{6} + 5.516(14)\right]$$

$$= 1 + 1.43\% - 0.44\% + 1.32\% = 1 + 2.31\%$$

Wilson Coefficient — Threshold effects

- Large shift of the branching ratio of the same order of the current error on V_{cb}
- How do we incorporate in the current datasets?
  - $\Rightarrow$ Possible only on BaBar data
  - $\Rightarrow$ A systematic approach is needed and foreseen for future experimental analysis
  - $\Rightarrow$ How to evaluate structure-dependent terms is an open task

Exclusive matrix elements

$$\langle H_c | J_\mu | H_b \rangle = \sum_i S_\mu^i \mathcal{F}_i$$

Exclusive matrix elements

$$\langle H_c | J_\mu | H_b \rangle = \sum_i S^i_\mu \mathcal{F}_i$$

← form factor

scale Λ_{QCD}

independent

Lorentz structures

Exclusive matrix elements

$$\langle H_c | J_\mu | H_b \rangle = \sum_i S^i_\mu \mathcal{F}_i$$

$\longleftarrow$ form factor

scale Λ_{QCD}

independent Lorentz structures

Form factors determinations

- Lattice QCD
- QCD SR, LCSR

only points at specific kinematic points

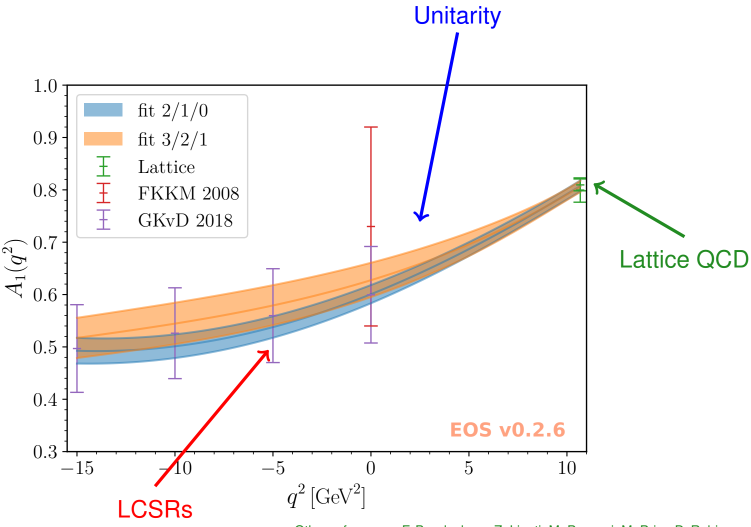
Form factors parametrisations

- HQET (CLN + improvements) $\Rightarrow$ reduce independent degrees of freedom
- Analytic properties $\rightarrow$ BGL

data points needed to fix the coefficients of the expansion

$B \to D^*$ **before 2021**

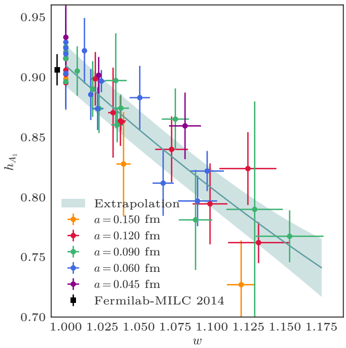
[MB, Gubernari, Jung, van Dyk, '19]



Other references: F. Bernlochner, Z. Ligeti, M. Papucci, M. Prim, D. Robinson, '22
P. Gambino, M. Jung, S. Schacht, '19

$B \to D^*$ from lattice away from zero recoil

[2105.14019]



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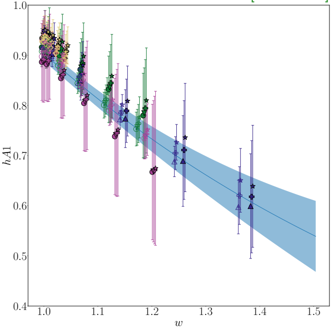

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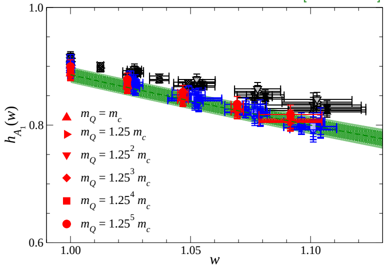
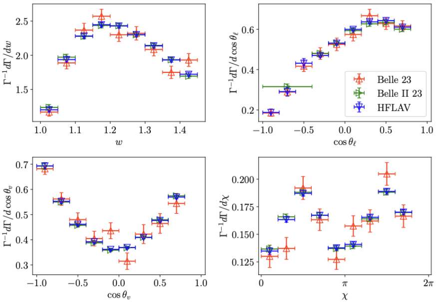
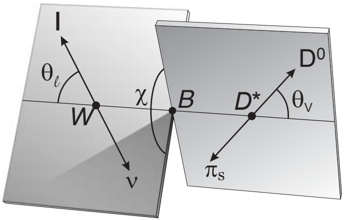


- Are these results compatible with each other?
- Are they compatible with experimental data?

New $B \to D^* \ell \bar{\nu}$ Belle and Belle II data





$$
\begin{aligned}
\frac{d\Gamma}{dw d\cos(\theta_\ell) d\cos(\theta_v) d\chi} =& \frac{3G_F^2}{1024\pi^4} |V_{cb}|^2 \eta_{EW}^2 M_B r^2 \sqrt{w^2-1} q^2 \\
&\times \{(1-\cos(\theta_\ell))^2 \sin^2(\theta_v) H_+^2(w) + (1+\cos(\theta_\ell))^2 \sin^2(\theta_v) H_-^2(w) \\
&+ 4\sin^2(\theta_\ell)\cos^2(\theta_v) H_0^2(w) - 2\sin^2(\theta_\ell)\sin^2(\theta_v)\cos(2\chi) H_+(w) H_-(w) \\
&- 4\sin(\theta_\ell)(1-\cos(\theta_\ell))\sin(\theta_v)\cos(\theta_v)\cos(\chi) H_+(w) H_0(w) \\
&+ 4\sin(\theta_\ell)(1+\cos(\theta_\ell))\sin(\theta_v)\cos(\theta_v)\cos(\chi) H_-(w) H_0(w)\}
\end{aligned}
$$

- Between 7 to 10 bins per kinematic variable
- Available on HEPData with correlations
- Angular observables analysis are available, data just newly released

Analysis strategies

Setup

- BGL parametrisation
- Bayesian inference to apply unitarity — Flynn, Jüttner, Tsang, '23

Questions

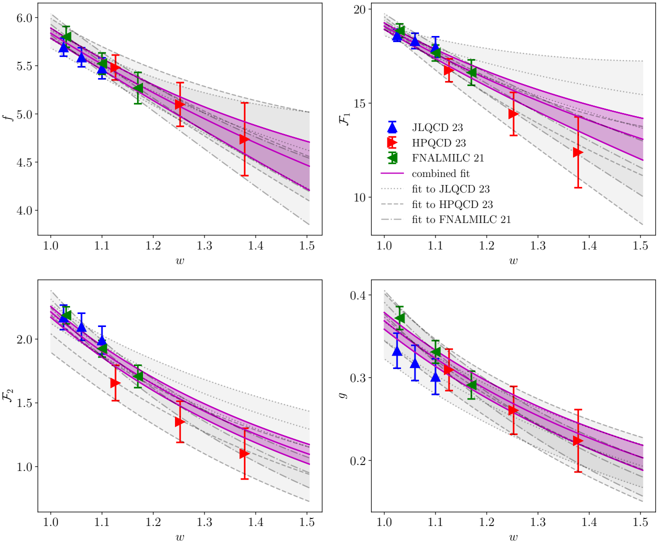
- Combine the three LQCD datasets

  $\Rightarrow$ Is the combination acceptable?

- Combine with experimental data
- What are the consequences for phenomenology?

Strategy A: Lattice only

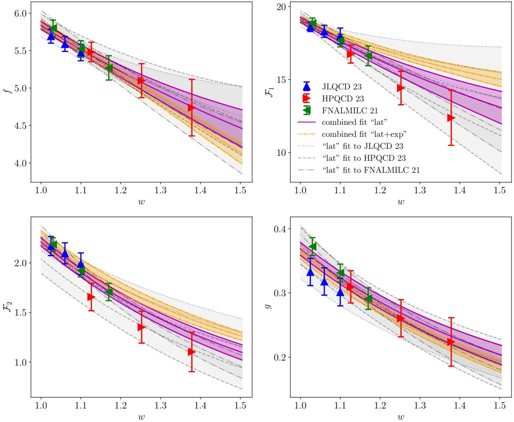
[MB, A. Jüttner, '24]



see also G. Martinelli, S. Simula, L. Vittorio, '23,'24

Strategy B: Lattice + experimental data

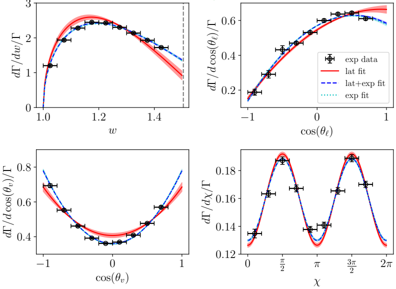
[MB, A. Jüttner, '24]



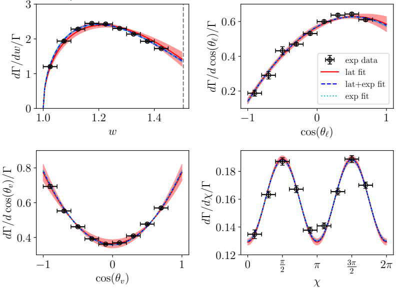
- Good fit quality for Strategy B (p-value $\sim 18\%$)
- Adding experimental data reduces the uncertainties, especially at large w
- Especially for $\mathcal{F}_1$ and $\mathcal{F}_2$, the shape changes between Strategy A and B

FNAL/MILC + HPQCD +HFLAV

[MB, A. Jüttner, '24]



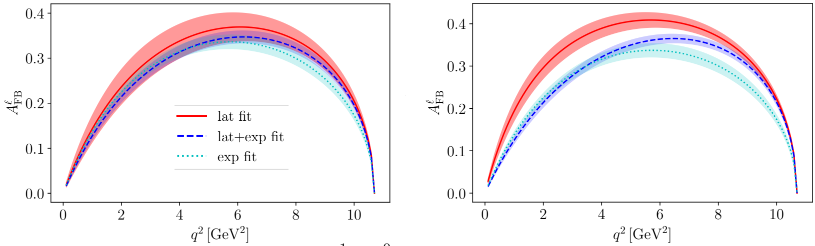
JLQCD +HFLAV



- Fit to HPQCD and FNAL/MILC misses experimental points
- BGL fit to experimental and lattice data has $p-$value $\sim 18\%$
- BGL coefficients shift of a few σ between Strategy A and B

Differential observables

[MB, A. Jüttner, '24]

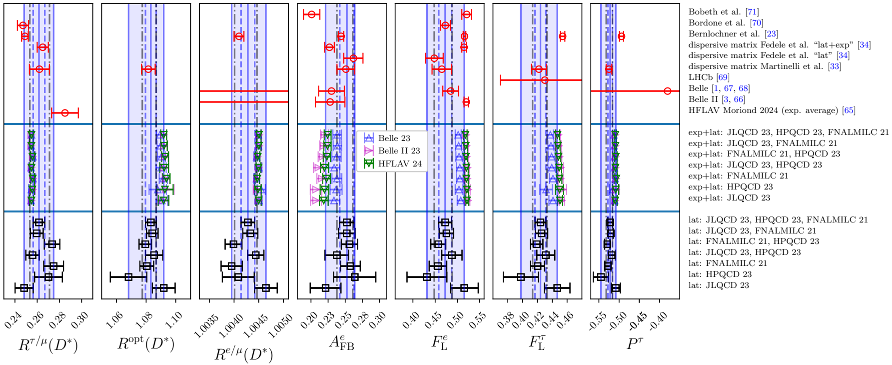


$$A^{\ell}_{\rm FB} = \frac{\int_0^1 - \int_{-1}^0 d\cos\theta_\ell d\Gamma/d\cos\theta_\ell}{\int_0^1 + \int_{-1}^0 d\cos\theta_\ell d\Gamma/d\cos\theta_\ell}$$

- The combined lattice + experimental precision makes it possible to study the differences in the shape
- It is clear that there is a distinct difference between JLQCD and FNAL/MILC+HPQCD
- Difficult to understand what is going on, JLQCD errors are also a bit larger

Integrated observables

[MB, A. Jüttner, '24]



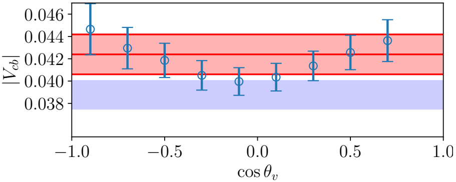
- Significant scatter between various combinations of lattice results
  - We apply a systematic error to account for the spread
- Consistent scatter of the experimental results independently of the lattice information

⇒ see also: Fedele et al, '23

$|V_{cb}|$ - Strategy A

$$|V_{cb}|_{\alpha,i} = \left(\Gamma_{\rm exp}\left[\frac{1}{\Gamma}\frac{d\Gamma}{d\alpha}\right]^{(i)}_{\rm exp} \Big/ \left[\frac{d\Gamma_0}{d\alpha}(\mathbf{a})\right]^{(i)}_{\rm lat}\right)^{1/2}$$

[MB, A. Jüttner, '24]



Blue band

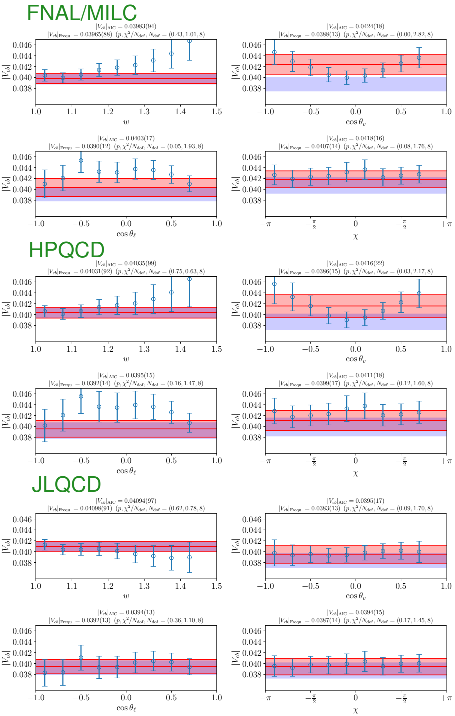
- Frequentist fit $p-$value $\sim 0\%$
- Affected by d'Agostini Bias

Red band

- Frequentist fit $p-$value $\sim 0\%$
- Akaike-Information-Criterion analysis: average over all possible fits with at least two data points and then weighted average

$$w_{\{\alpha,i\}} = \mathcal{N}^{-1}\exp\left(-\frac{1}{2}(\chi^2_{\{\alpha,i\}} - 2N_{{\rm dof},\{\alpha,i\}})\right) \qquad \text{where} \quad \mathcal{N} = \sum_{\text{sets}\,\{\alpha,i\}} w_{\rm set}$$

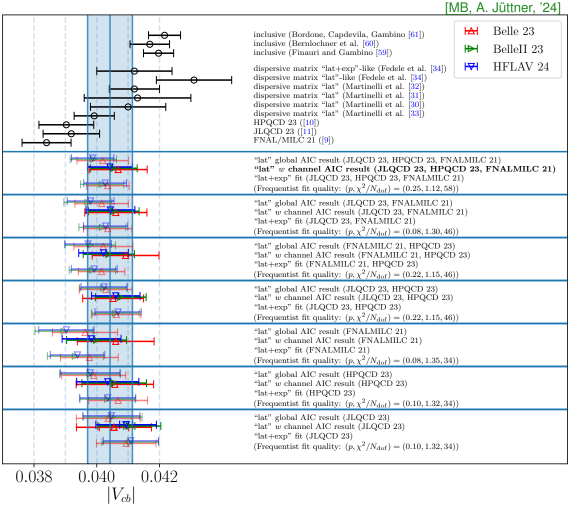
$$|V_{cb}| = \langle|V_{cb}|\rangle \equiv \sum_{\text{sets}\,\{\alpha,i\}} w_{\rm set}|V_{cb}|_{\rm set}$$



- Analysis based on Strategy A
- The AIC nicely reduces the d'Agostini bias
- Some lattice data behave strangely
- Would it be safer to discard the angular distributions?
- Combining the three lattice datasets doesn't help, shape driven by FNAL/MILC and HPQCD
- Good compatibility with Strategy B

see also G. Martinelli, S. Simula, L. Vittorio, '23,'24

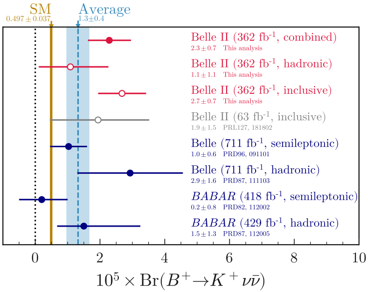
$|V_{cb}|$ - Summary

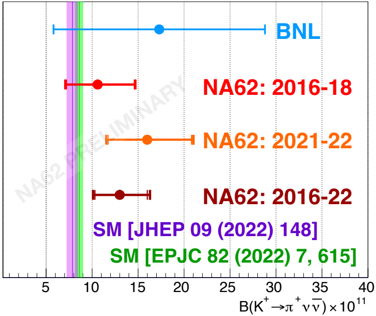


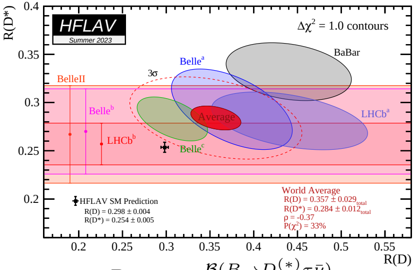
- Residual 2σ difference with inclusive
- The AIC produces slightly larger uncertainties, overall all results are quite consistent

Outlook and prospects on BSM

What about BSM?

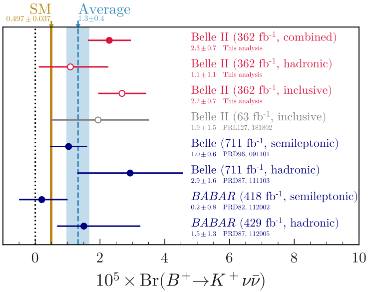


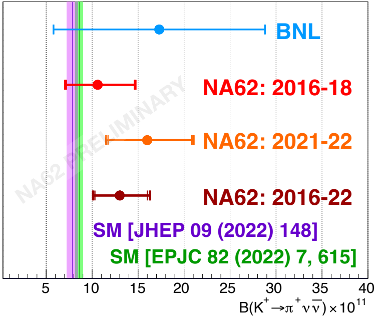


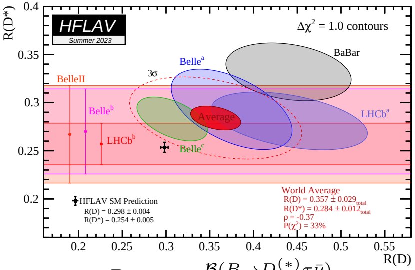


$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau\bar{\nu})}{\mathcal{B}(B \to D^{(*)}\ell\bar{\nu})}$$

What about BSM?







$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)} \tau \bar{\nu})}{\mathcal{B}(B \to D^{(*)} \ell \bar{\nu})}$$

Can we accommodate all these deviations together?

The EFT approach

- Since we haven't observed any clear sign of NP yet at low energies, we can work in an EFT context

  $\Rightarrow$ Agnostic of the nature of new physics, describe more than one UV model with the same operators

  $\Rightarrow$ Try to derive model-independent bounds

- We use the SMEFT

  $\Rightarrow$ Build all possible operators with SM fields and respecting SM symmetries

- The remnant of high-energy new physics is contained in the Wilson Coefficients

  $\Rightarrow$ With flavour, we have a lot of free degrees of freedom

  $\Rightarrow$ We need a criterium to infer their magnitude

The $U(2)^n$ symmetry for BSM

$$q_{3L} \sim (\mathbf{1}, \mathbf{1}) \qquad \ell_{3L} \sim (\mathbf{1}, \mathbf{1})$$

$$Q_L = (Q_L^1, Q_L^2) \sim (\bar{\mathbf{2}}, \mathbf{1}) \qquad L_L = (\ell_L^1, \ell_L^2) \sim (\mathbf{1}, \bar{\mathbf{2}})$$

Unbroken $U(2)^5$

$$Y_u = y_t \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad \Longrightarrow$$

$\bar{q}_{3L} \Gamma q_{3L}$ ✓

$\bar{q}_{3L} \Gamma Q$ ✗

The $U(2)^n$ symmetry for BSM

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$$Q_L = (Q_L^1, Q_L^2) \sim (\bar{\mathbf{2}},\mathbf{1}) \qquad L_L = (\ell_L^1, \ell_L^2) \sim (\mathbf{1},\bar{\mathbf{2}})$$

$$V_q \sim (\mathbf{2},\mathbf{1}) \qquad V_\ell \sim (\mathbf{1},\mathbf{2})$$

Unbroken $U(2)^5$

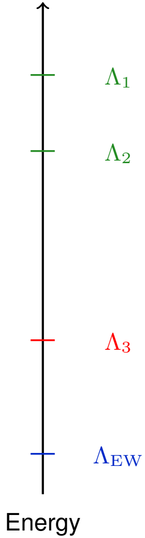
$$Y_u = y_t \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \Longrightarrow \quad \bar{q}_{3L}\Gamma q_{3L} \ ✓ \qquad \bar{q}_{3L}\Gamma Q \ ✗$$

Soft symmetry breaking

$$Y_u = y_t \begin{pmatrix} \Delta & V_q \\ 0 & 1 \end{pmatrix} \Longrightarrow \quad \bar{q}_{3L}\Gamma q_{3L} \ ✓ \qquad \bar{q}_{3L}\Gamma (V_q Q) \ ✓$$

Flavour Non-Universal New Physics

Dvali, Shifman, '00
Panico, Pomarol, '16
MB, Cornella, Fuentes-Martin, Isidori '17
Allwicher, Isidori, Thomsen '20
Barbieri, Cornella, Isidori, '21
Davighi, Isidori '21



Basic idea:

- 1st and 2nd have small masses and small couplings to NP because they are generated by dynamics at a heavier scale
- 3rd generation is linked to dynamics at lower scales and has stronger couplings

Flavour deconstruction:
fermion families interact with different gauge groups and flavour hierarchies emerge as accidental symmetries

Flavour Non-Universal New Physics

Dvali, Shifman, '00
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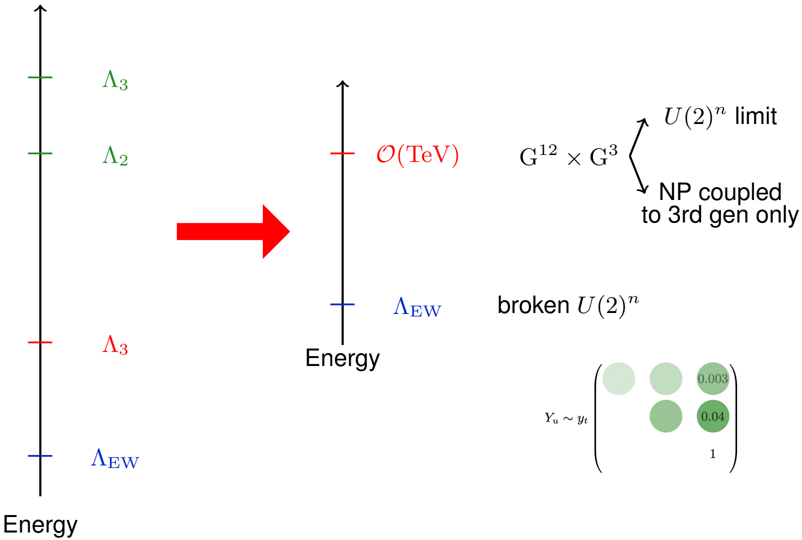


Which operators?

$$Q_{\ell q}^{\pm} = (\bar{q}_L^3 \gamma^\mu q_L^3)(\bar{\ell}_L^3 \gamma_\mu \ell_L^3) \pm (\bar{q}_L^3 \gamma^\mu \sigma^a q_L^3)(\bar{\ell}_L^3 \gamma_\mu \sigma^a \ell_L^3) \quad Q_S = (\bar{\ell}_L^3 \tau_R)(\bar{b}_R q_L^3)$$

Which operators?

$$Q_{\ell q}^{\pm} = (\bar{q}_L^3 \gamma^\mu q_L^3)(\bar{\ell}_L^3 \gamma_\mu \ell_L^3) \pm (\bar{q}_L^3 \gamma^\mu \sigma^a q_L^3)(\bar{\ell}_L^3 \gamma_\mu \sigma^a \ell_L^3) \quad Q_S = (\bar{\ell}_L^3 \tau_R)(\bar{b}_R q_L^3)$$

↑ $SU(2)$ singlet

↑ $SU(2)$ triplet

↑ scalar

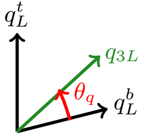
Which operators?

$$Q^{\pm}_{\ell q} = (\bar{q}^3_L \gamma^\mu q^3_L)(\bar{\ell}^3_L \gamma_\mu \ell^3_L) \pm (\bar{q}^3_L \gamma^\mu \sigma^a q^3_L)(\bar{\ell}^3_L \gamma_\mu \sigma^a \ell^3_L) \qquad Q_S = (\bar{\ell}^3_L \tau_R)(\bar{b}_R q^3_L)$$

$SU(2)$ singlet ↑ $SU(2)$ triplet ↑ scalar ↑

- Only left-handed neutrinos
- $q_{3L} \equiv q^b_L + \hat{V} \cdot Q_L$

$$q^b_L = \begin{pmatrix} V^*_{j3} u^j_L \\ b_L \end{pmatrix} \qquad Q^i_L = \begin{pmatrix} V^*_{ji} u^j_L \\ d^i_L \end{pmatrix} \qquad \hat{V}_q \equiv -\epsilon V_{ts} \begin{pmatrix} \kappa V_{td}/V_{ts} \\ 1 \end{pmatrix}$$



$$(\bar{q}_{3L}\gamma_\mu q_{3L})(\bar{\ell}_{3L}\gamma^\mu \ell_{3L})$$

CKM or $\hat{V}$

$\hat{V}$

$$(\bar{c}_L\gamma_\mu b_L)(\bar{\nu}_\tau\gamma^\mu \tau_L)$$

$$(\bar{s}_L\gamma_\mu b_L)(\bar{\nu}_\tau\gamma^\mu \nu_\tau)$$

two insertion of $\hat{V}$

$$(\bar{s}_L\gamma_\mu d_L)(\bar{\nu}_\tau\gamma^\mu \nu_\tau)$$

$$(\bar{q}_{3L}\gamma_\mu q_{3L})(\bar{\ell}_{3L}\gamma^\mu \ell_{3L})$$

CKM or $\hat{V}$ → $(\bar{c}_L\gamma_\mu b_L)(\bar{\nu}_\tau \gamma^\mu \tau_L)$

$\hat{V}$ → $(\bar{s}_L\gamma_\mu b_L)(\bar{\nu}_\tau \gamma^\mu \nu_\tau)$

two insertion of $\hat{V}$ → $(\bar{s}_L\gamma_\mu d_L)(\bar{\nu}_\tau \gamma^\mu \nu_\tau)$

Correlations among all these modes is essential to prove NP scenarios

What do we expect in the SMEFT?

$$\mathcal{L}_{\rm EFT} \supset \boxed{\frac{C_{bc\tau\tau}}{\Lambda^2}} (\bar{b}_L \gamma_\nu c_L)(\bar{\nu}_\tau \gamma^\mu \tau_L)$$

From $U(2)^n \Rightarrow C_{bc\tau\tau} \sim V_{cb} \mathcal{O}(1)$

From $R_{D^{(*)}} \Rightarrow \Lambda \sim \mathcal{O}(\mathrm{TeV})$

Using $SU(2)_L$ invariance, we have

$$\mathcal{L}_{\rm EFT} \supset \frac{C_{ij\tau\tau}}{\Lambda^2} (\bar{d}^i_L \gamma_\nu d^j_L)(\bar{\nu}_\tau \gamma^\mu \nu_\tau)$$

$B^+ \to K^+ \nu \bar{\nu}$

From $U(2)^n \Rightarrow C_{bs\tau\tau} \sim V_{cb} \mathcal{O}(1)$

$K^+ \to \pi^+ \nu \bar{\nu}$

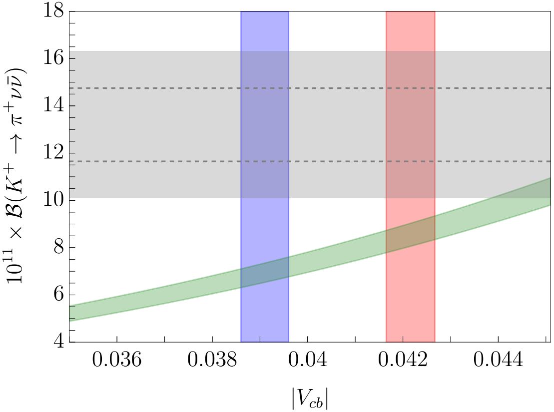
From $U(2)^n \Rightarrow C_{sd\tau\tau} \sim 10^{-1} V_{cb} \mathcal{O}(1)$

On the V_{cb} puzzle (again)

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) \propto |\lambda_{ts}|^2 \quad \lambda_{ts} \equiv \lambda |V_{cb}| \left[(\bar{\rho} - 1) \left(1 - \frac{\lambda^2}{2} \right) + i\bar{\eta} \left(1 + \frac{\lambda^2}{2} \right) \right] + \mathcal{O}(\lambda^4)$$

On the V_{cb} puzzle (again)

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) \propto |\lambda_{ts}|^2 \quad \lambda_{ts} \equiv \lambda |V_{cb}| \left[(\bar{\rho} - 1)\left(1 - \frac{\lambda^2}{2}\right) + i\bar{\eta}\left(1 + \frac{\lambda^2}{2}\right)\right] + \mathcal{O}(\lambda^4)$$



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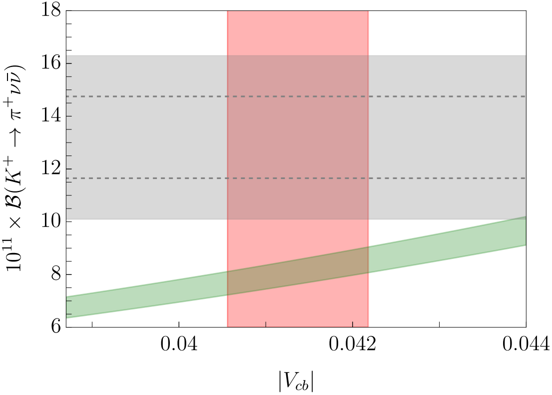
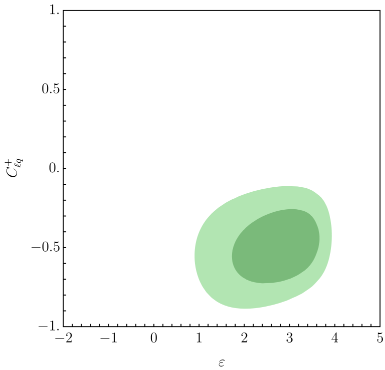
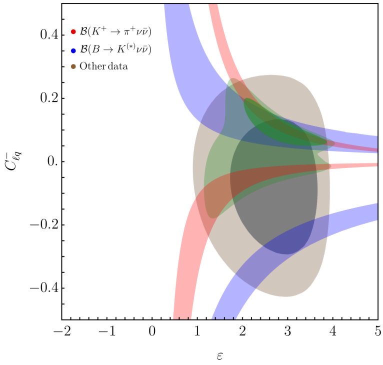


$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})^{\rm SM} = (8.09 \pm 0.63) \times 10^{-11}$$

$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})^{\rm SM} = (2.58 \pm 0.30) \times 10^{-11}$$

[L. Allwicher, MB, G. Isidori, G. Piazza, A. Stanzione, '24]

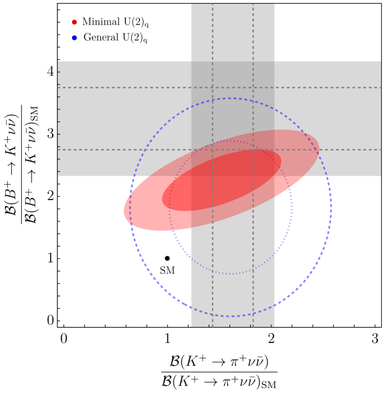


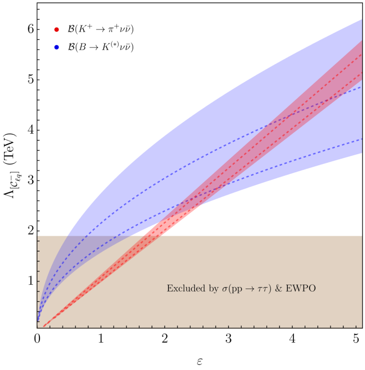


- EWPO and direct searches
- $R_{D^{(*)}}$
- $B \to K^{(*)} \mu^+ \mu^-$

- $B \to K^{(*)} \nu\bar{\nu}$
- $K^+ \to \pi^+ \nu\bar{\nu}$

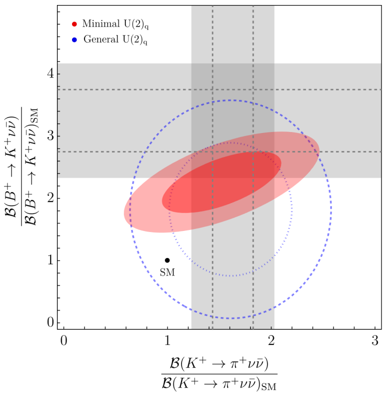
[L. Allwicher, MB, G. Isidori, G. Piazza, A. Stanzione, '24]

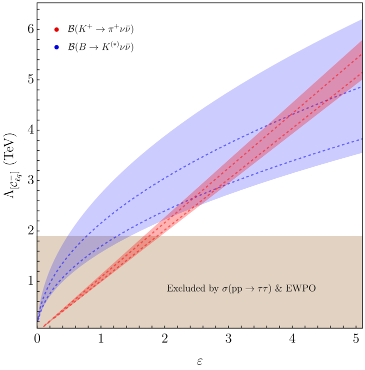




- The $U(2)^n$ symmetry creates a natural link between all this observables
- The complementarity between low- and high-energy data is useful to probe the parameter space

[L. Allwicher, MB, G. Isidori, G. Piazza, A. Stanzione, '24]





- The $U(2)^n$ symmetry creates a natural link between all this observables
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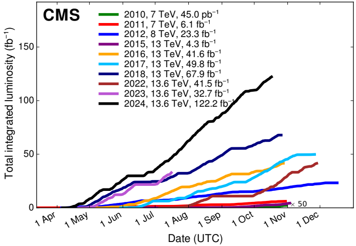
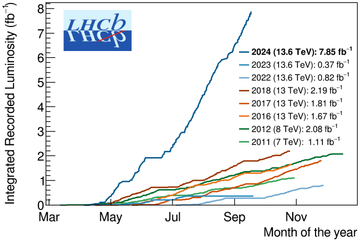
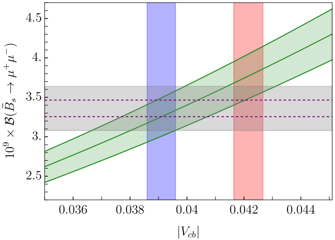
Further data is essential!

Experimental prospects





- Experimental facilities are delivering unprecedented datasets
- The experimental reach supported by new analysis techniques already superseded the expectations
- Theoretical advancements are crucial for achieving greater precision in understanding flavor processes and evaluating potential signs of new physics



Summary

- Flavour physics has the potential to test for possible hints of extensions of the SM

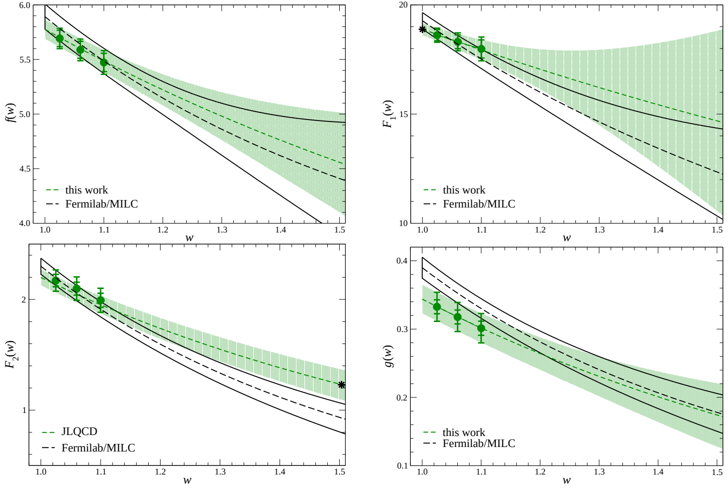
- The main showstopper is the theoretical precision

- A lot of progress has been made, but a few pivotal puzzles persist

- There are hints for possible BSM directions, but more efforts and more data are needed to shed light on their nature

Appendix

Compatiblity of lattice data



- Similar results with HPQCD
- There are some differences in the slopes
- How good is the compatibility?
- Do the differences yield significant pheno consequences?

Strategy A

[MB, A. Jüttner, '24]

Frequentist fit

K_f	$K_{\mathcal{F}_1}$	$K_{\mathcal{F}_2}$	K_g	$a_{g,0}$	$a_{g,1}$	$a_{g,2}$	$a_{g,3}$	p	χ^2/N_{dof}	N_{dof}
2	2	2	2	0.03138(87)	-0.059(24)	-	-	0.95	0.62	30
3	3	3	3	0.03131(87)	-0.046(36)	-1.2(1.8)	-	0.90	0.67	26
4	4	4	4	0.03126(87)	-0.017(48)	-3.7(3.3)	49.9(53.6)	0.79	0.75	22

- good fit quality
- lattice data are compatible
- no unitarity

Bayesian Fit

K_f	$K_{\mathcal{F}_1}$	$K_{\mathcal{F}_2}$	K_g	$a_{g,0}$	$a_{g,1}$	$a_{g,2}$	$a_{g,3}$
2	2	2	2	0.03018(76)	-0.101(21)	-	-
3	3	3	3	0.03034(78)	-0.087(24)	-0.34(45)	-
4	4	4	4	0.03035(77)	-0.089(23)	-0.27(41)	-0.04(45)

- unitarity regulates higher orders
- truncation dependent

Despite the SM successes,
there are open problems:

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there are open problems:

Hierarchy problem

dark matter/dark energy

flavour hierarchies

neutrino masses

gravity

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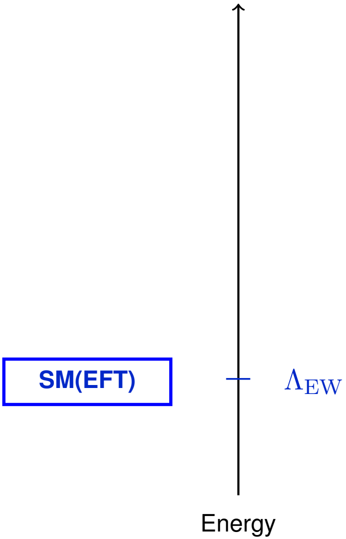
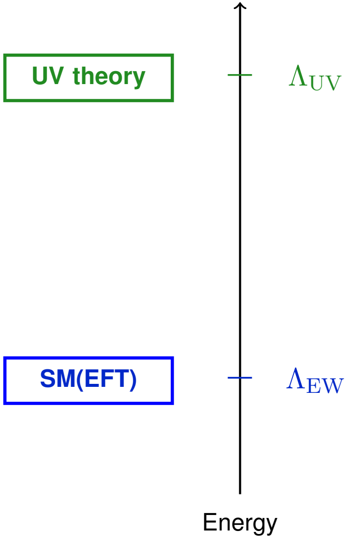


Despite the SM successes,
there are open problems:

- Hierarchy problem
- dark matter/dark energy
- flavour hierarchies
- neutrino masses
- gravity



Despite the SM successes,
there are open problems:

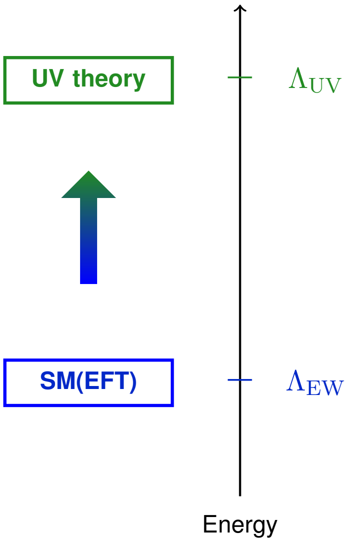
Hierarchy problem

dark matter/dark energy

flavour hierarchies

neutrino masses

gravity



What's the problem for BSM?

B-physics

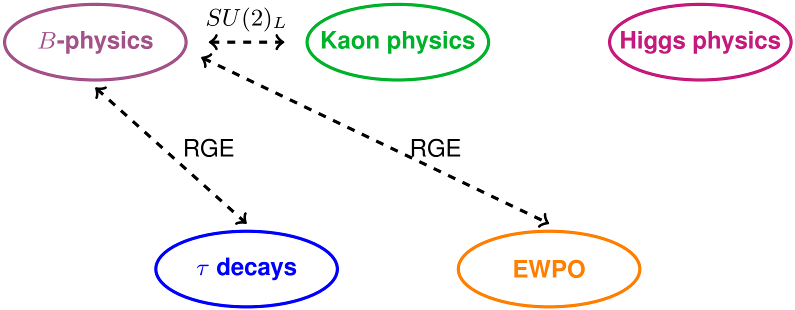
Kaon physics

Higgs physics

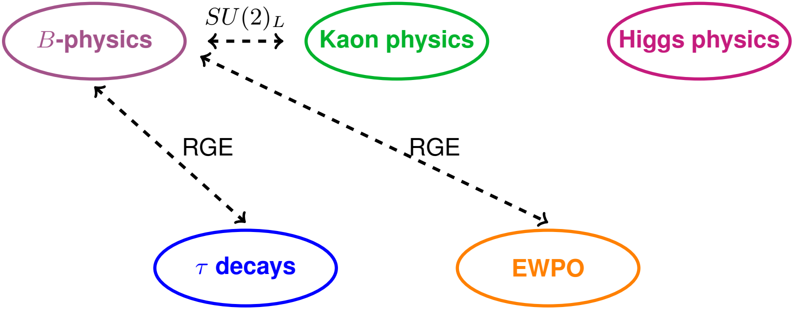
τ decays

EWPO

What's the problem for BSM?



What's the problem for BSM?



How to satisfy all the constraints at the same time?

The NP flavour problem

$$\mathcal{L} = \boxed{\mathcal{L}_{\rm gauge}} + \boxed{\mathcal{L}_{\rm Higgs}} + \boxed{\sum_{d,i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^d}$$

Large Flavour symmetry → $\mathcal{L}_{\rm gauge}$

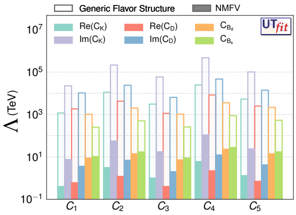
Three replica of the same fermion fields

↓

$U(3)^5$ symmetry

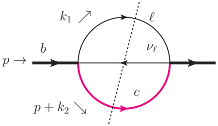
Flavour degeneracy is broken → $\mathcal{L}_{\rm Higgs}$

The breaking is peculiar



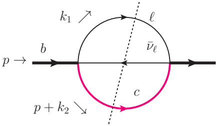
- In the SM: accidental $U(3)^5 \rightarrow$ approx $U(2)^n$
- **What happens when we switch on NP?**

Theory framework



$$\Gamma = \frac{1}{m_B} \mathrm{Im} \int d^4x \langle B(p) | T \left\{ \mathcal{H}^\dagger_{\mathrm{eff}}(x) \mathcal{H}_{\mathrm{eff}}(0) \right\} | B(p) \rangle$$

Theory framework



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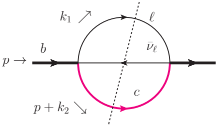
$$\uparrow$$

$$\sum_{n,i} \frac{1}{m_b^n} \mathcal{C}_{n,i} \mathcal{O}_{n+3,i}$$

Theory framework



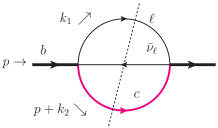
$$\Gamma = \frac{1}{m_B} \mathrm{Im} \int d^4x \langle B(p)| T \left\{ \mathcal{H}^\dagger_{\mathrm{eff}}(x) \mathcal{H}_{\mathrm{eff}}(0) \right\} |B(p)\rangle$$

$$\uparrow$$

$$\sum_{n,i} \frac{1}{m_b^n} \mathcal{C}_{n,i} \mathcal{O}_{n+3,i}$$

- The Wilson coefficients are calculated perturbatively
- The matrix elements $\langle B(p)|\mathcal{O}_{n+3,i}|B(p)\rangle$ are non perturbative
  - $\Rightarrow$ They need to be determined with non-perturbative methods, e.g. Lattice QCD
  - $\Rightarrow$ They can be extracted from data
  - $\Rightarrow$ With large n, large number of operators

Theory framework



$$\Gamma = \frac{1}{m_B} \mathrm{Im} \int d^4x \langle B(p) | T \left\{ \mathcal{H}^\dagger_{\text{eff}}(x) \mathcal{H}_{\text{eff}}(0) \right\} | B(p) \rangle$$

↑

$$\sum_{n,i} \frac{1}{m_b^n} \mathcal{C}_{n,i} \mathcal{O}_{n+3,i}$$

- The Wilson coefficients are calculated perturbatively
- The matrix elements $\langle B(p) | \mathcal{O}_{n+3,i} | B(p) \rangle$ are non perturbative

$\Rightarrow$ They need to be determined with non-perturbative methods, e.g. Lattice QCD

$\Rightarrow$ They can be extracted from data

$\Rightarrow$ With large n, large number of operators

↑

loss of predictivity

Theory framework for $B \to X_c \ell \bar{\nu}$

Double expansion in $1/m$ and α_s

$$\Gamma_{sl} = \Gamma_0 f(\rho)\Big[1 + a_1\left(\frac{\alpha_s}{\pi}\right) + a_2\left(\frac{\alpha_s}{\pi}\right)^2 + a_3\left(\frac{\alpha_s}{\pi}\right)^3 - \left(\frac{1}{2} - p_1\left(\frac{\alpha_s}{\pi}\right)\right)\frac{\mu_\pi^2}{m_b^2}$$
$$+ \left(g_0 + g_1\left(\frac{\alpha_s}{\pi}\right)\right)\frac{\mu_G^2(m_b)}{m_b^2} + d_0\frac{\rho_D^3}{m_b^3} - g_0\frac{\rho_{LS}^3}{m_b^3} + \ldots\Big]$$

- The coefficients are known
- $\mu_\pi^2(\mu) = \frac{1}{2m_B}\langle B|\bar{b}_v(i\vec{D})^2 b_v|B\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2m_B}\langle B|\bar{b}_v \frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b_v|B\rangle_\mu$

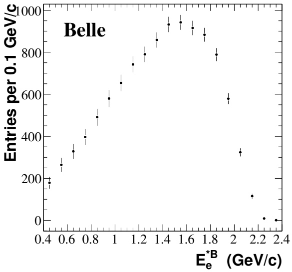
  $\Rightarrow$ No Lattice QCD determinations are available yet

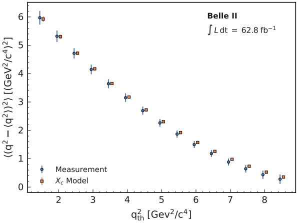
- Use for the first time of α_s^3 corrections [Fael, Schönwald, Steinhauser, '20]
- Ellipses stands for higher orders

  $\Rightarrow$ proliferation of terms and loss of predictivity

How do we constrain the hadronic parameters?

We need information from kinematic distributions



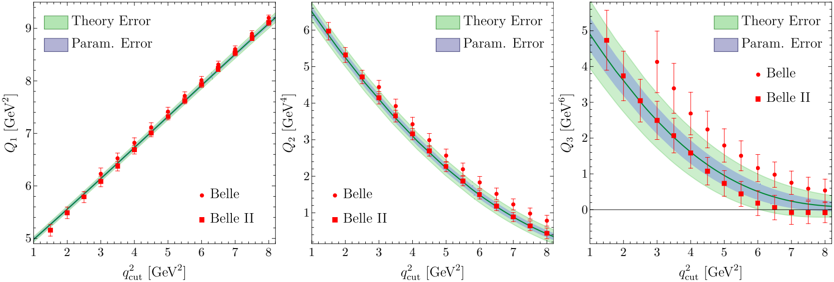


- Traditional method: Extract the hadronic parameters from moments of kinematic distributions in E_l and M_X
- New idea: Use q^2 moments to exploit the reduction of free parameters due to RPI [Fael, Mannel, Vos, '18, Bernlochner et al, '22]
- Measurements of branching fractions are needed and are at the moment quite old
- Can we do it on the lattice? [Gambino, Hashimoto, '20, '23, Hashimoto, Jüttner, et al, '23]

Global fit

[MB, Capdevila, Gambino, '21, Finauri, Gambino, '23]

	$m_b^{\rm kin}$	$\overline{m}_c$	μ_π^2	μ_G^2	ρ_D^3	ρ_{LS}^3	$10^2 {\rm BR}_{c\ell\nu}$	$10^3 \lvert V_{cb}\rvert$	$\chi^2_{\rm min}(/{\rm dof})$
without	4.573	1.092	0.477	0.306	0.185	−0.130	10.66	42.16	22.3
q^2-moments	0.012	0.008	0.056	0.050	0.031	0.092	0.15	0.51	0.474
Belle II	4.573	1.092	0.460	0.303	0.175	−0.118	10.65	42.08	26.4
	0.012	0.008	0.044	0.049	0.020	0.090	0.15	0.48	0.425
Belle	4.572	1.092	0.434	0.302	0.157	−0.100	10.64	41.96	28.1
	0.012	0.008	0.043	0.048	0.020	0.089	0.15	0.48	0.476
Belle &	4.572	1.092	0.449	0.301	0.167	−0.109	10.65	42.02	41.3
Belle II	0.012	0.008	0.042	0.048	0.018	0.089	0.15	0.48	0.559



Two calculation approaches

1. Splitting Functions

$$\left(\frac{d\Gamma}{dy}\right)^{(1)} = \frac{\alpha}{2\pi}\,\bar{L}_{b/e}\int_y^{1-\rho}\frac{dx}{x}\,P_{ee}^{(0)}\left(\frac{y}{x}\right)\left(\frac{d\Gamma}{dx}\right)^{(0)}$$

$\log(m_b^2/m_e^2)$ plus distribution

- Correction vanishes for the inclusive branching fraction
- Suitable for evaluating $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha/m_b^n)$ corrections

2. Full $\mathcal{O}(\alpha)$ corrections

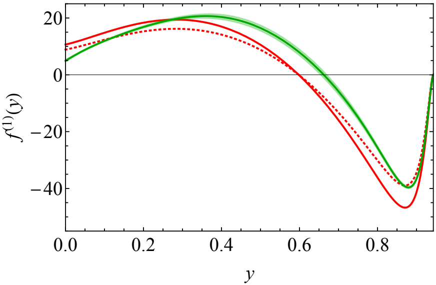
- Access all corrections, not only the one that factorise
- Real corrections are computationally expensive
  - $\Rightarrow$ Cuba library employed to carry out the 4-body integration
  - $\Rightarrow$ Phase space splitting used to reduce the size of the integrands

Lepton Energy spectrum

[Bigi, MB, Gambino, Haisch, Piccione, '23]

- We compute bins in the lepton energy using the full $\mathcal{O}(\alpha)$ calculation
- We compare them to the results given by the splitting functions
- The difference the two calculations for the lepton energy spectrum and obtain a full analytic formula for the radiative corrections

  $\Rightarrow$ Relatively small, easy-to-use formula to obtain branching fractions, lepton energy moments w/o cuts



$$f^{(1)}(y) = \frac{\bar{L}_{b/e}}{2}\, f^{(1)}_{\rm LL}(y) + \Delta f^{(1)}(y)$$

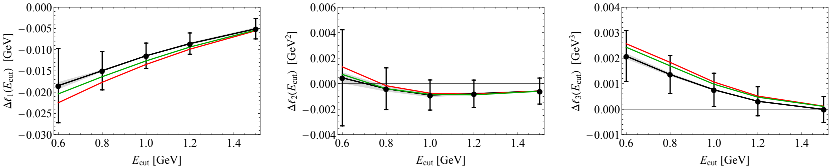
Comparison with data

[Bigi, MB, Gambino, Haisch, Piccione, '23]

- Babar provides data with and without applying PHOTOS to subtract QED effects

  ⇒ Perfect ground to test our calculations

  ⇒ Not the same for Belle at the moment, could be possible for future analysis

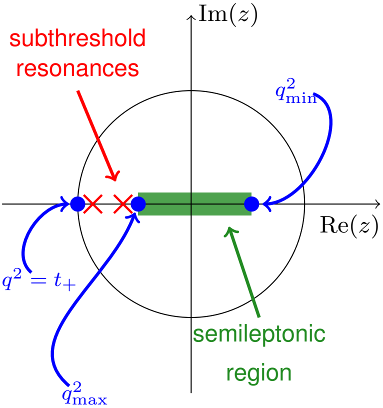


- The moments, since they are normalised, are not affected by the large threshold corrections
- The agreement with BaBar is very good

$$\langle E_\ell^n \rangle = \frac{\int_{E_\ell > E_{\ell,\rm cut}} dE_\ell E_\ell^n \frac{d\Gamma}{dE_\ell}}{\Gamma_{E_\ell > E_{\ell,\rm cut}}}$$

The z-expansion and unitarity

[Boyd, Grinstein, Lebed, '95, Caprini, Lellouch, Neubert, '98]



- in the complex plane form factors are real analytic functions
- q^2 is mapped onto the conformal complex variable z

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

- q^2 is mapped onto a disk in the complex z plane, where $|z(q^2, t_0)| < 1$

$$F_i = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k$$

$$\sum_{k=0}^{n_i} |a_k^i|^2 < 1$$

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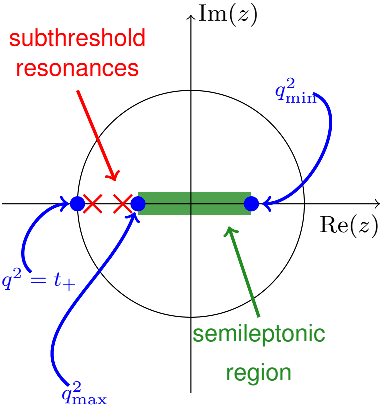

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BGL

How to apply unitarity

- Penalty function in the χ^2 or likelihood [P. Gambino, M. Jung, S. Schacht, '19]

$$\chi^2 \to \chi^2(a_k^i, a_k^i|_{\mathrm{data}}) + w_i \theta \left(\sum_{k=0}^{n_i} |a_k^i|^2 - 1 \right)$$

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- Dispersive Matrix Method [M. Di Carlo, G. Martinelli, M. Naviglio, F. Sanfilippo, S. Simula, L. Vittorio, '21] [G. Martinelli, S. Simula, L. Vittorio, '21,'23]

$$\mathbf{M} = \begin{pmatrix} \chi & \phi f & \phi_1 f_1 & \phi_2 f_2 & \dots & \phi_N f_N \\ \phi f & \frac{1}{1-z^2} & \frac{1}{1-z z_1} & \frac{1}{1-z z_2} & \dots & \frac{1}{1-z z_N} \\ \phi_1 f_1 & \frac{1}{1-z_1 z} & \frac{1}{1-z_1^2} & \frac{1}{1-z_1 z_2} & \dots & \frac{1}{1-z_1 z_N} \\ \phi_2 f_2 & \frac{1}{1-z_2 z} & \frac{1}{1-z_2 z_1} & \frac{1}{1-z_2^2} & \dots & \frac{1}{1-z_2 z_N} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \phi_N f_N & \frac{1}{1-z_N z} & \frac{1}{1-z_N z_1} & \frac{1}{1-z_N z_2} & \dots & \frac{1}{1-z_N^2} \end{pmatrix}$$

$$\det \mathbf{M} > 0 \Rightarrow \beta - \sqrt{\gamma} \leq f_0 \leq \beta + \sqrt{\gamma}$$

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$$\det \mathbf{M} > 0 \Rightarrow \beta - \sqrt{\gamma} \leq f_0 \leq \beta + \sqrt{\gamma}$$

- Bayesian inference

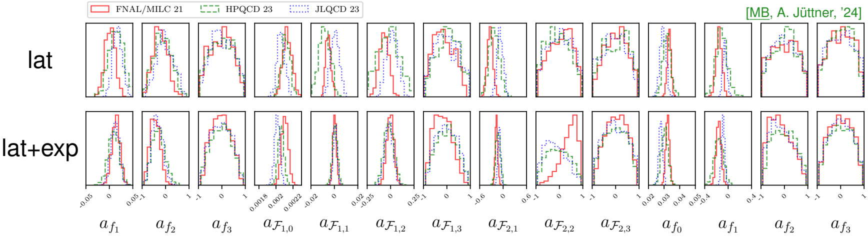
[J. Flynn, A. Jüttner, T. Tsang, '23]

$$\langle g(\mathbf{a}) \rangle = \mathcal{N} \int d\mathbf{a}\, g(\mathbf{a})\, \pi(\mathbf{a}|\mathbf{f}, C_{\mathbf{f}}) \pi_{\mathbf{a}}$$

$\theta(1 - |\mathbf{a}|^2)$

contains the lattice χ^2

Posterior distribution



- Small shifts between lattice only and lattice + data
- Higher order coefficients well constrained by unitarity
- $a_{\mathcal{F}_{2,2}}$ has a strange behaviour, maybe kinematic constraints?