From Physics to Medical Image Segmentation: Is What We are Doing Random?

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Medical Image Segmentation

Image Segmentation

Decomposition of an Image into Non-overlapping and Meaningful Regions

$f: \Omega \rightarrow \{\Omega_0, \Omega_1\}$ s.t:

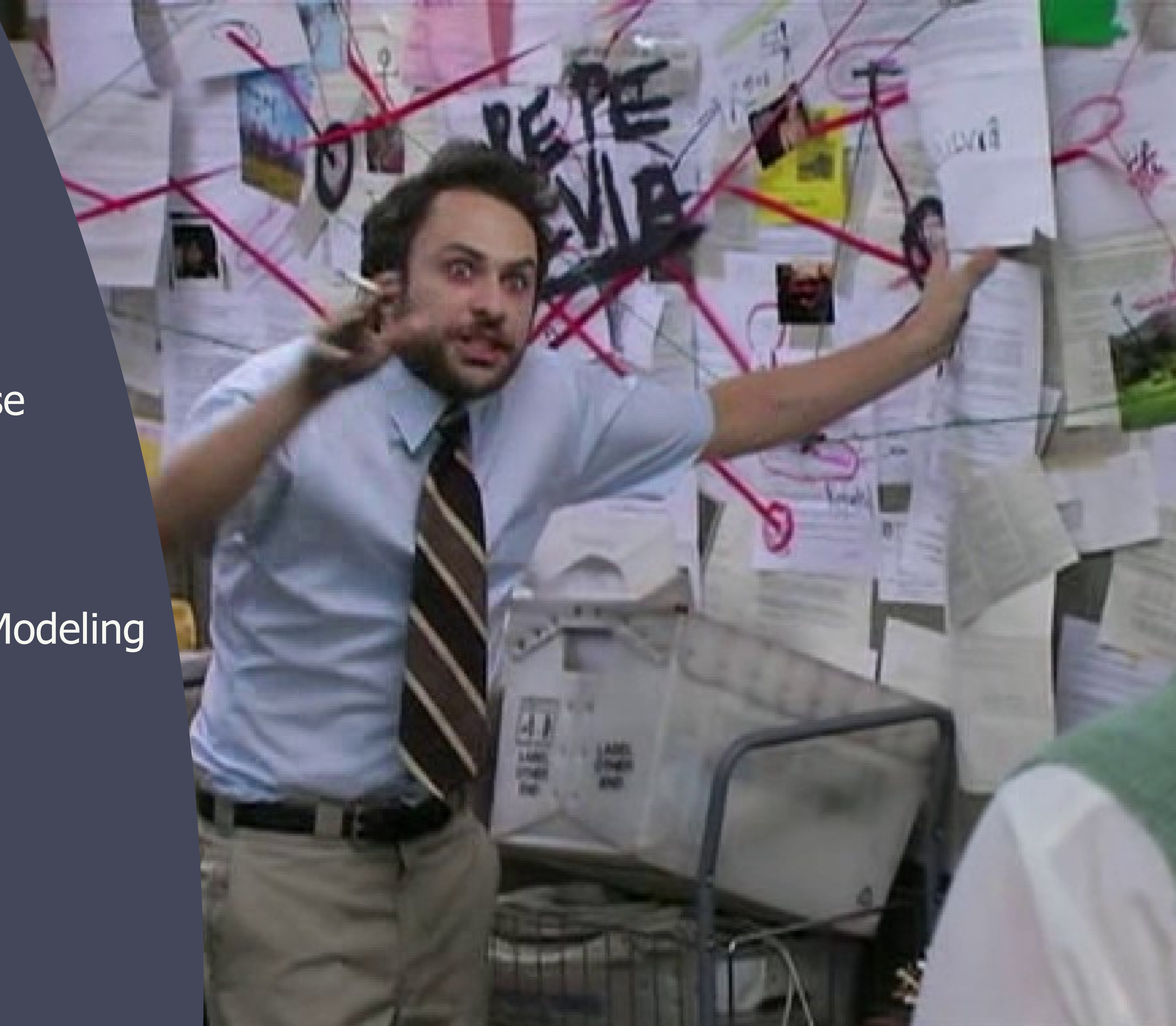
1. $\Omega_0 \cap \Omega_1 = \emptyset$
2. $\Omega_0 \cup \Omega_1 = \Omega$
3. Ω_0, Ω_1 *are connected regions*

Outline

Define The Study Case

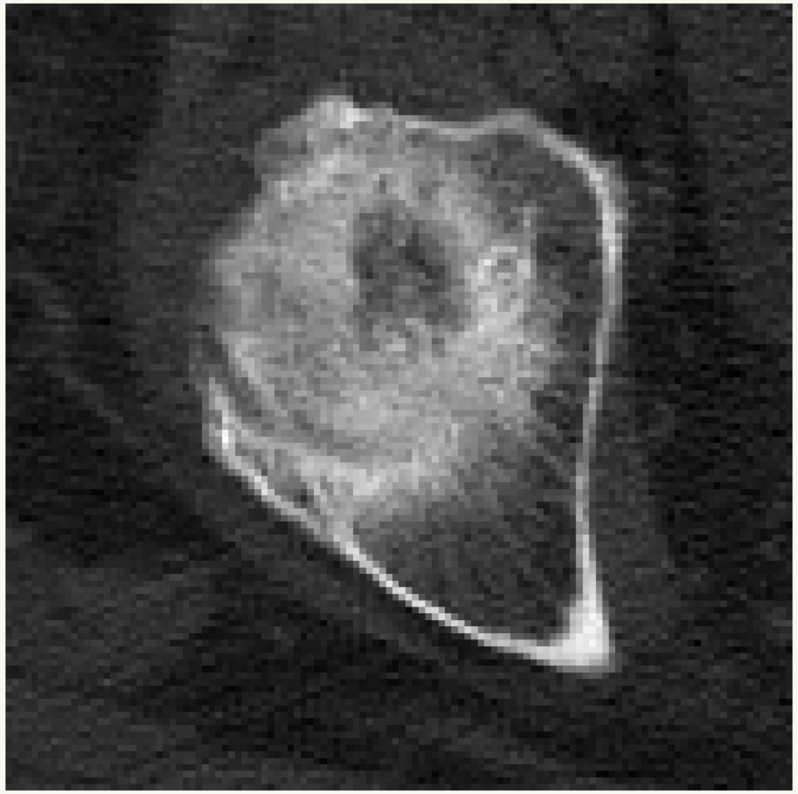
Image Acquisition Modeling

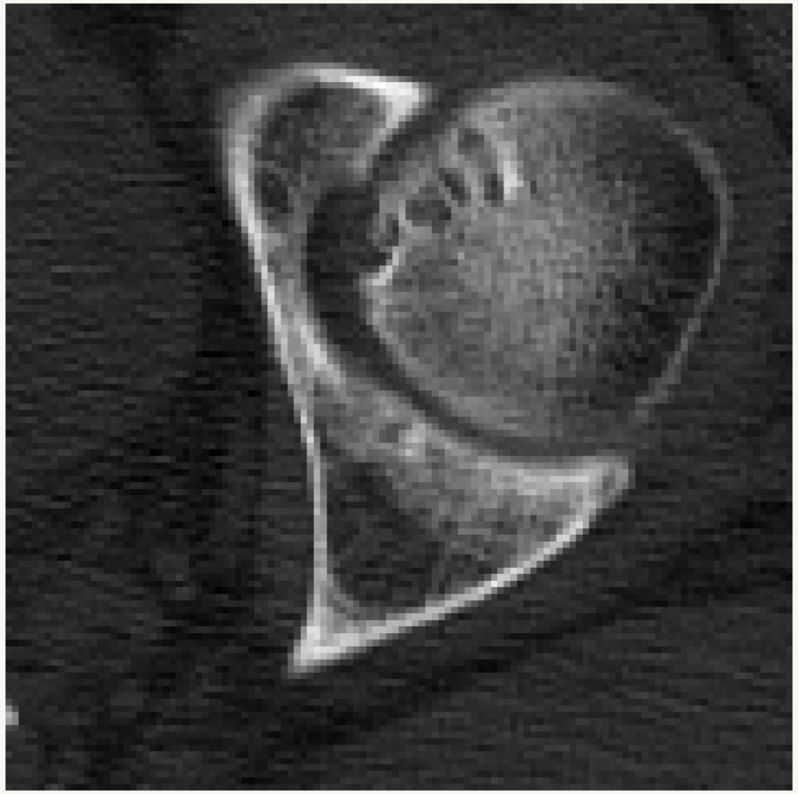
Image Modeling



Define the Study Case

Femur segmentation from CT for risk of fracture computation





Define the Study Case

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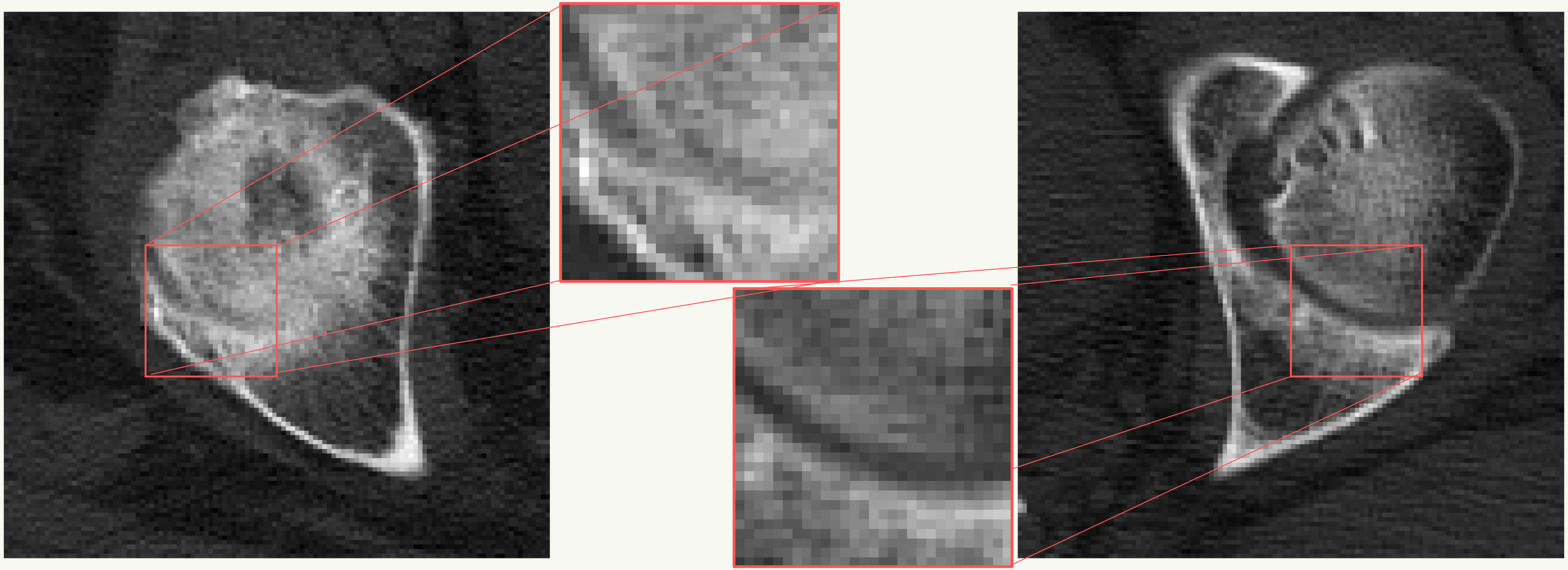
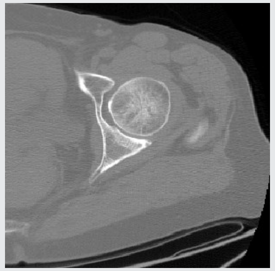


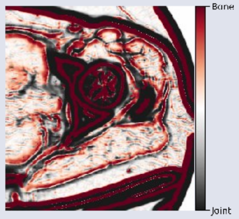
Define the Study Case: The Pipeline

Semi-Automated

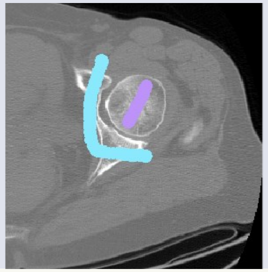
CT



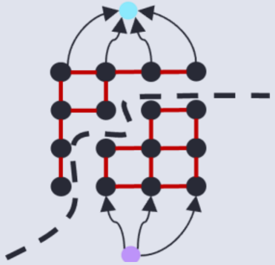
Boneness



Init



Graph-Cut

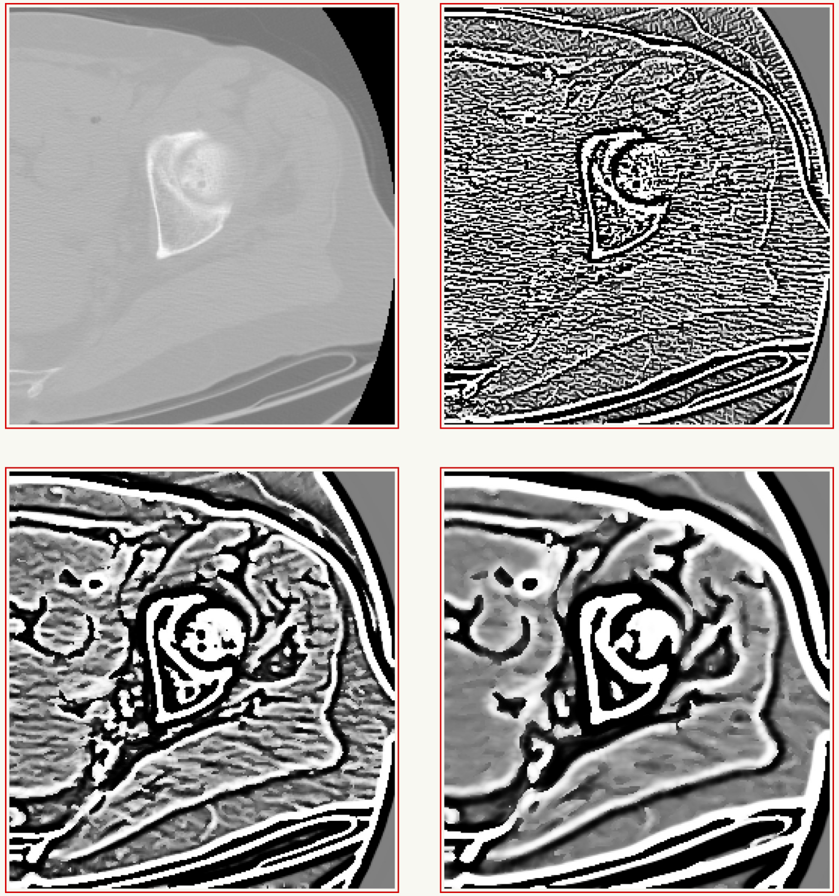


Refine

Validate

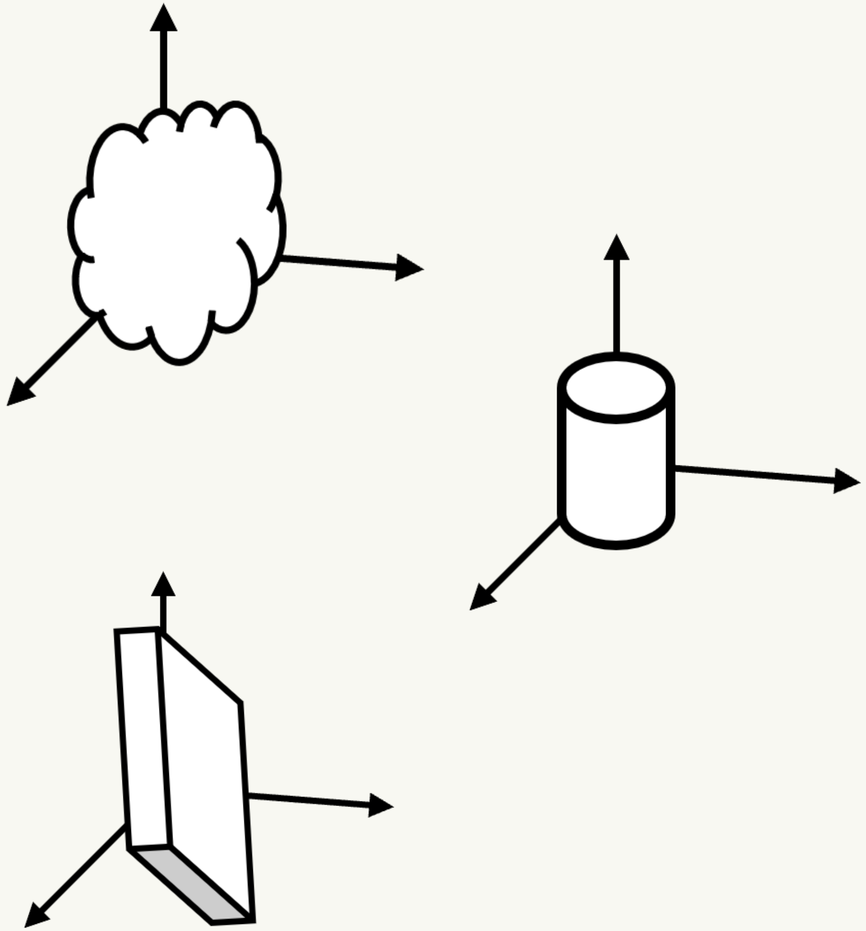
Aldieri, A., Biondi, R., et al. (2024). Development and validation of a semi-automated and unsupervised method for femur segmentation from CT. SCIENTIFIC REPORTS, 14(1), 1-13 [10.1038/s41598-024-57618-6].

Define the Study Case: Joint Enhancement



$$BJE(p) = -sign(\lambda_3)\, exp\left[-\frac{R_{bones}^2}{c}\right]\left(1 - exp\left[-4R_{noise}^2\right]\right)$$

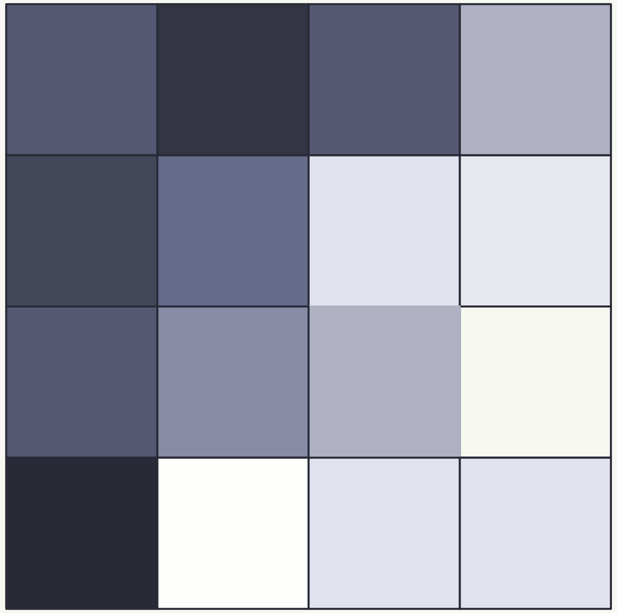
Define the Study Case: Joint-Enhancement



$$|\lambda_3| \geq |\lambda_2| \geq |\lambda_1|$$

	Tube	Sheet	Blob
$R_{bone} = \frac{|\lambda_1 \lambda_2|}{\lambda_3^2}$	0	0	1
$R_{tube} = \frac{|\lambda_1|}{\sqrt{|\lambda_2 \lambda_3|}}$	0	$^1/_{\lambda_3}$	$^1/_{\lambda_3}$
$R_{blob} = \frac{\left|2|\lambda_3| - |\lambda_2| - |\lambda_1|\right|}{|\lambda_3|}$	1	2	0
$R_{noise} = |\lambda_1| + |\lambda_2| + |\lambda_3|$	$2|\lambda_3|$	$|\lambda_3|$	$3|\lambda_3|$

Define the Study Case: Segmentation



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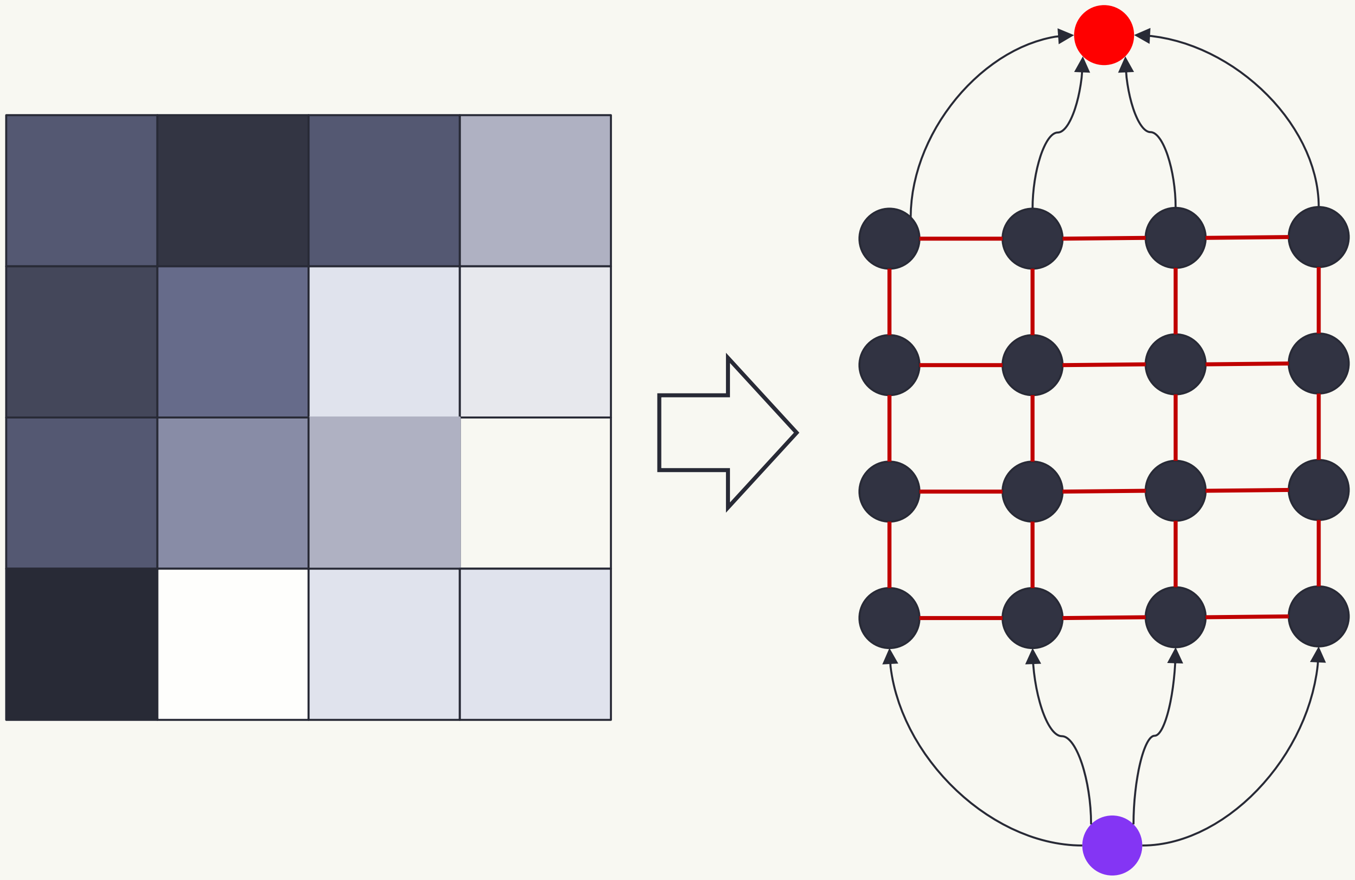

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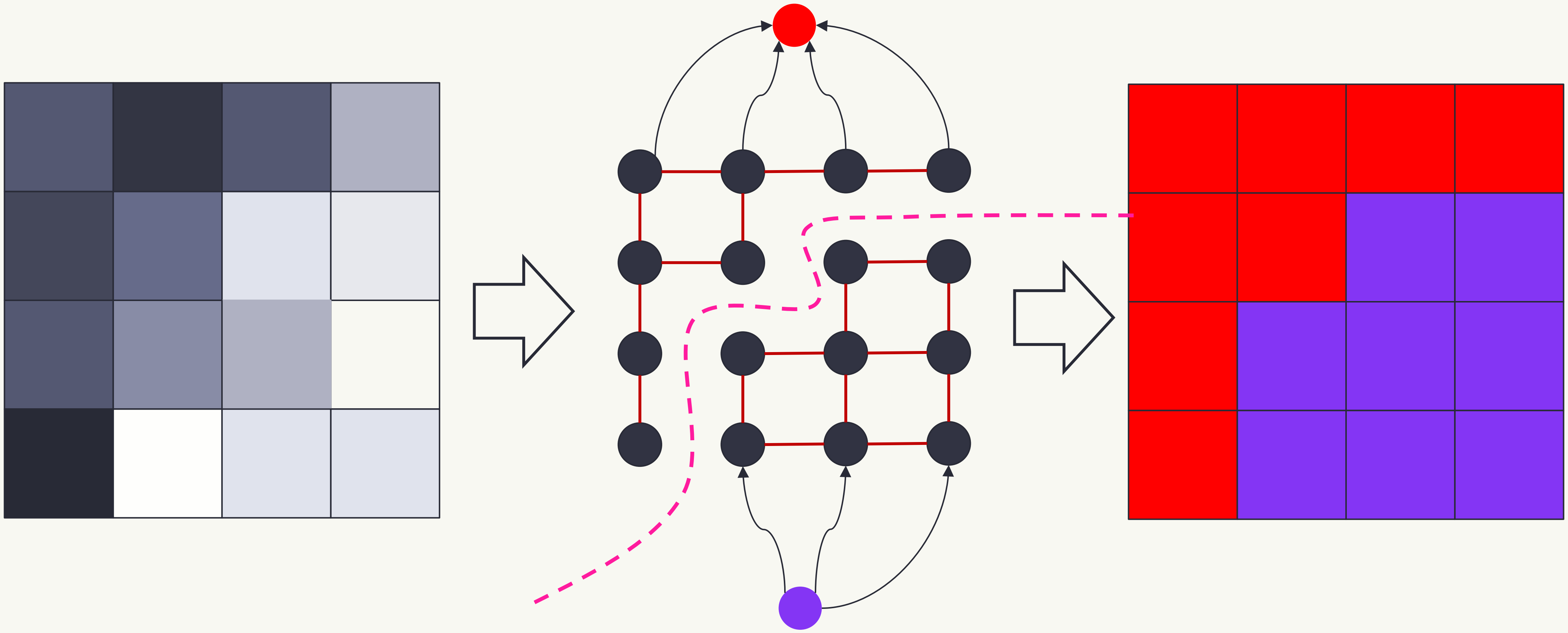
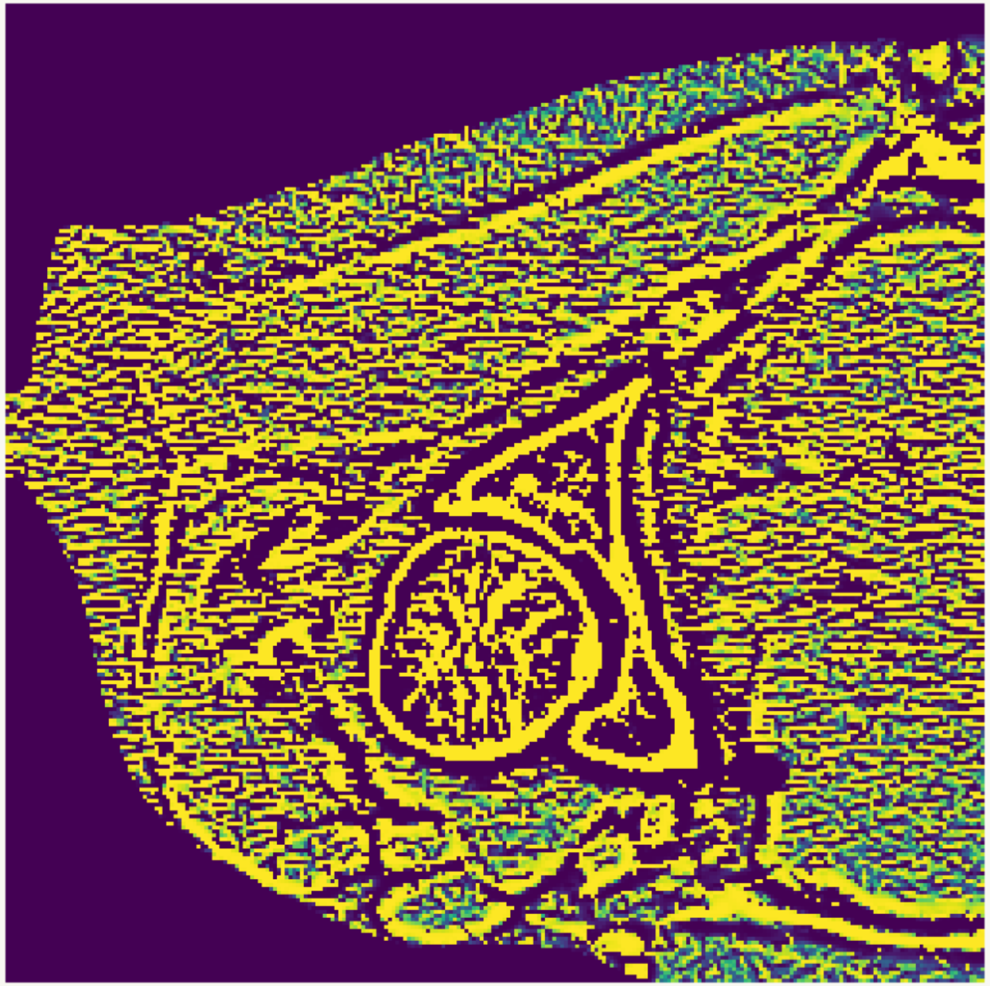


Define the Study Case: Boundary Term



Boundary Term

$$H(\sigma) = B(p,q) - \mu(R_f(p) + R_b(p))$$

$$B(p.q) = \begin{cases} e^{\lambda\frac{(BJE(p)-BJE(q))^2}{2\sigma^2}}, if\ BJE(p) < BJE(q) \\ \lambda, otherwise \end{cases}$$

Allows Spatial Coherence

Define the Study Case: Segmentation

Per-Pixel Term

$$H(\sigma) = B(p,q) - \mu(R_f(p) + R_b(p))$$

$$R_b(p) = \begin{cases} \lambda, if\ p \in bkg \\ 0, if\ p \in frg \\ 1, otherwise \end{cases}$$

$$R_f(p) = \begin{cases} \lambda, if\ p \in frg \\ 1\ if\ p \in bkg \\ 0, otherwise \end{cases}$$

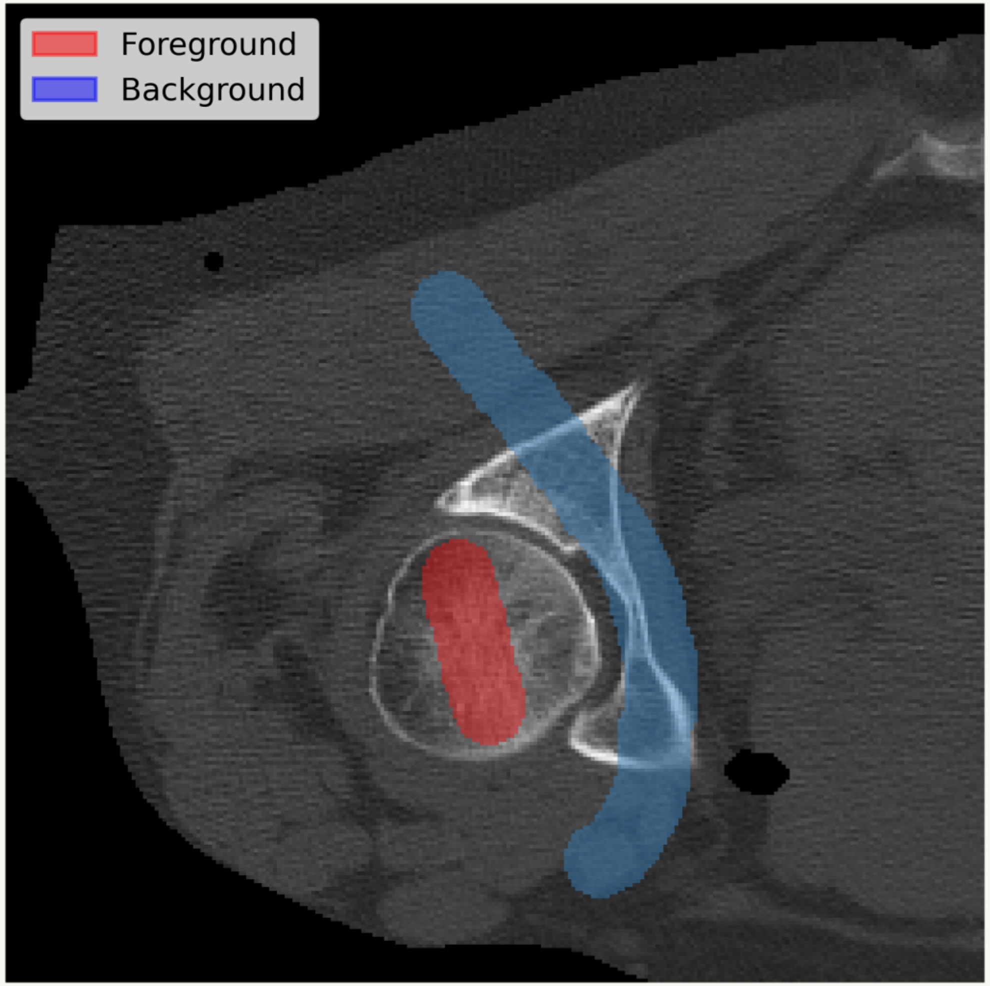
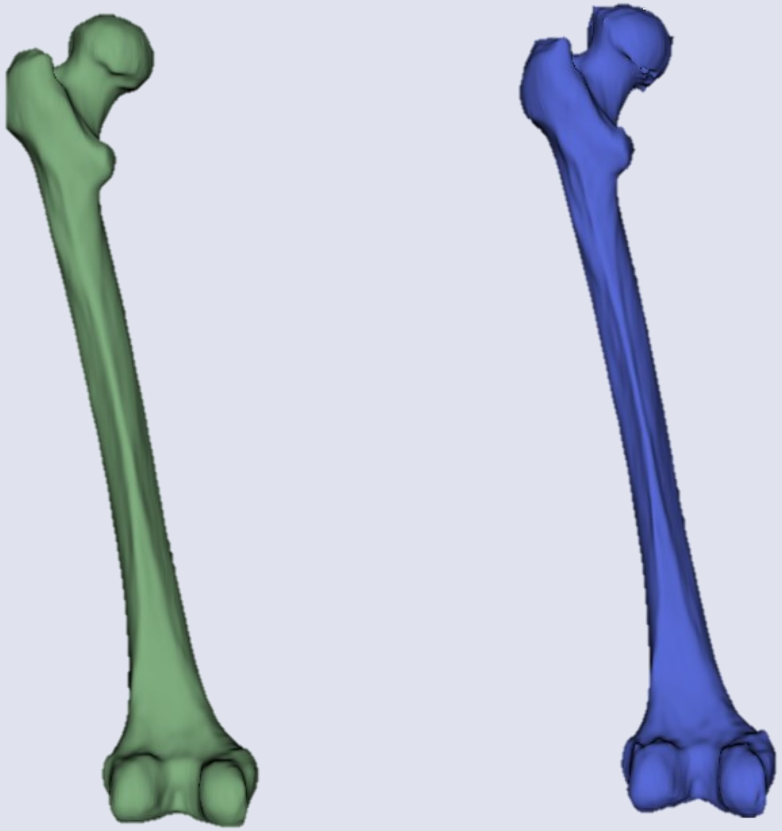
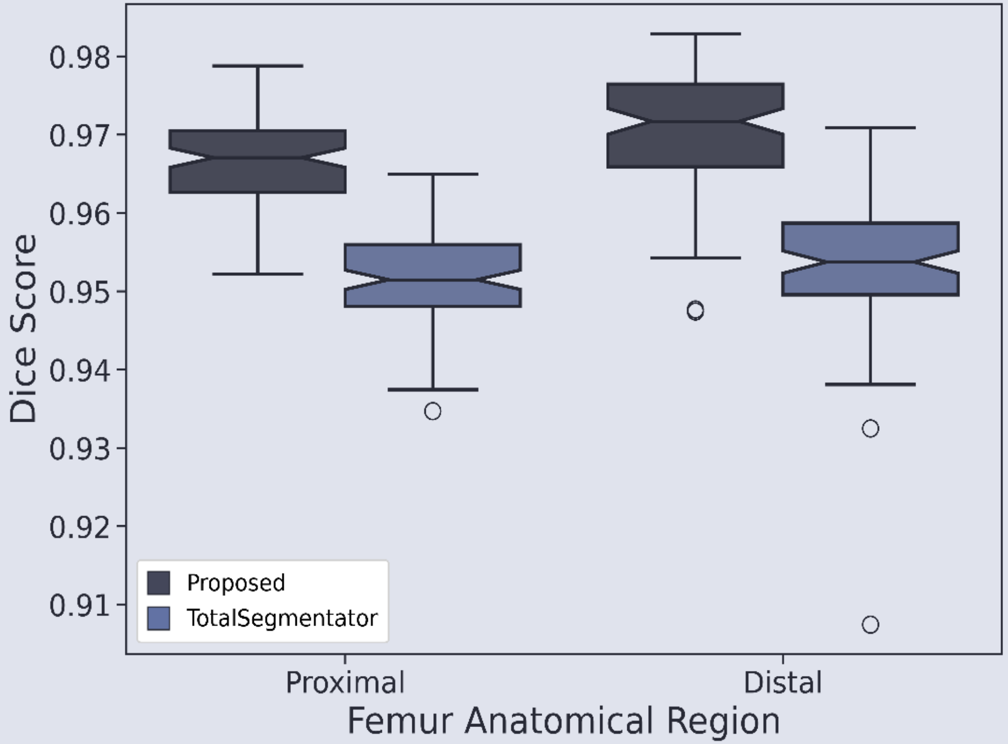


Image Acquisition Modling: Results

Results



Overlapping

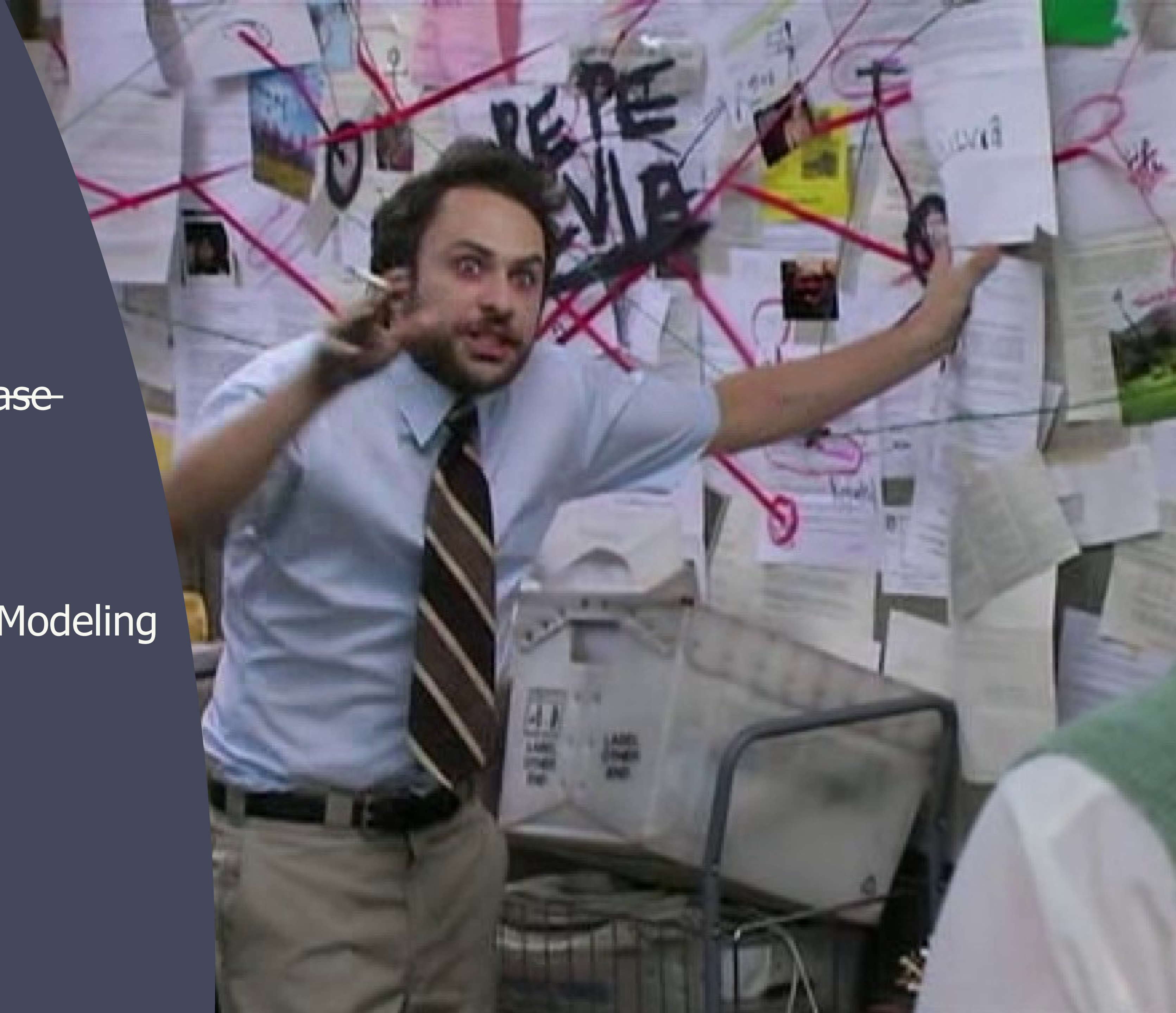


Outline

~~**Define** Real World Case~~

Image Acquisition Modeling

Image Modelling



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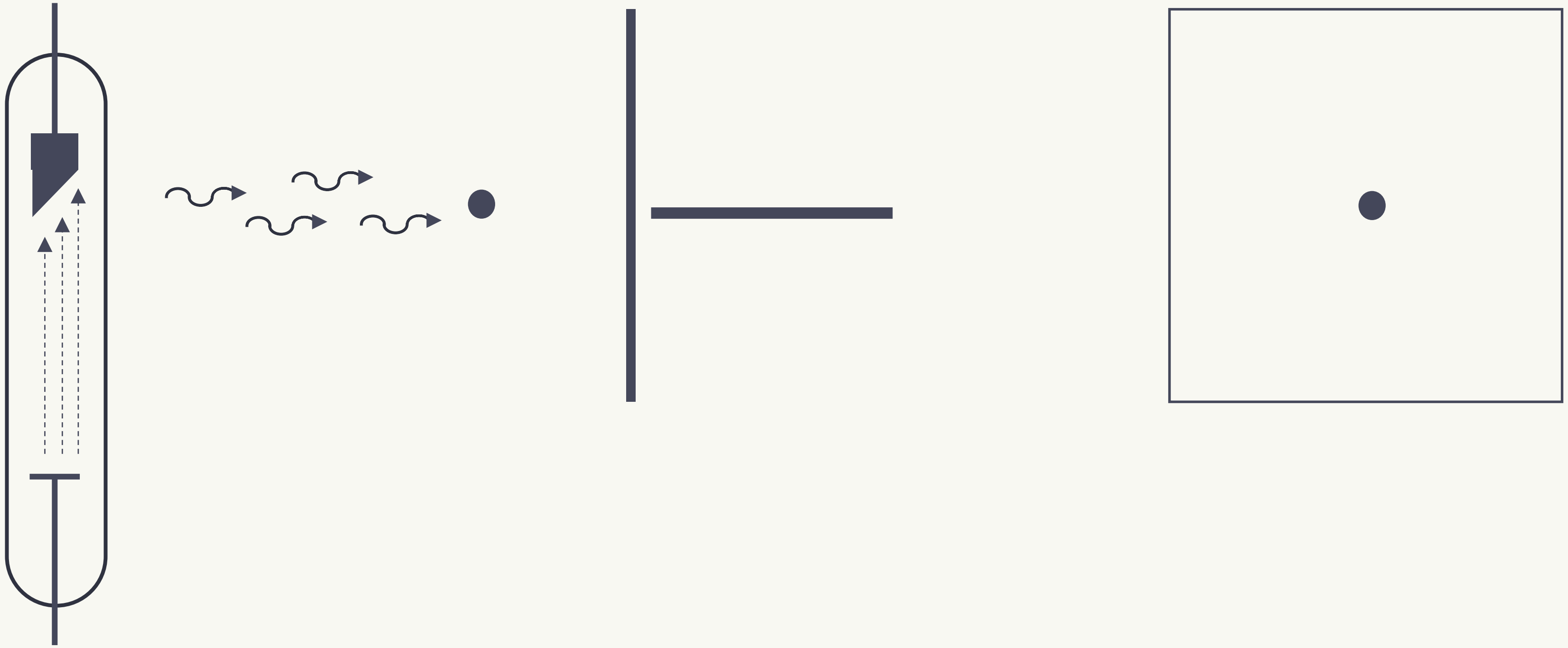

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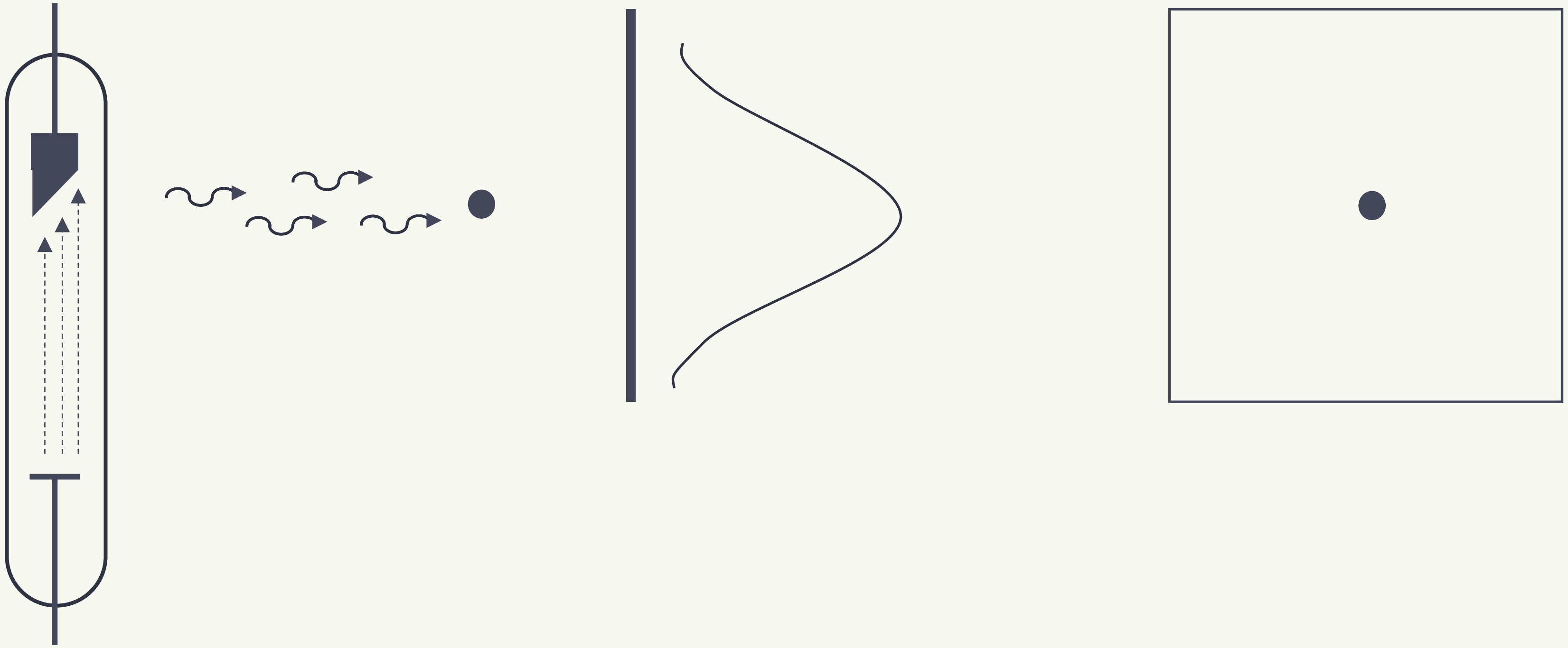

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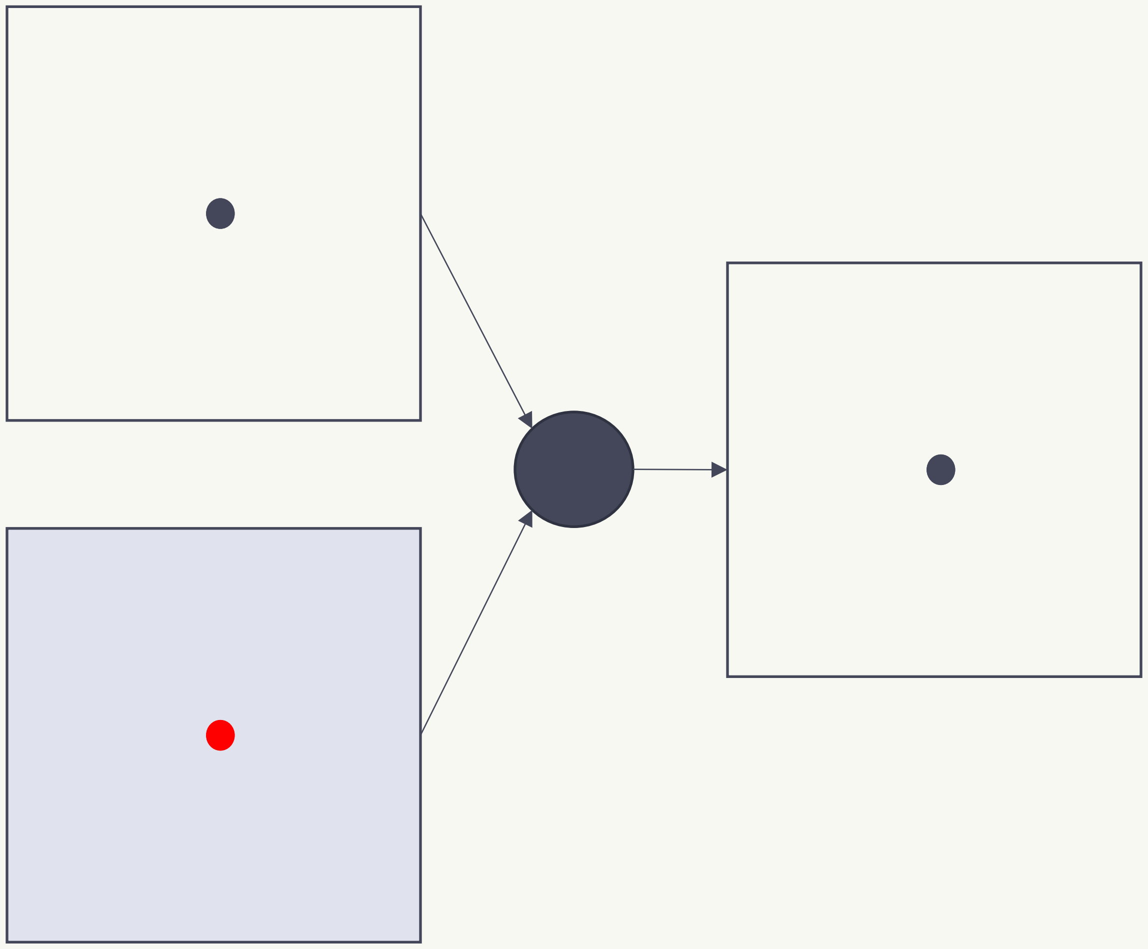


$$p(x) = \ p^t(x) * PSF(x)$$

$$L(x;t) = p(x) * G(x;t)$$

$$L(x_0 + \partial x_0) =$$
$$L(x_0) +$$
$$\partial x_0^T \nabla_{0,t}$$
$$+ \frac{1}{2} \partial x_0^T \mathcal{H}_{0;t} \partial x_0^T + \epsilon(||\partial x_0^3||)$$

Image Acquisition Modeling: Application

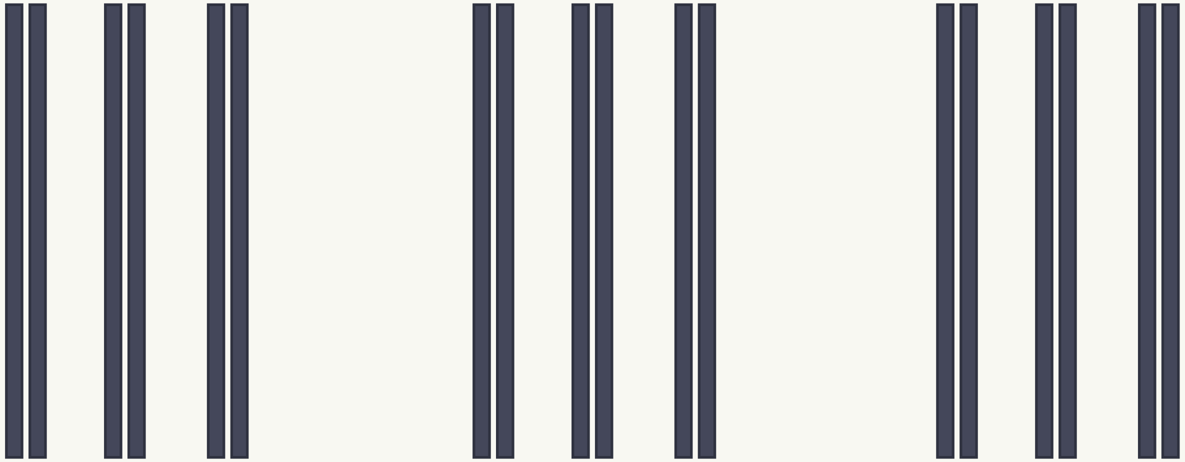


Image Acquisition Modeling: Application

$$L(x;t) = p(x) * G(x;t)$$

t Object Scale

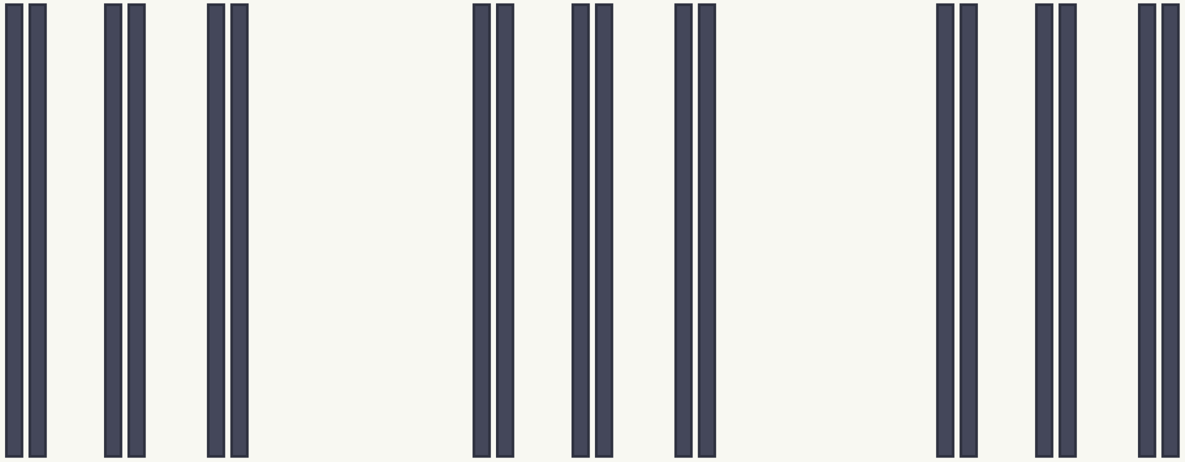
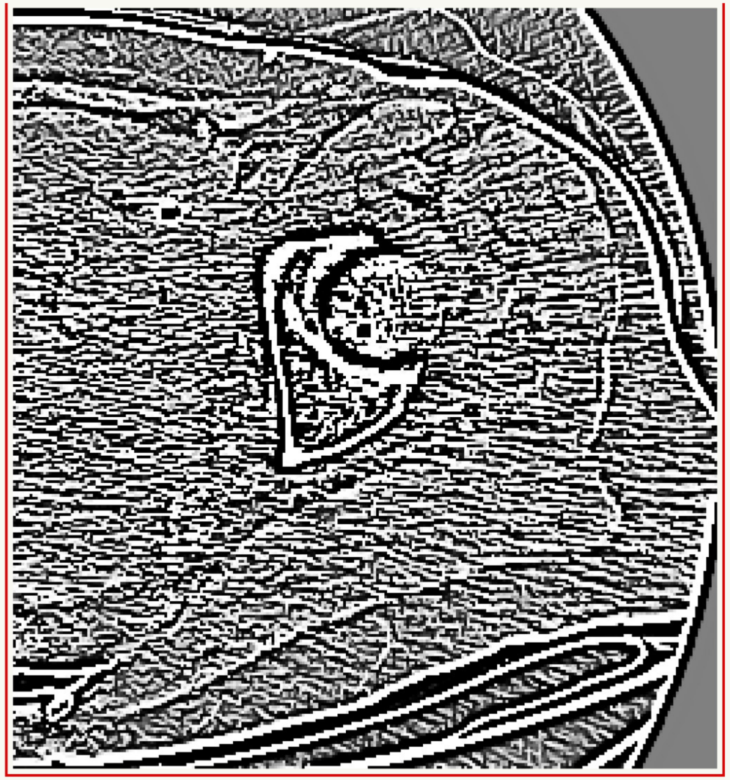


Image Acquisition Modeling: Application

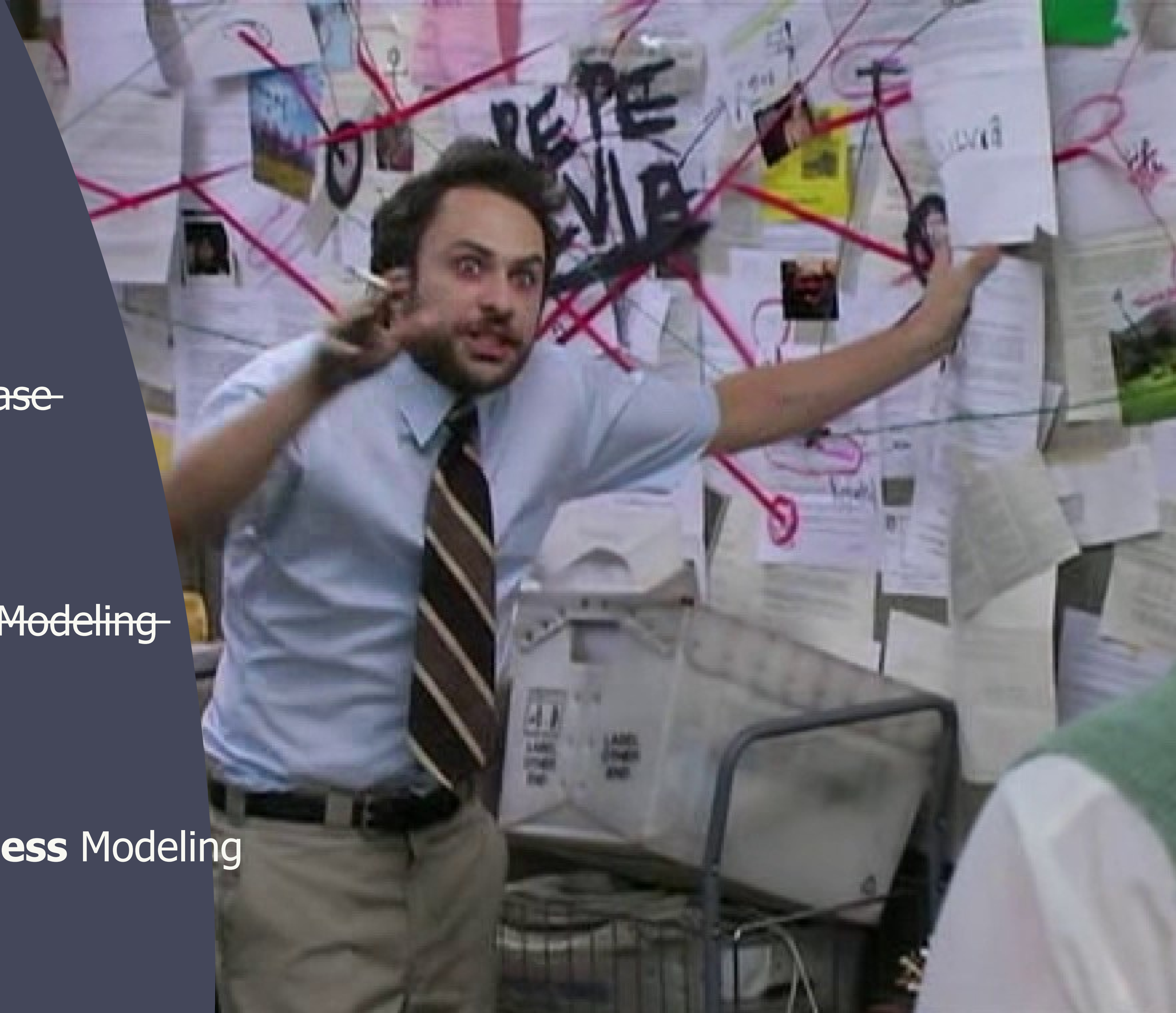


Outline

~~**Define** Real World Case~~

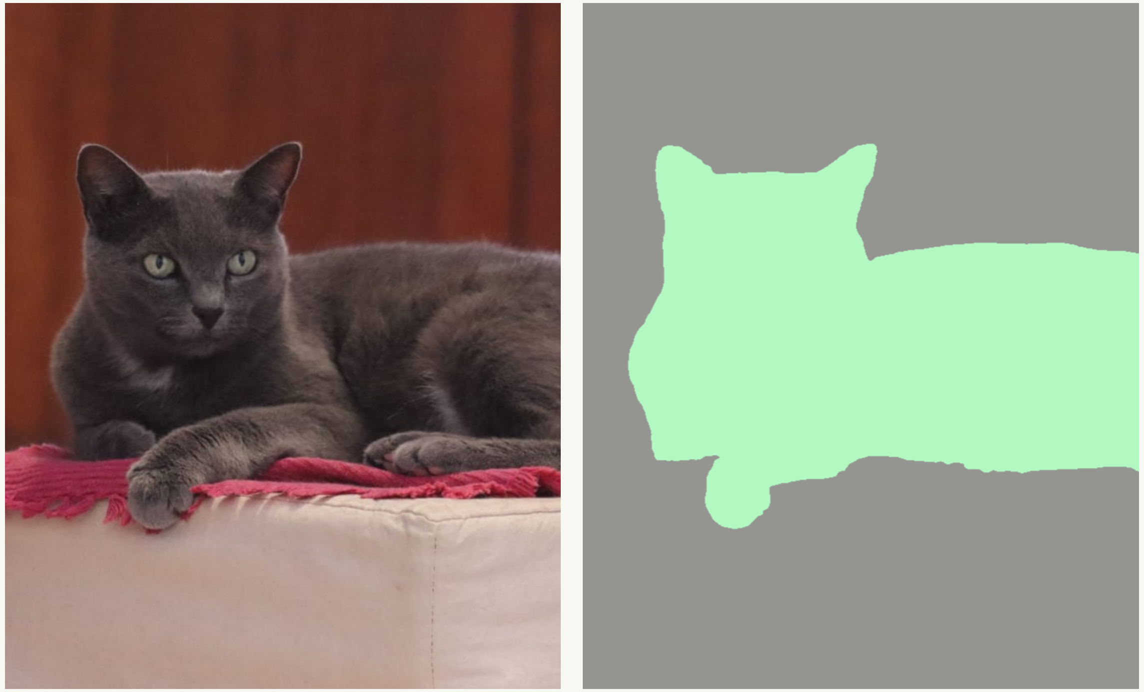
~~**Image Acquisition** Modeling~~

Segmentation Process Modeling



Segmentation Process Modeling

Assing to each voxel a probability to belong to a class. The probability change in time till an equilibrium that is the segmentation



- Each pixel has a probability to belong to one of two classes
- The probability evolves till equilibrium, That is the segmentation
- This process could be described by Fokker Plank Equation

Segmentation Process Modeling

Fokker-Plank Equation

$$\frac{\partial p}{\partial t} = -\nabla(p \times B) + \frac{1}{2}\nabla[D\nabla p]$$

- $p(x,t)$ pdf for voxel x at time t
- B(x,t) drift vector, representing deterministic movements (its a force)
- $D(x,t)$ diffusion tensor (representing diffusion rates)

Segmentation Process Modeling

Fokker-Plank Equation

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- $p(x,t)$ pdf for voxel x at time t
- B(x,t) drift vector, representing deterministic movements (its a force)
- $D(x,t)$ diffusion tensor (representing diffusion rates)

- We are interested in the stationary state
- Our potential will be not time-dependent
- The drift vector read as $\mathrm{B(x)} = -\nabla V(x)$
- And considering $D(x,t) = D$ and writing $\frac{1}{T} = \frac{D}{2}$

FP Equation Form

$$0 = -\nabla(p \times B) + \frac{1}{2}\nabla[D\nabla p]$$

$$0 = -\nabla(p \times B) + \frac{1}{2}\nabla[D\nabla p]$$

$$\frac{1}{T}\nabla V(x) = \frac{1}{p^2}\nabla p^2 = \nabla \ln(p^2)$$

Segmentation Process Modeling

Maxwell-Boltzmann

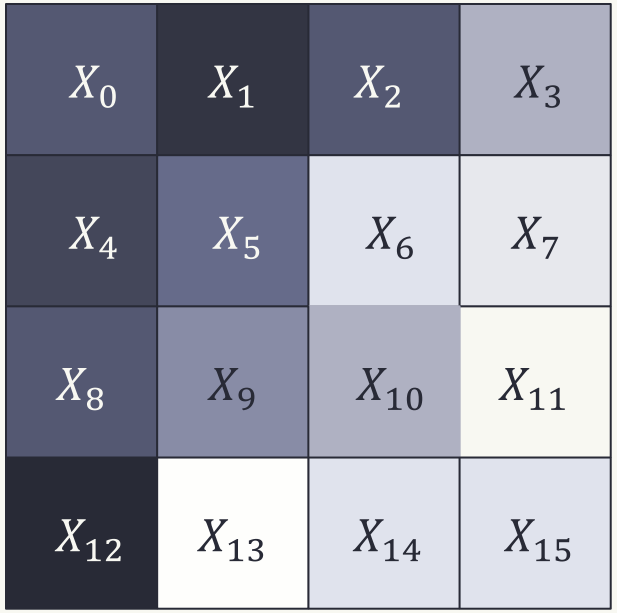
$$\frac{1}{T}\nabla V(x) = \frac{1}{p^2}\nabla p^2 = \nabla \ln(p^2)$$

$$p^2(x) \propto \exp[-\frac{V(x)}{T}]$$

FP to MB Equation

- The potential could be unknown for the specific task, and learned
- The potential could be computed for each specific task and image modelling the image, eg. As a lattice, using models as the Ising or Potts model

Image Modeling



- $\boldsymbol{X} = \{X_0, \dots, X_{15}\}$
- $\mathrm{P}(X_0, \dots, X_{15}) = f_c^{\theta}(\boldsymbol{X})$
- $\mathrm{P}(\sigma|\theta) = \frac{1}{Z(\theta)} e^{-\beta V(\sigma,\theta)}$

Image Modeling: Graph Approach

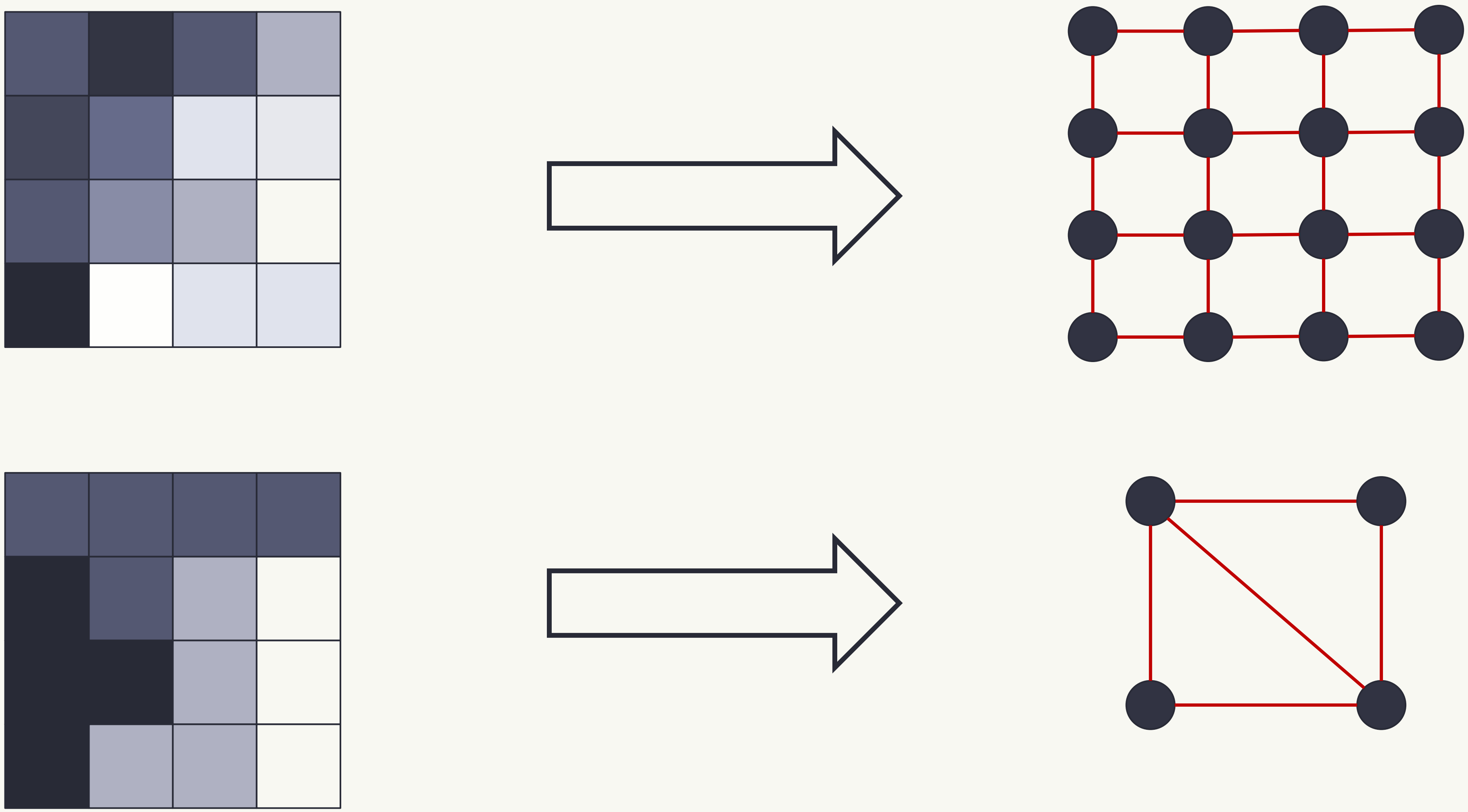
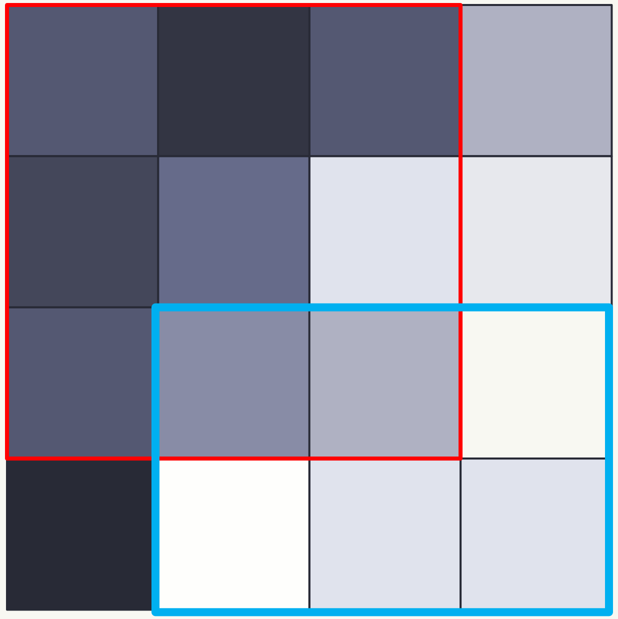
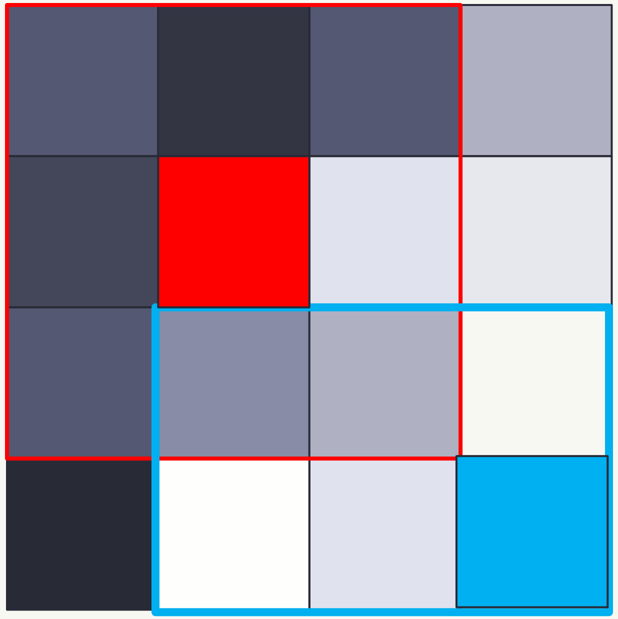


Image Modeling: Markow Random Field



- **Pairwise Markov Property** Any two non-adjacent variables are conditionally independent given all other variables
- **Local Markov Property** A variable is conditionally independent of all other variables given its neighbors
- **Global Markov Property** Any two subsets of variables are conditionally independent given a separating subset

Image Modeling: Markow Random Field



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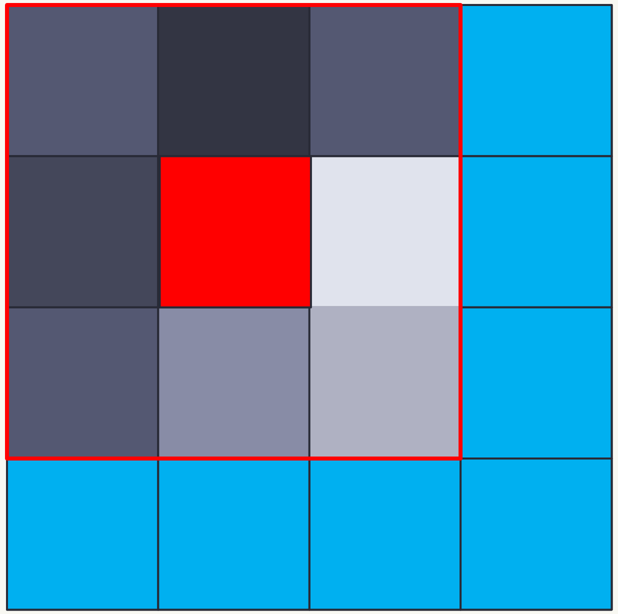

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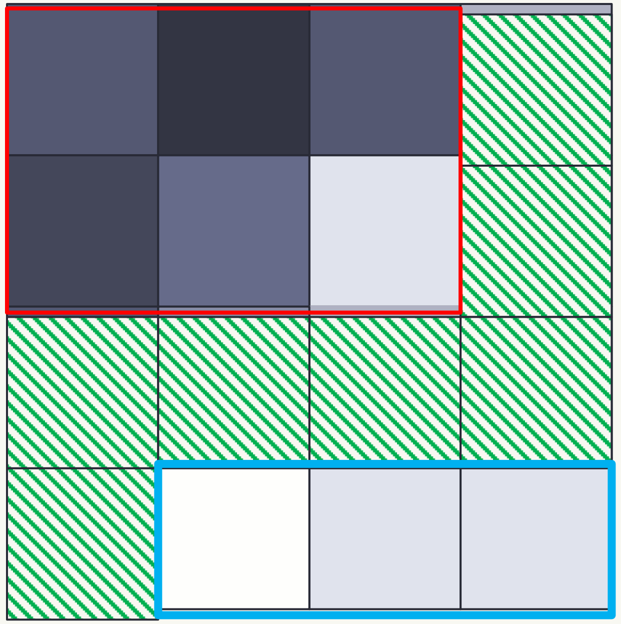

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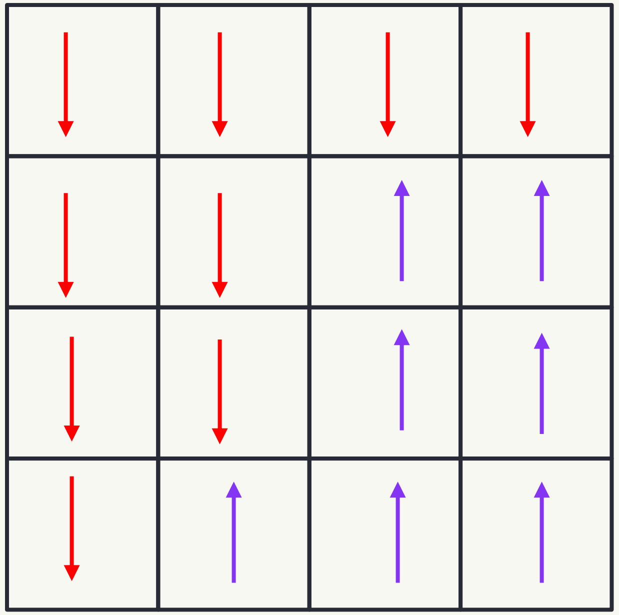
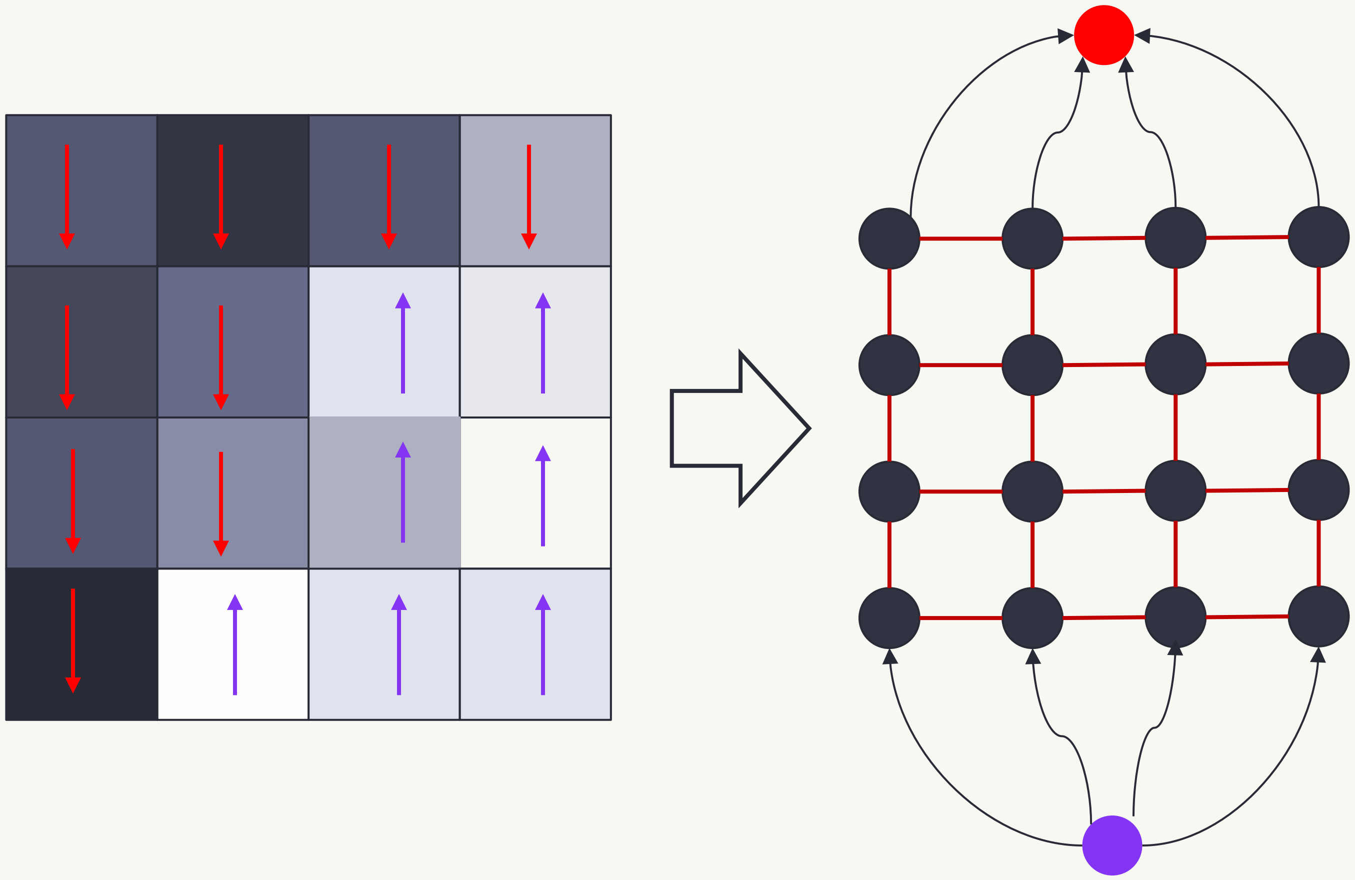
Image Modeling: Ising Model



Describe Spin distribution in a Lattice. Spin could have two states: Up and Down

$$V(\sigma) = -\sum_{<i,j>} J_{i,j}\sigma_i\sigma_j - \mu\sum_j h_j\sigma_j$$

Image Modeling: Ising Model

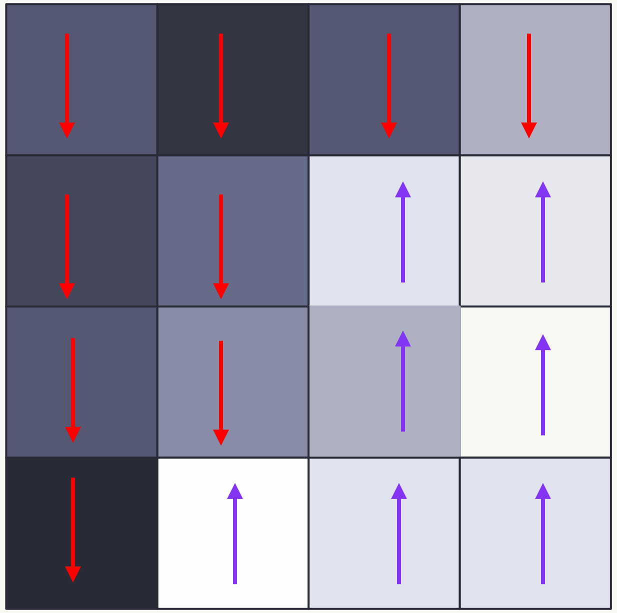


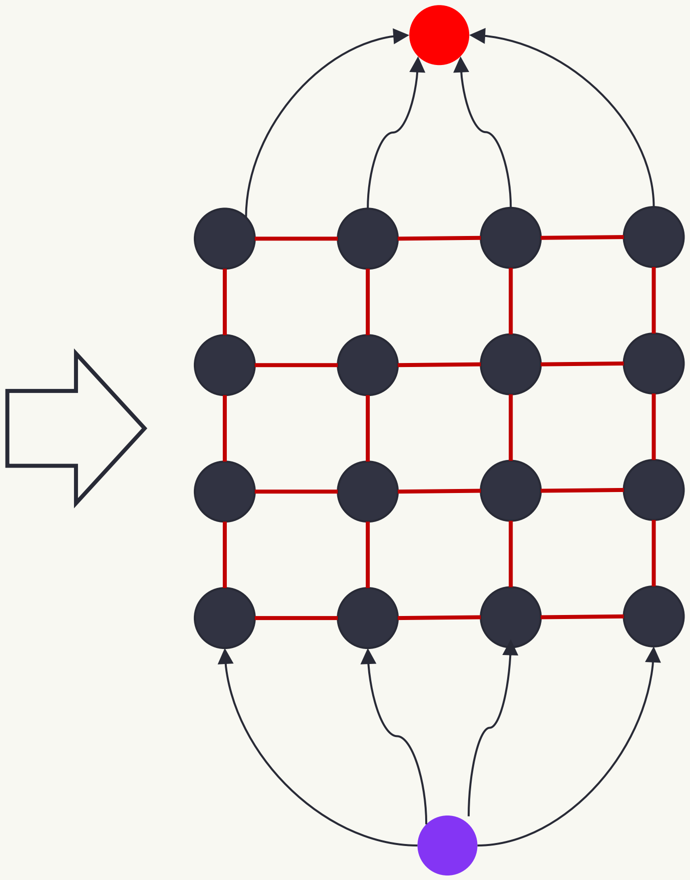
$$V(\sigma) = B(p,q) - \mu(R_f(p) + R_b(p))$$

Find the configuration that minimize the Energy

Y. Boykov, O. Veksler, and R. Zabih. Fast approximate energy minimization via graph cuts. IEEE Transactions on Pattern Analysis and Machine Intelligence, 23(11):1222–1239, November 2001.

Image Modeling: Ising Model





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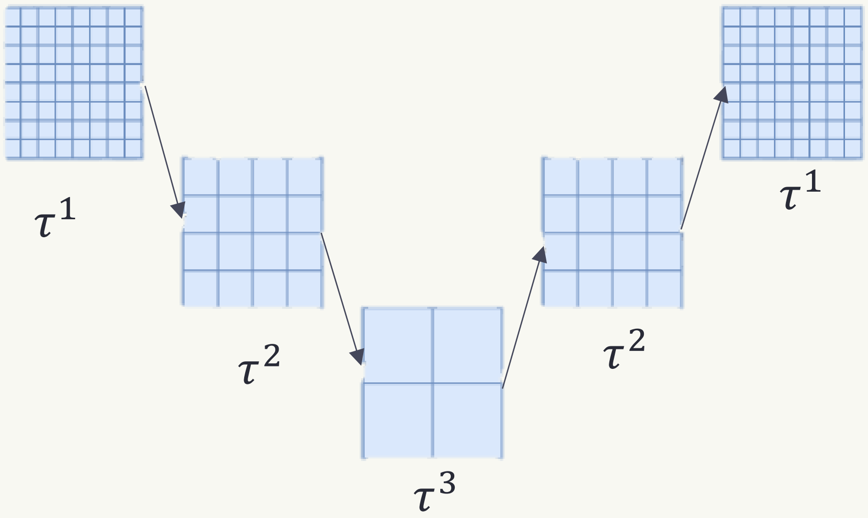
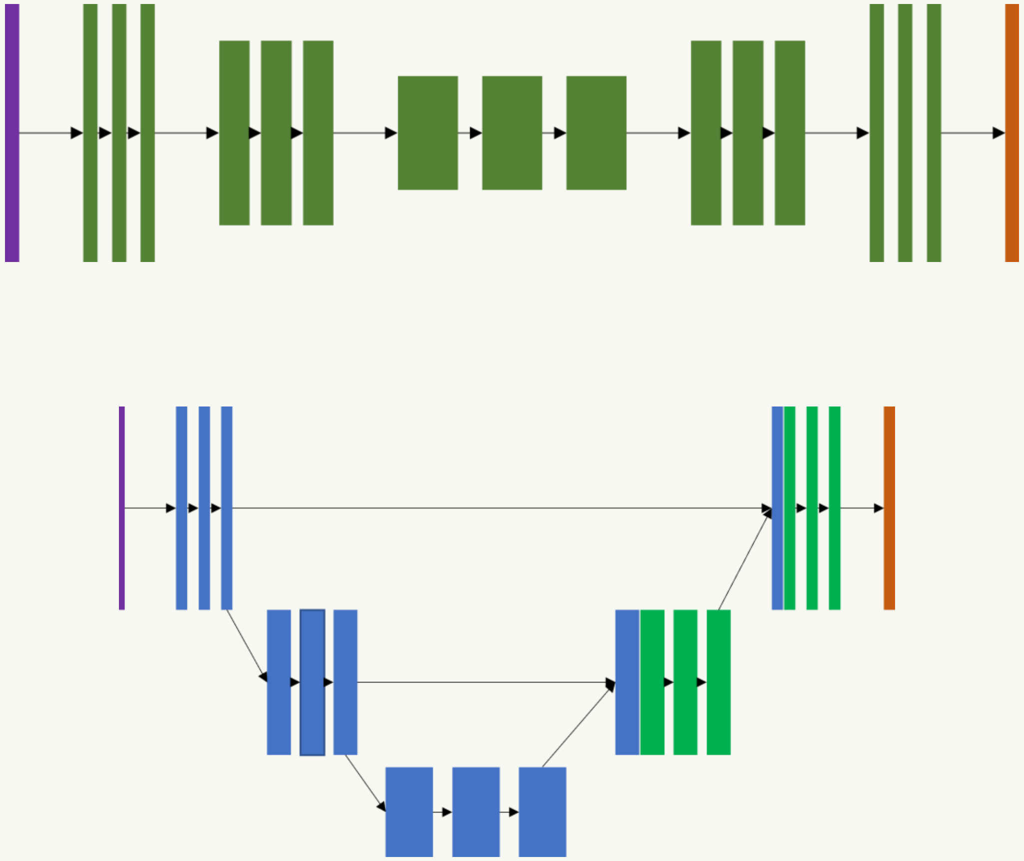
Neural Network derivation





Xue-Cheng Tai, Hao Liu, and Raymond Chan. PottsMGNet: A Mathematical Explanation of Encoder-Decoder Based Neural Networks, September 2023. arXiv:2307.09039

Conclusion

Take Home

Many method works well also out of the box, however sometimes it is worth to step back and try to understand why they work

Get in Touch

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