EXCEPTIONAL STATIONARY STATE IN A DEPHASING MANY-BODY OPEN QUANTUM SYSTEM

New preprint [arXiv:2412.13820]: joint work
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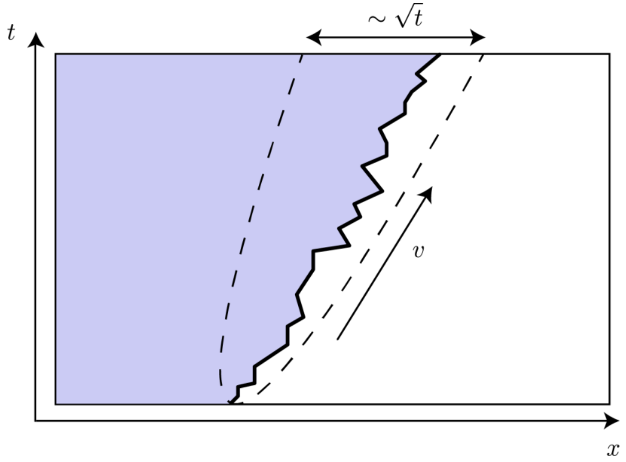

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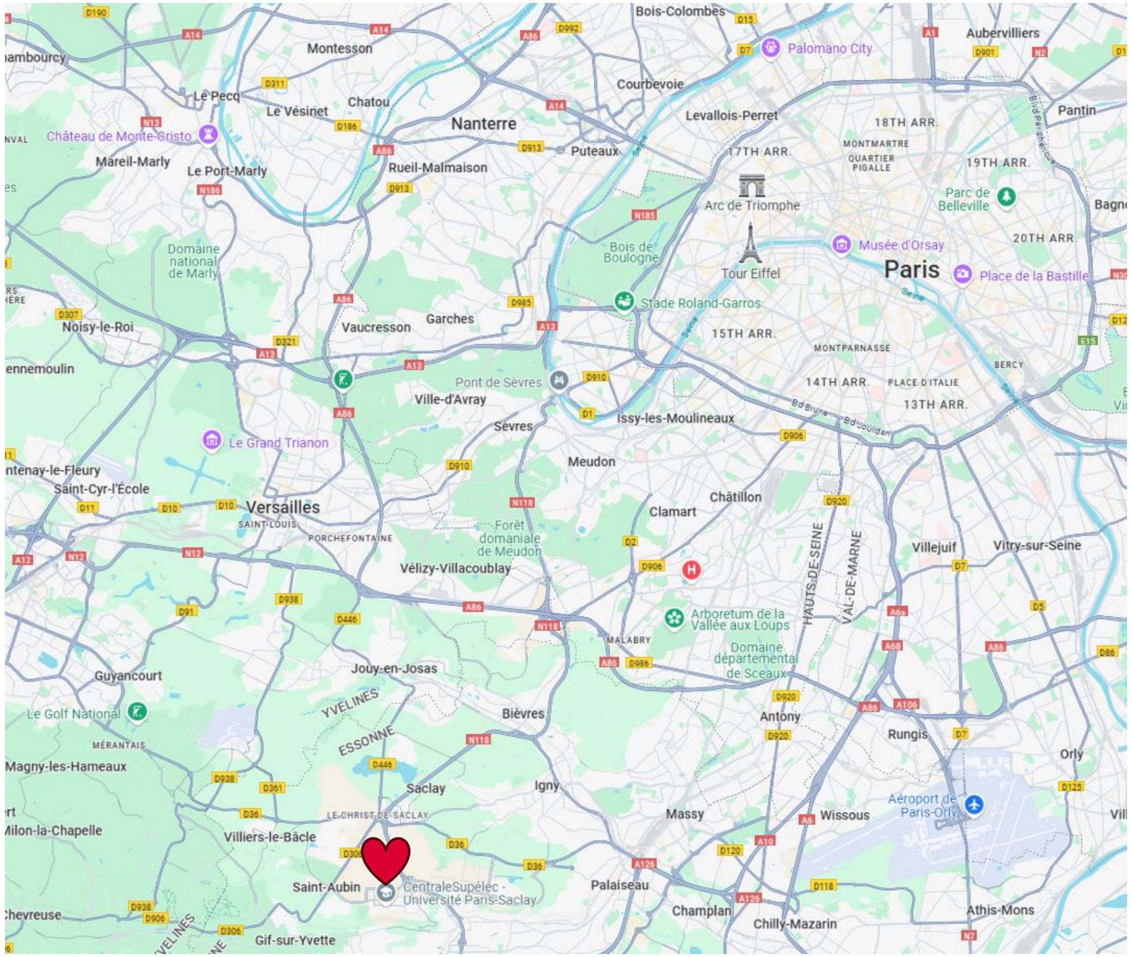

Thanks!!

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OUTLINE

INTRODUCTION

- Scars, Eigenstate thermalization hypothesis (**ETH**) and violations thereof.
- **Hydrodynamics**, Gibbs ensembles and transport.

OPEN QUANTUM SYSTEM WITH A "SCAR"

- Paradigmatic model with two stationary states: the infinite temperature state (typical), and an additional one (exceptional, ”scar”).
- Spectral properties of the Lindbladian (**finite gap**).

MEMBRANE PICTURE

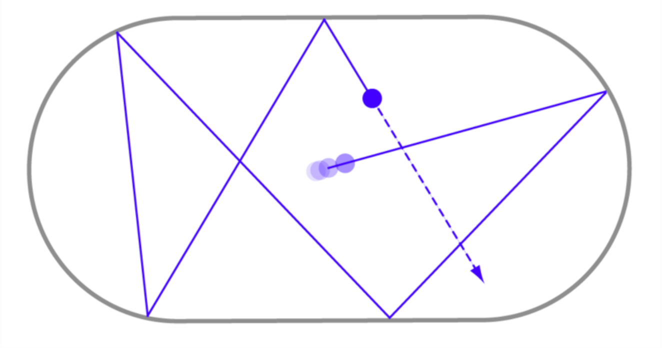
- Bipartition protocol: dynamics arising after "joining" the two stationary states.
- **Fluctuating interface** that follows a driven Brownian motion

INTRODUCTION

QUANTUM SCARS

- **Quantum scars:** (weak) **violations of ergodicity**. First studied in ‘’single-body systems’’ (**Quantum Biliards**).

Rare periodic (non-ergodic orbits) → ? **Rare non-ergodic eigenfunctions**



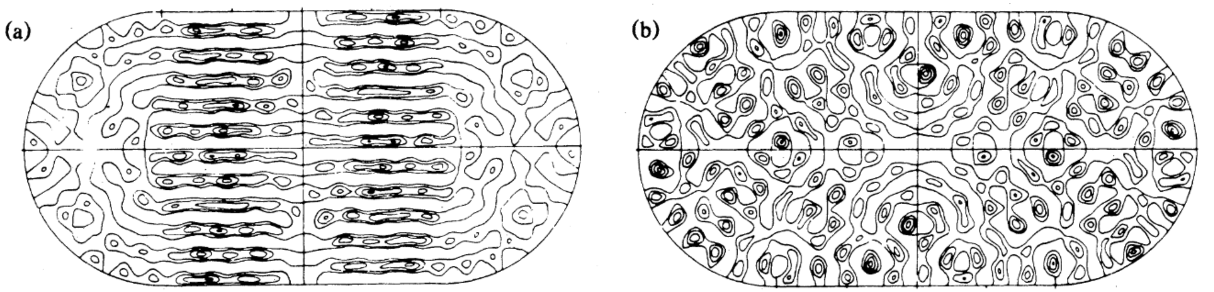


FIG. 1. (a) Localized and (b) chaotic states of the stadium potential; only the negative contours are shown. From

Heller, Eric J. (1984). *Bound-State Eigenfunctions of Classically Chaotic Hamiltonian Systems: Scars of Periodic Orbits. PRL, 53(16) (1984).*

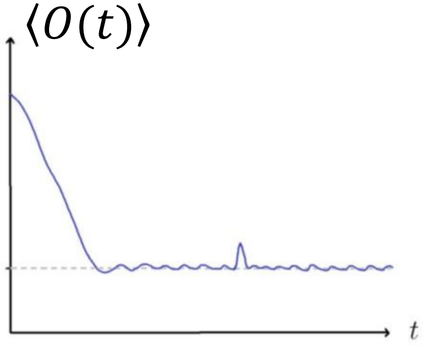
… Analogous mechanism in **quantum many-body systems**??

EIGENSTATE THERMALIZATION HYPOTHESIS (ETH)

How do close systems relax toward equilibrium?

“Generic” local Hamiltonian $H = \sum_x h(x)$. Local observable O.

$$\langle \psi_0 | O(t) | \psi_0 \rangle \overset{?}{\rightarrow} \langle O \rangle_{\text{stationary}} = \frac{\mathrm{Tr}\left[\mathrm{e}^{-\beta H} O\right]}{\mathrm{Tr}\left[\mathrm{e}^{-\beta H}\right]}$$



How is it possible?

Possible Problems (at finite size): Revivals??! Many eigenvectors of H, thus many stationary states..?!

ETH postulates

$$\langle E_j | O | E_j \rangle \simeq O(E/V)$$

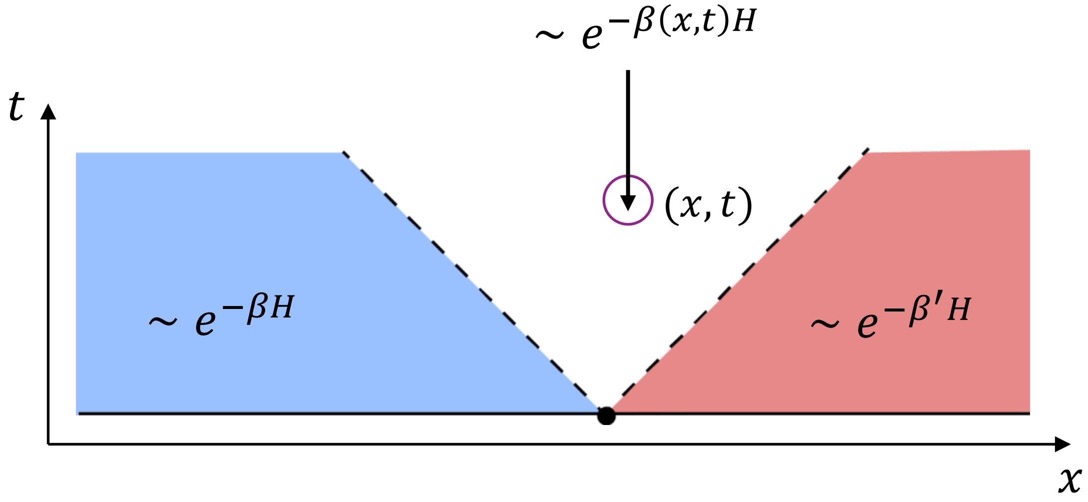
Microcanonical (≃ thermal) expectation value

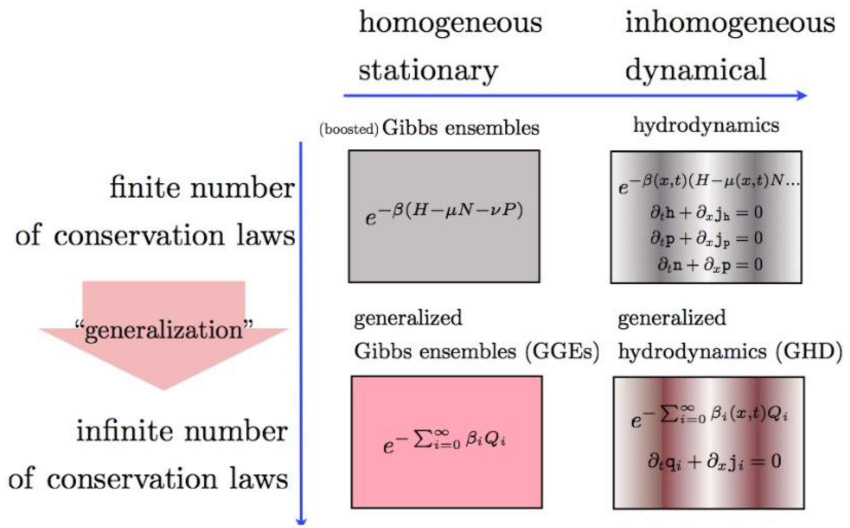
… In particular: **Thermal states** $\mathrm{Tr}\left[\mathrm{e}^{-\beta H} \ldots\right]$ are the **only stationary states**

Mark Srednicki, *The approach to thermal equilibrium in quantized chaotic systems*, Journal of Physics A: Mathematical and General 32, 1163 (1999).

HYDRODYNAMICS

Postulate of hydrodynamics: «**Local temperature**» completely describe local properties!





Benjamin Doyon, *Lecture notes on Generalised Hydrodynamics* [SciPost Phys. Lect. Notes 18 (2020)]

Predictive for integrable systems

O. A. Castro-Alvaredo et. Al., *Emergent hydrodynamics in integrable quantum systems out of equilibrium*, Phys. Rev. X 6, 041065 (2016).

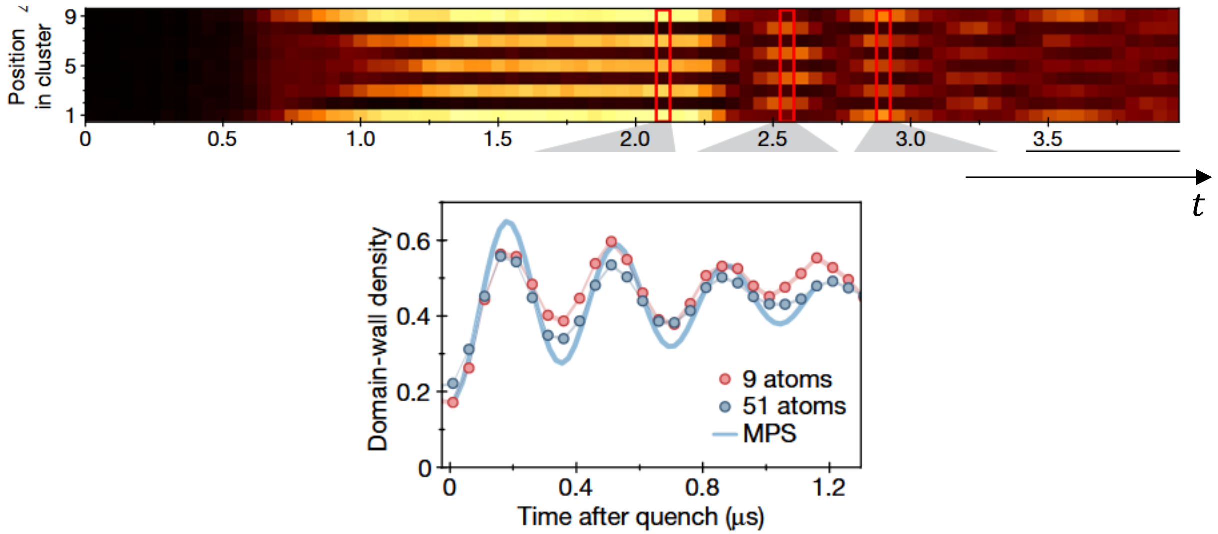
B. Bertini et al., *Transport in out-of-equilibrium XXZ chains: Exact profiles of charges and currents*, Phys. Rev. Lett. 117, 207201 (2016).

… Anything else???

VIOLATION OF ETH

Experiments

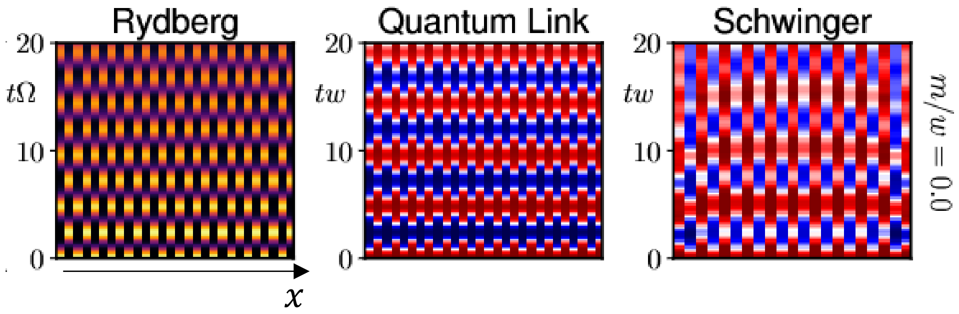
$$\frac{\mathcal{H}}{\hbar} = \sum_i \frac{\Omega_i}{2}\sigma_x^i - \sum_i \Delta_i n_i + \sum_{i<j} V_{ij} n_i n_j$$



From: Bernien et al., "*Probing many-body dynamics on a 51-atom quantum simulator,*" Nature **551**, 579–584 (2017).

Numerical simulations (constrained models)

$$H = \sum_j P_j X_{j+1} P_{j+2} \ \ldots$$



F.M. Surace et al., *Lattice gauge theories and string dynamics in Rydberg atom quantum simulators*, Phys. Rev. X **10**, 021041 (2020)

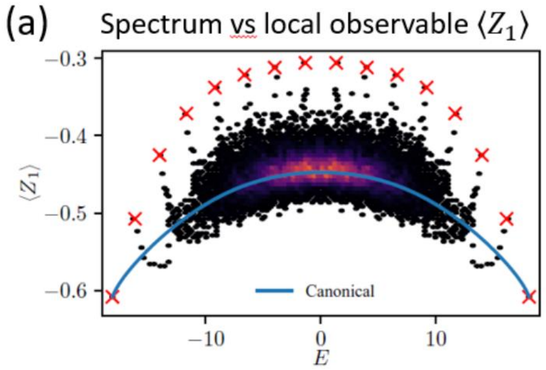
VIOLATION OF ETH

- "Other **extensive conserved quantities**" (beside H) $\{Q_a = \sum q_a(x)\}$ $[Q_a, H] = 0$. **Generalized Gibbs ensembles** $\mathrm{Tr}\left[\mathrm{e}^{-\sum \beta_a Q_a} \ldots\right]$ are stationary. "Typical scenario" in the presence of global symmetries or integrability.

- Many-body localized (**MBL**) phases: conserved (quasi) local operator $q_a(x)$… Sensibility to initial condition and absence of transport. Conjectured for quantum disordered systems.

… Anything else???
Additional stationary states BEYOND usual conservation laws

- **Quantum many-body scars**: Some "rare" eigenstates violate ETH. [motivated by H. Bernien et al., Nature 551, (2017)]



From: Turner et al. "*Quantum scarred eigenstates in a Rydberg atom chain: entanglement, breakdown of thermalization, and stability to perturbations*". PRB 98, 155134 (2018)

…Attempt for a "general theory":

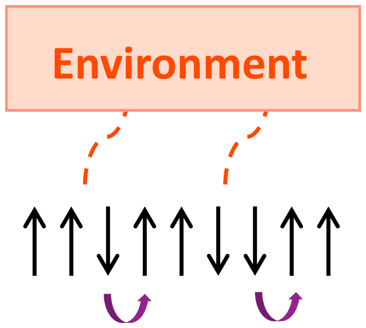
S. Moudgalya et al., *Quantum Many-Body Scars and Hilbert Space Fragmentation: A Review of Exact Results.* Rep. Prog. Phys. 85 086501 (2022)

OPEN QUANTUM SYSTEM
WITH A “SCAR”

OPEN QUANTUM SYSTEM WITH SCARS

Lindblad evolution

$$\frac{d}{dt}\rho = \mathcal{L}[\rho] = -i[H,\rho] + \sum_j L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\}$$



$$H = J\sum_j \sigma_j^+\sigma_{j+1}^- + \sigma_j^-\sigma_{j+1}^+ + g\sum_j (1-\sigma_j^z)\sigma_{j+1}^x + (j \leftrightarrow j+1)$$

Hopping term: it conserves magnetization S^z

East-West term: it breaks S^z conservation, preserving $|\uparrow \cdots \uparrow\rangle$ as stationary state.

Dissipator: $L_j = \sqrt{\gamma}\sigma_j^z$ Decoherence, it tries to “kills” coeherences in the z basis.

Saverio Bocini and Maurizio Fagotti, “*Growing Schrödinger’s cat states by local unitary time evolution of product states*” Phys. Rev. Res. 6, 033108 (2024).

Nicola Pancotti, Giacomo Giudice, J. Ignacio Cirac, Juan P. Garrahan, and Mari Carmen Bañuls, *“Quantum East model: Localization, nonthermal eigenstates, and slow dynamics”,* Phys. Rev. X 10, 021051 (2020)

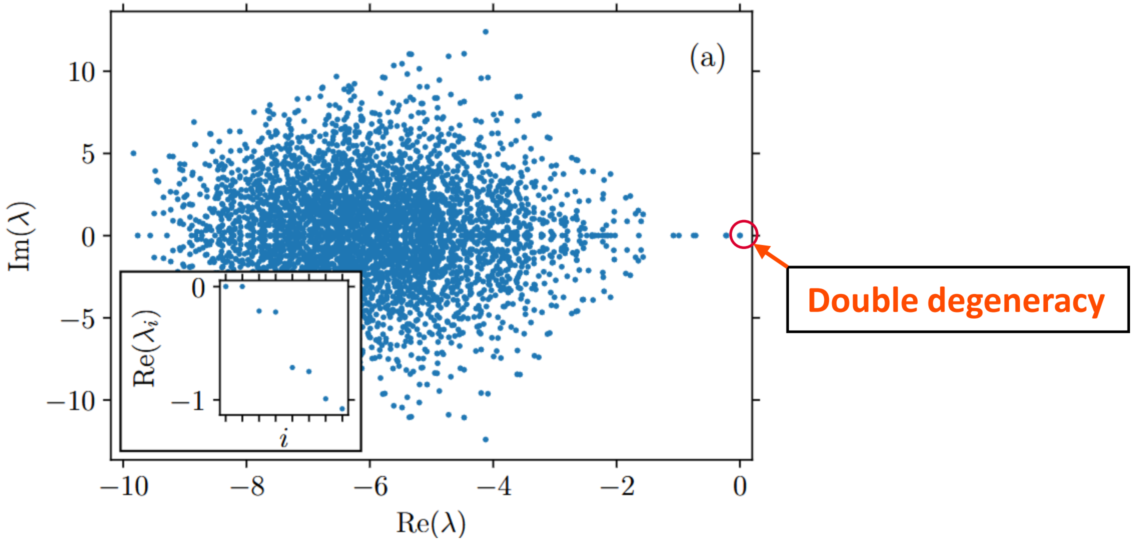
OPEN QUANTUM SYSTEM WITH SCARS: STATIONARY STATES

$$\mathcal{L}[\rho] = 0$$

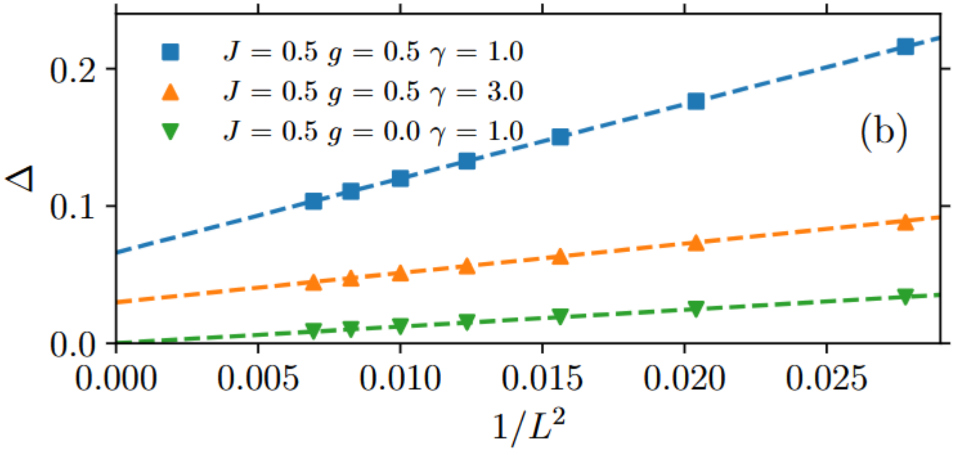
Infinite temperature state: $\sim \mathrm{Tr}[\ldots]$: «Typical» stationary state, as a consequence of $L_j = L_j^\dagger$

All spins up ⟨↑↑ | … | ↑↑⟩ : «Exceptional» stationary state, coming from *'kinetical constraints'*.

Spectrum $\mathcal{L}[O] = \lambda\, O$



Spectral gap $\Delta = \min_{\lambda \neq 0} |\mathrm{Re}[\lambda]|$



OPEN QUANTUM SYSTEM: SYMMETRIES AND CONSERVATION LAW

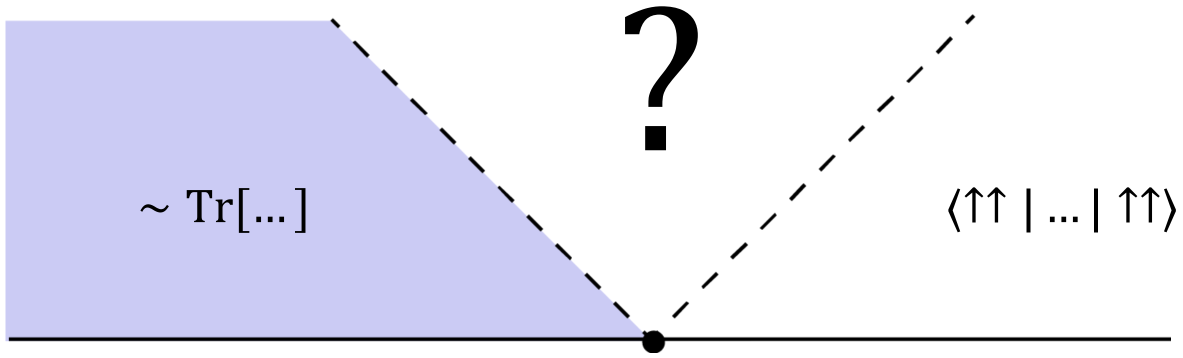
- **Extensive conserved quantities** in **open systems:** $\{Q_a = \sum q_a(x)\}$ $[Q_a, H] = 0$ and $[L_j, Q_a] = 0$. Absent here!! (unless $g = 0$).

- $\mathcal{L}[Q_a] = 0$, $\mathcal{L}[Q_a^2] = 0$, … **Large degeneracy** ($\sim L$) in the kernel of $\mathcal{L}$.

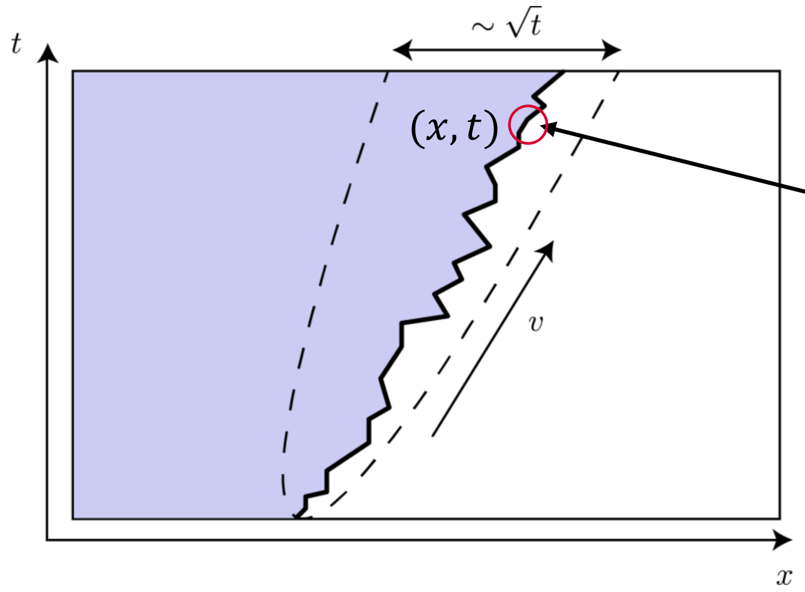
- $\mathrm{Tr}\left[\mathrm{e}^{-\sum \beta_a(x,t) Q_a} \dots\right]$ typical 'local stationary states' in the presence of conserved quantities: analogous to closed system under the 'hydrodynamic scenario'

…In THAT case one expects ''usual Hydrodynamics'' to hold

BIPARTITION PROTOCOL:



What happens here??



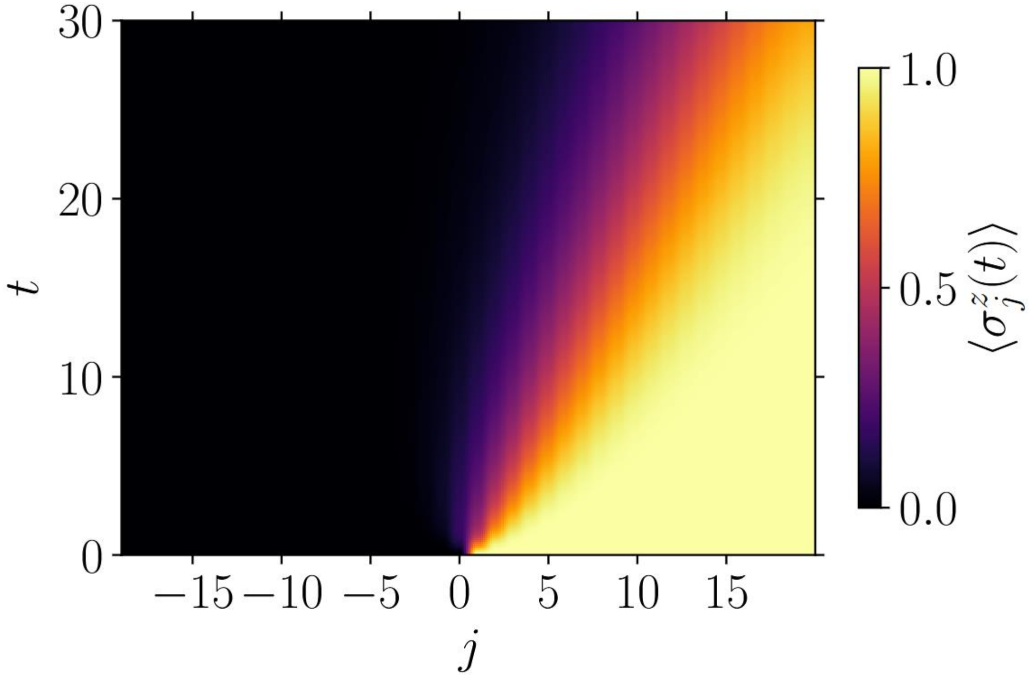
Local statistical mixture

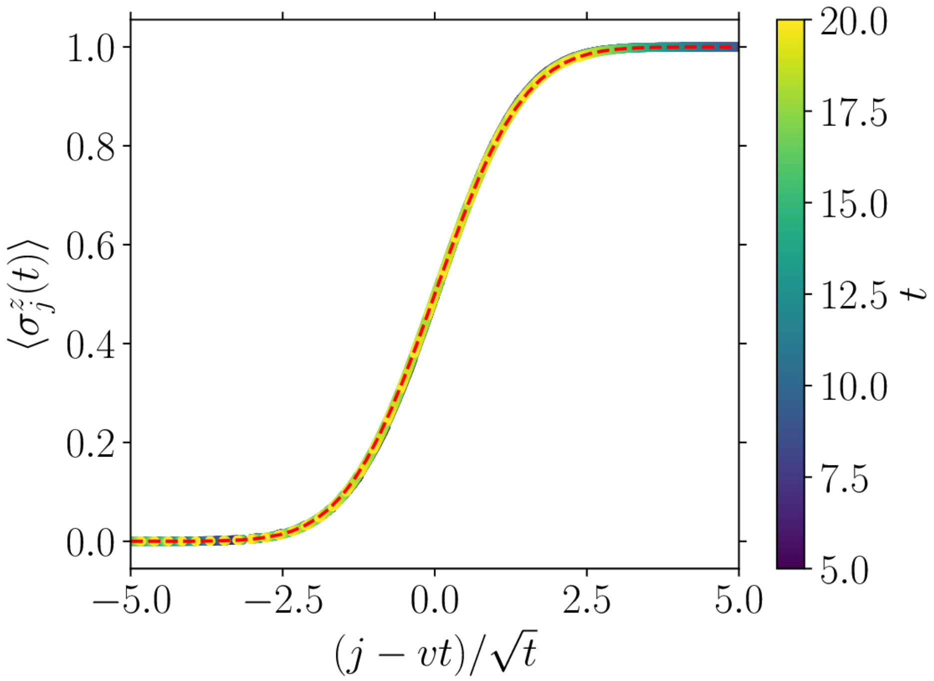
$$P(x,t)\langle\uparrow\uparrow|\ldots|\uparrow\uparrow\rangle + \big(1 - P(x,t)\big)\frac{Tr[\ldots]}{Tr[1]}$$

MEMBRANE PICTURE

- An fluctuating interface separates the two stationary states.
- The «scar» is progressively ‘eaten’ at finite velocity.
- The interface distribution broadens diffusively (as for the **driven random walk**).

BIPARTITION PROTOCOL: NUMERICAL CHECKS





$$\langle \sigma^z(x,t) \rangle \simeq \int_{-\infty}^{\frac{x-vt}{\sqrt{Dt}}} \frac{du}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right)$$

v (**velocity**) D (**diffusion constant**): fitting parameters.
They depends non-trivially on the microscopic couplings.

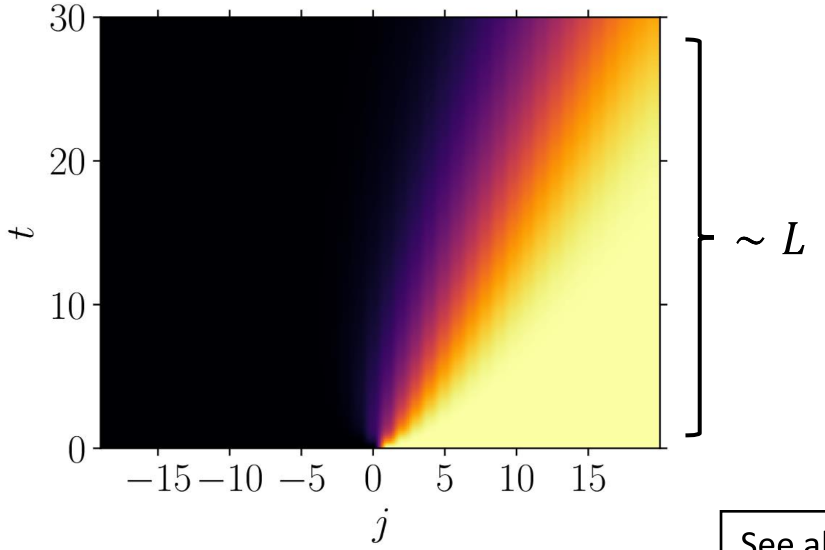
RELAXATION TIME AND SPECTRAL GAP

Spectral gap $\overset{?}{\rightarrow}$ finite relaxation time

Naïve (MISLEADING) argument:

$$\rho(t) = e^{\mathcal{L}t}[\rho] = \rho(t = \infty) + \sum_{n\neq 0} e^{-\epsilon_n t} A_n \ , |\epsilon_n| \geq \Delta \quad \overset{?}{\rightarrow} \quad \rho(t) \text{ relaxes to the stationary state for } t > \Delta^{-1}$$

NO



Reason:

$$|\langle O(t)\rangle - \langle O(t=\infty)\rangle| \leq e^{-\Delta t} \underbrace{\sum_{n\neq 0} |Tr[A_n O]|}_{\sim e^L}$$

Useless for large systems!!!

See also: Takashi Mori and Tatsuhiko Shirai, "*Resolving a discrepancy between Liouvillian gap and relaxation time in boundary-dissipated quantum many-body systems*," PRL. **125**, 230604 (2020).

MICROSCOPIC ORIGIN OF THE MEMBRANE PICTURE

MICROSCOPIC ORIGIN OF THE MEMBRANE PICTURE

1. The dissipation kills the coeherences… Perturbative treatmente for **large dissipation rate** $\frac{\gamma}{J}, \frac{\gamma}{g} \gg 1$. **Effective classical stochastic model** (**Markov chain**) arising from 2° order perturbation theory. [Phys. Rev. Lett. **111**, 150403 (2013)]

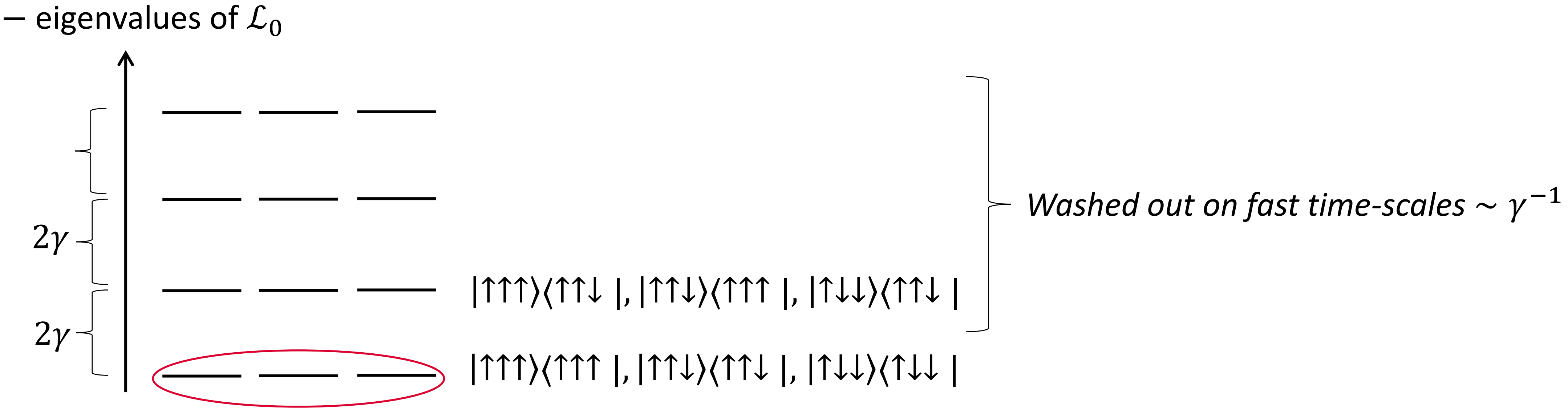
2. Identify a «**solvable point**»: same general physics, the membrane picture arises from the calculation (and v, D can be predicted).

3. Arguments toward the '**universality**' of our results.

MARKOV CHAIN

At 0 order

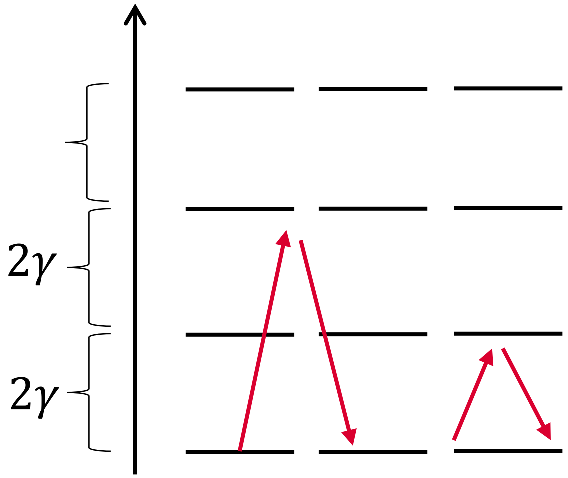
$$\mathcal{L}_0[\rho] = -\gamma \sum_j \rho - \sigma_j^z \rho \sigma_j^z$$



«Unperturbed stationary states»: diagonal matrices

MARKOV CHAIN

At 2nd order



Mixing between diagonal matrix elements: coming from 'virtual transitions'

$$\left|\rho(t)\right) = \sum_{\boldsymbol{\sigma}} p_{\boldsymbol{\sigma}}(t)|\boldsymbol{\sigma}) = e^{-Wt}\left|\rho(0)\right)$$

$$W = -\frac{J^2}{4\gamma}\sum_j \left(\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y + \sigma_j^z\sigma_{j+1}^z - 1\right) +$$
$$\frac{2g^2}{\gamma}\sum_j (1-\sigma_j^x)(1-\sigma_{j+1}^z) + (1-\sigma_j^z)(1-\sigma_{j+1}^x) + (1-\sigma_j^z)(1-\sigma_{j+1}^x)(1-\sigma_{j+2}^z)$$

…Predictive at large γ, but difficult to «solve»

MARKOV CHAIN

$$W = -\frac{J^2}{4\gamma}\sum_j \left(\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y + \sigma_j^z\sigma_{j+1}^z - 1\right) +$$

$$\frac{2g^2}{\gamma}\sum_j (1-\sigma_j^x)(1-\sigma_{j+1}^z) + (1-\sigma_j^z)(1-\sigma_{j+1}^x) + (1-\sigma_j$$ [illegible] $$-\sigma_{j+2}^z)$$

+ Fine tuning: $\frac{J^2}{4\gamma} = \frac{2g^2}{\gamma} = \frac{1}{6}$

$|\bullet) = \frac{1}{2}(|\uparrow) + |\downarrow))$ $\quad |x) \equiv |\bullet \cdots \bullet \underset{x}{\bullet} \uparrow\uparrow \ldots \uparrow)$ *Interface (mixed) state at position x*

$$W|x) = |x) - \frac{2}{3}|x+1) - \frac{1}{3}|x-1)$$

*It reduces to a 'single-particle problem': **Random walk with a drift…***
*It gives EXACTLY the **membrane picture**!*

DOMAIN WALLS

… So far, the membrane picture is derived for a very specific point and under some approximations…

WHY DOES IT ACTUALLY WORK?

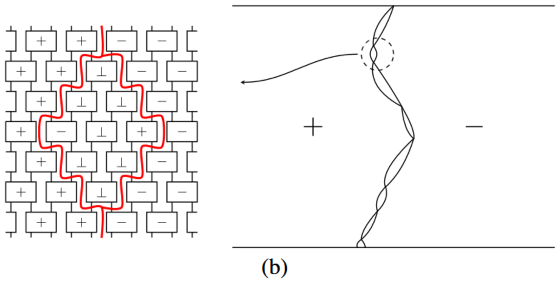
Important features:

- «Slow modes» given by **domain-wall** states **interpolating** the two stationary states.
- At the solvable point, the shape of the domain wall is simple $\sim \sum_x e^{ikx}|x\rangle$. Slightly away from it, one expects $\sim \sum_x e^{ikx}\,|\bullet\bullet\bullet\rangle \otimes \underbrace{O}_{x} \otimes |\uparrow\uparrow\uparrow\rangle$

$\sim O(1)$ 'length of the interpolating region'

DOMAIN WALLS

…Same mechanism found for the **entanglement** dynamics of **chaotic systems**:
«discrete» structure of stationary states in the **replica models**.

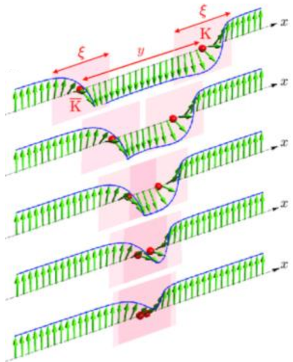


$$|\psi_k\rangle \approx \sum_x e^{-ikx} \, |\eta\rangle \dots |\eta\rangle_x \, |\phi\rangle_{x+1,\dots,x+\Delta} \, |e\rangle_{x+\Delta+1} \dots |e\rangle$$

T. Zhou and A. Nahum, *Entanglement Membrane in Chaotic Many-Body Systems*, PRX 10, 031066 (2020).

S. Vardhan, and S. Moudgalya, *Entanglement dynamics from universal low-lying modes* [arXiv:2407.16763] *(2024)*

…Analogous to «**kinks**» when Spontaneous Symmetry Breaking (**SSB**) occurs



[From: PRB **102**, 024437 (2020)]

CONCLUSIONS

- Perspective on «scars in open systems» as **exceptional stationary states**.

- **Interface diffusion**: Physical mechanism for large-scale dynamics (in the presence of a gapped Lindbladian, and few stationary states).

OPEN PROBLEMS

- What happens when multiple scars (and/or Gibbs ensembles) are present?? Interplay between transport of conserved charges and fluctuating interfaces??

- Notion of **scars** in the presence of **non-hermitian dissipators** ($L_j \neq L_j^\dagger$) ???: E.g. «lossy systems» $\mathcal{L}[\rho] = \sum_j \sigma_j^+ \rho \sigma_j^- - \frac{1}{2}\{\sigma_j^- \sigma_j^+, \rho\}$. $|\uparrow \cdots \uparrow\rangle$ is the ONLY stationary state…

THANKS!