ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

Summer School on Physical Sensing & Processing
VII Edition, 7-11 July 2025

PHYSICS FOR A BETTER PLANET - II EDITION

EXPLORING THE SUBSURFACE (WITHOUT A SHOVEL…): SEISMIC REFRACTION

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PROGRAM OF THE ACTIVITIES

WEDNESDAY, 9th JULY

10:30 – ca. 11:00 (Room BP-2B) **LECTURE**

- Geophysics: a brief overview
- Seismic waves and interaction with a discontinuity
- Seismic prospection: refraction

ca. 11:00 – ca. 11:30 (outside BP6/2) **EXAMPLE OF GEOPHYSICAL SURVEY**

14:00 – 16:00 (Room BP-2B) **LABORATORY**

- Refraction: data analysis

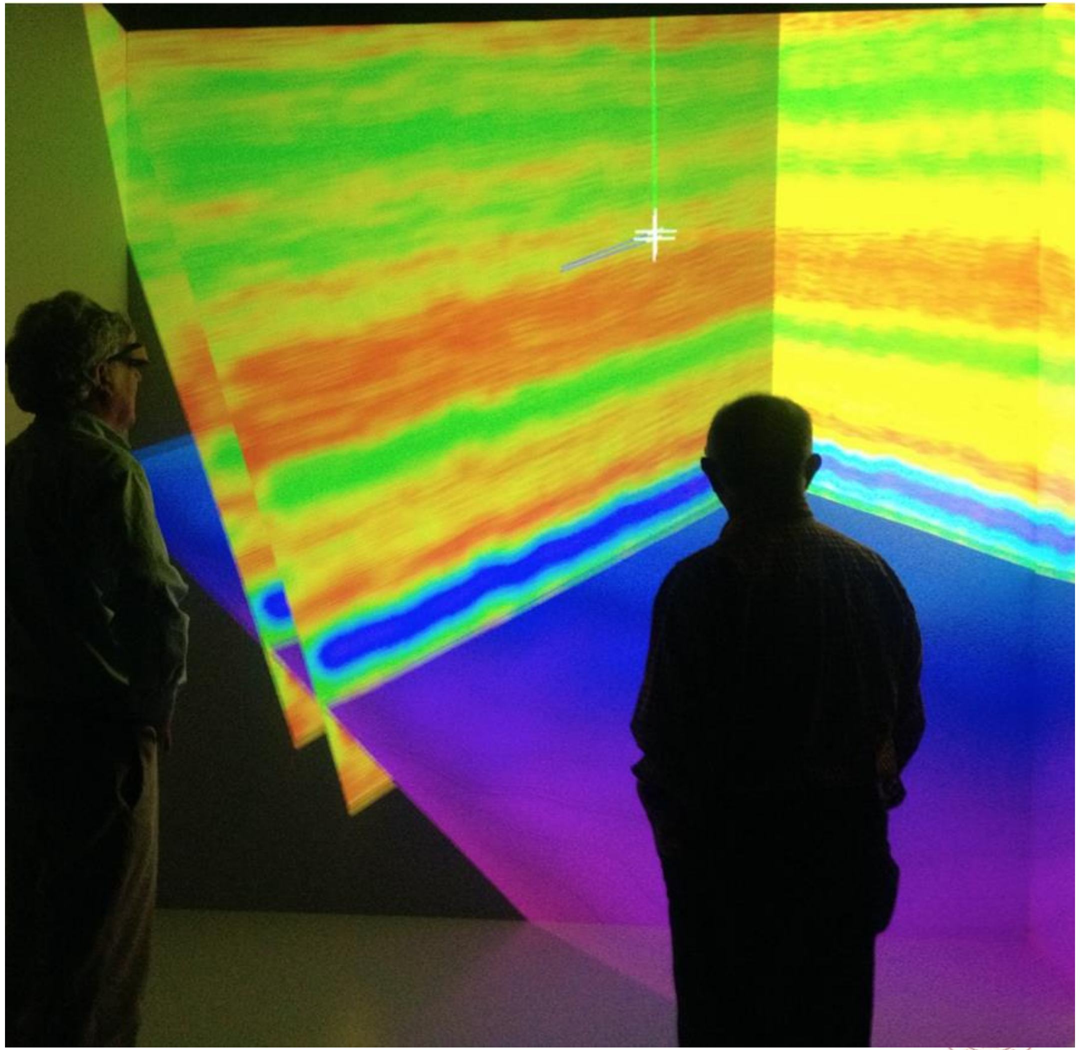
GEOPHYSICS: A BRIEF OVERVIEW

GEOPHYSICS: A BRIEF OVERVIEW

Geophysics is the study of the **PHYSICS OF THE EARTH'S INTERIOR** using both physical measurements and mathematical models.

GEOPHYSICAL MEASUREMENTS ARE (almost) ALWAYS INDIRECT: the phenomena are measured from the Earth surface (or above it…).

Data elaboration and analysis provide 2D/3D maps of the subsurface.

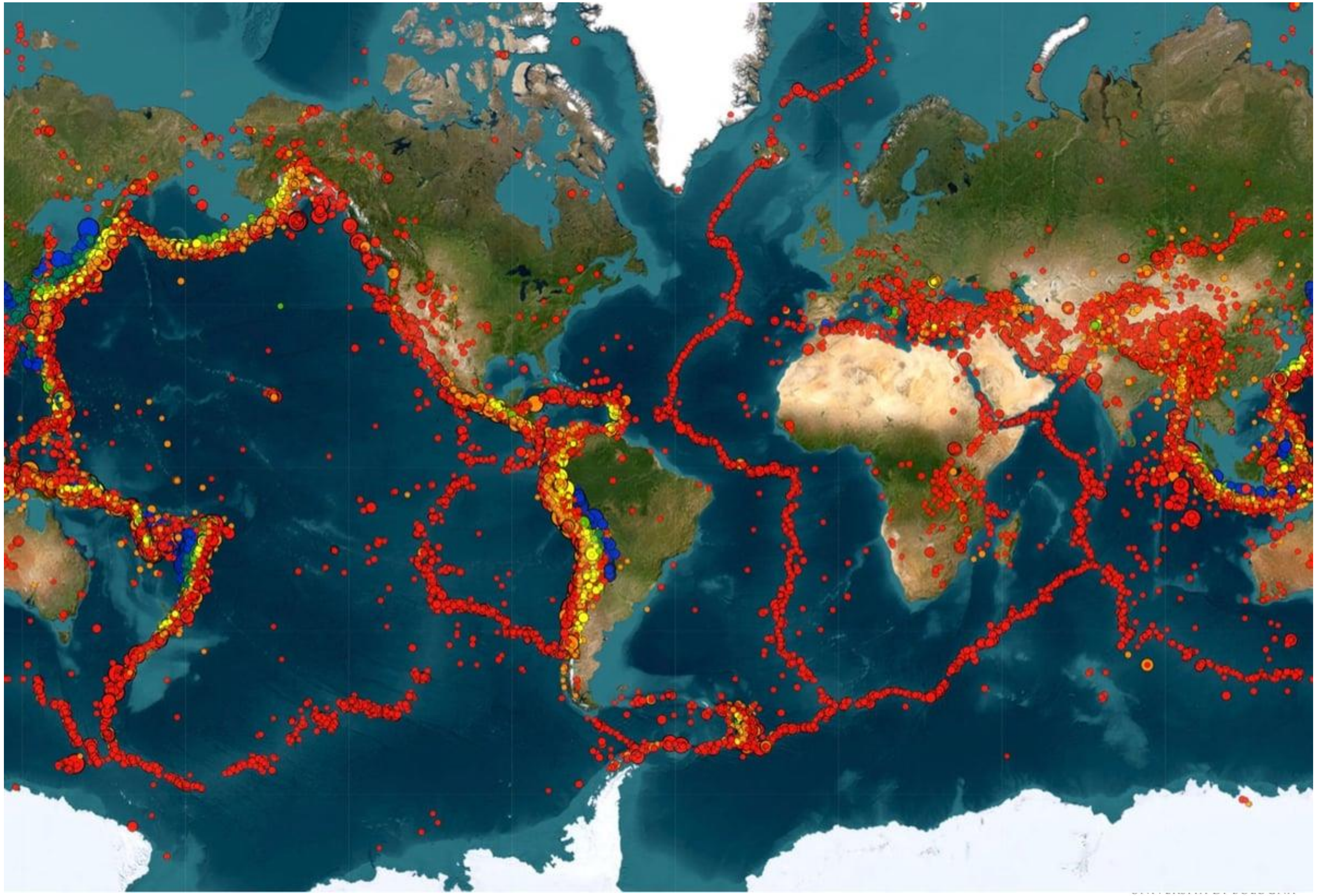


GEOPHYSICS: A BRIEF OVERVIEW

SEISMOLOGY

Study of the EARTHQUAKES

- Causes
- Wave propagation
- Effects
- Event statistics

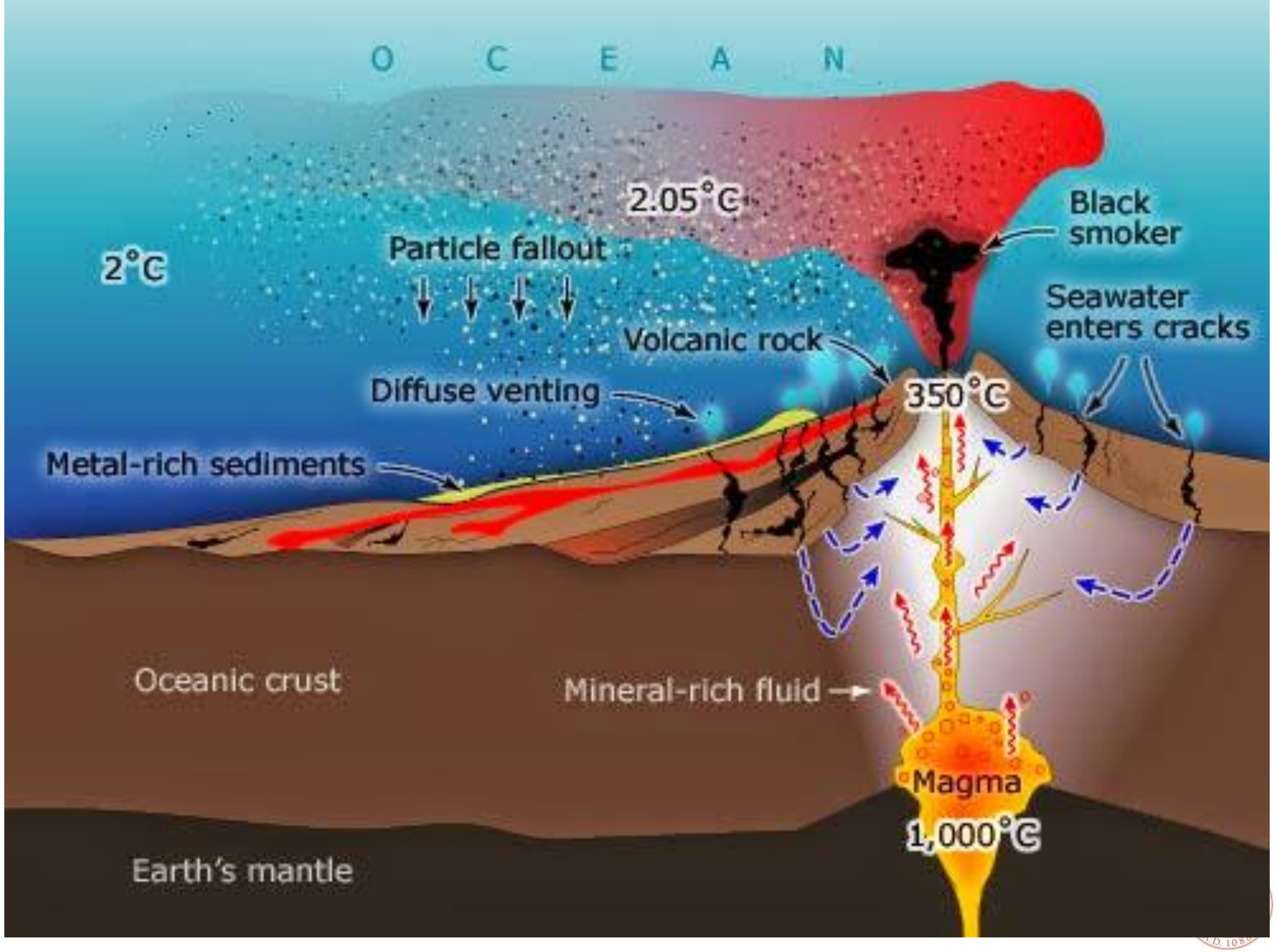


GEOPHYSICS: A BRIEF OVERVIEW

VOLCANOLOGY

Study of VOLCANOES:

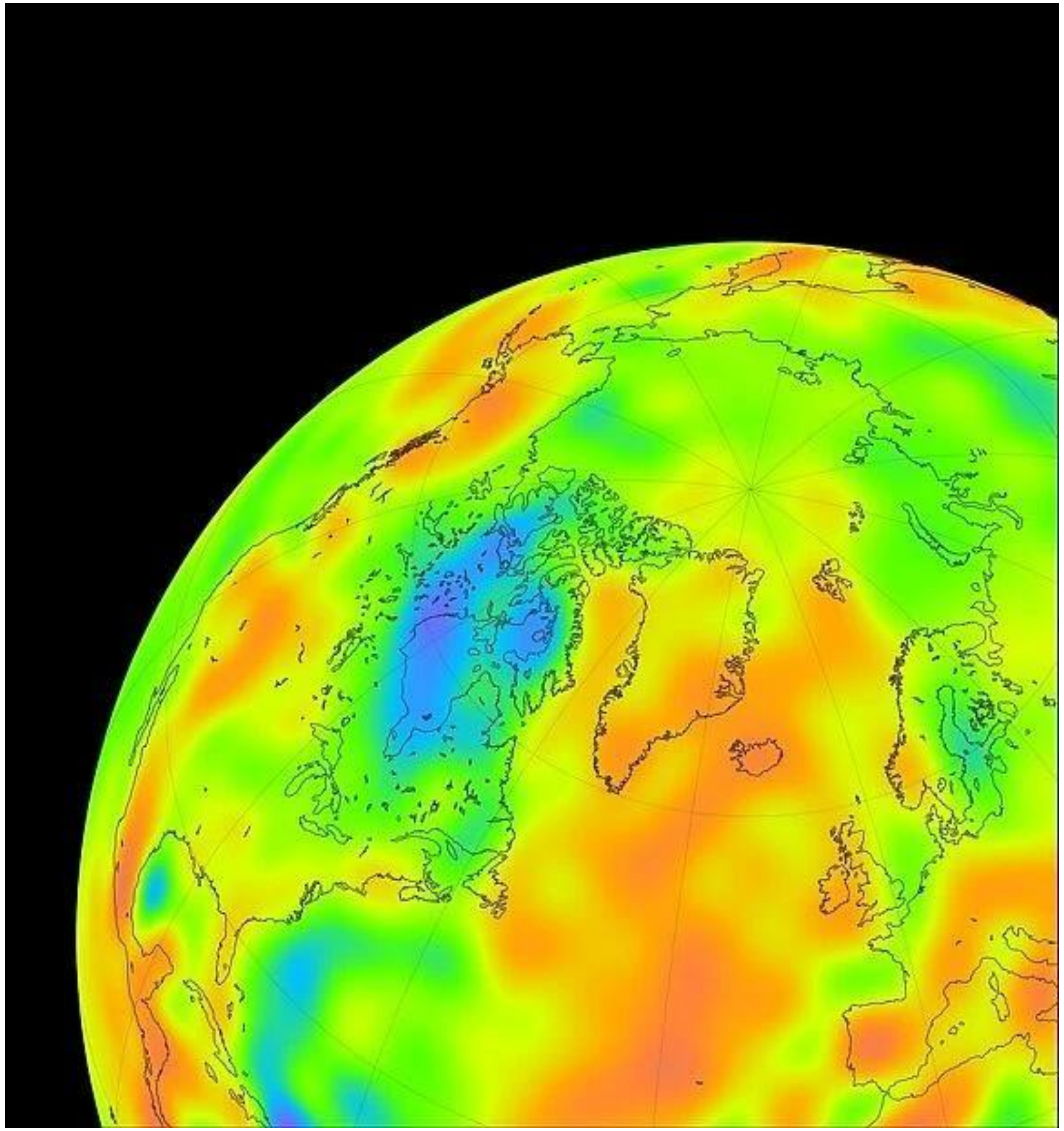
- Formation
- Structure
- Eruption processes
- Materials produced (lava, ash, gases)



GEOPHYSICS: A BRIEF OVERVIEW

GRAVIMETRY

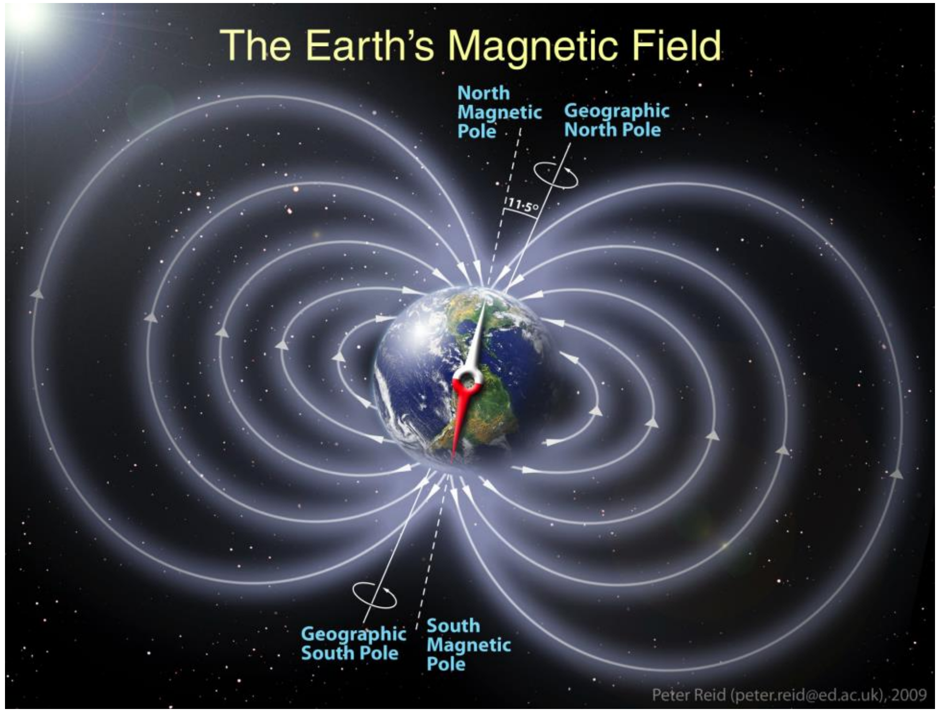
Measuring the ANOMALIES IN THE GRAVITY field.

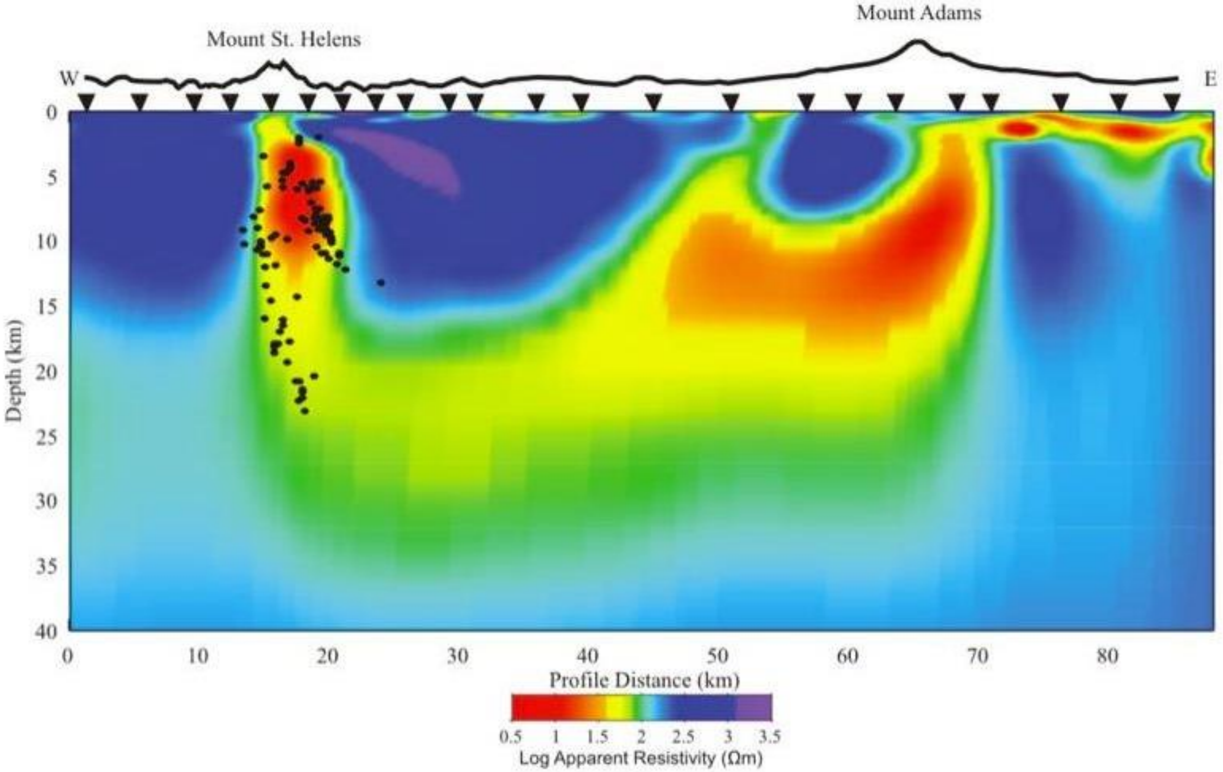


GEOPHYSICS: A BRIEF OVERVIEW

MAGNETOMETRY

Measuring PERTURBATIONS in the ambient magnetic field caused by contrasts in magnetic susceptibility

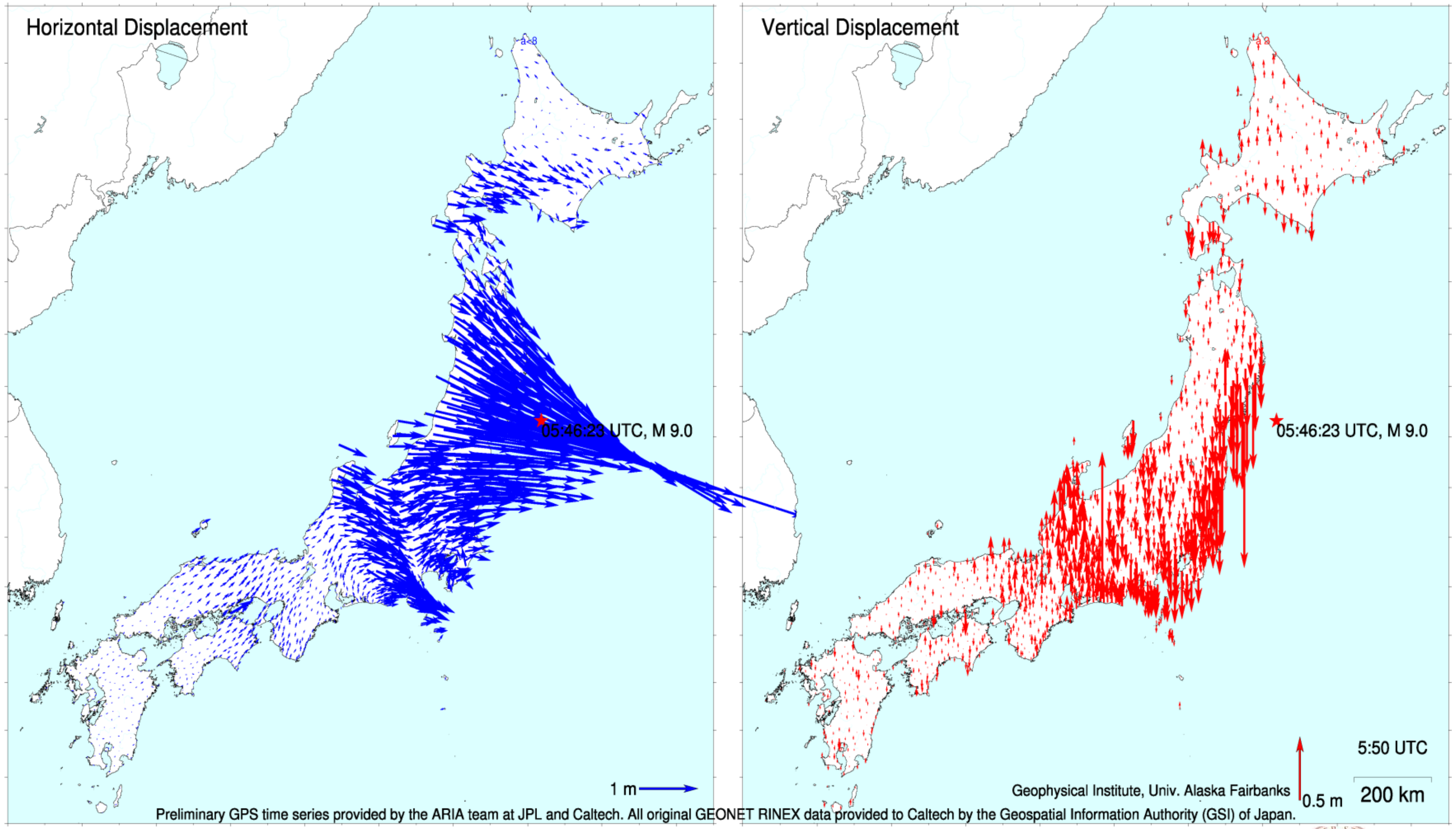




GEOPHYSICS: A BRIEF OVERVIEW

GEODESY

GPS measures for SURFACE DEFORMATION (displacement, velocity, acceleration).



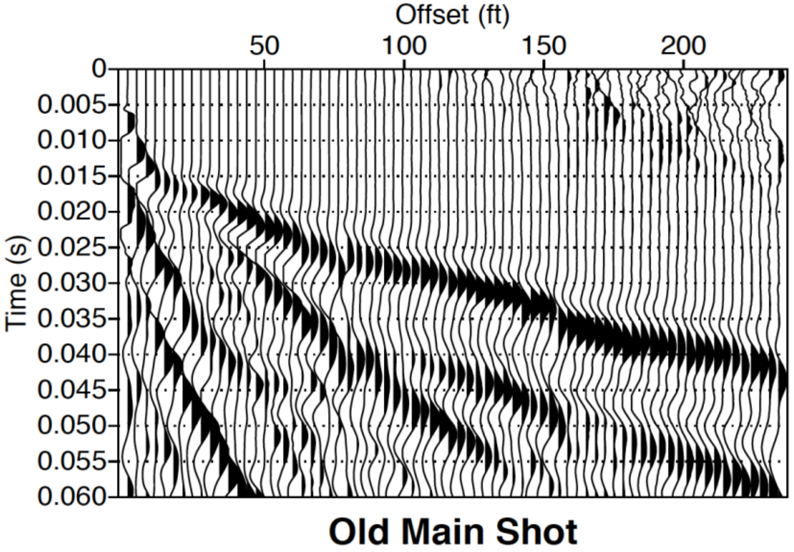
GEOPHYSICS: A BRIEF OVERVIEW

GEOPHYSICAL EXPLORATION

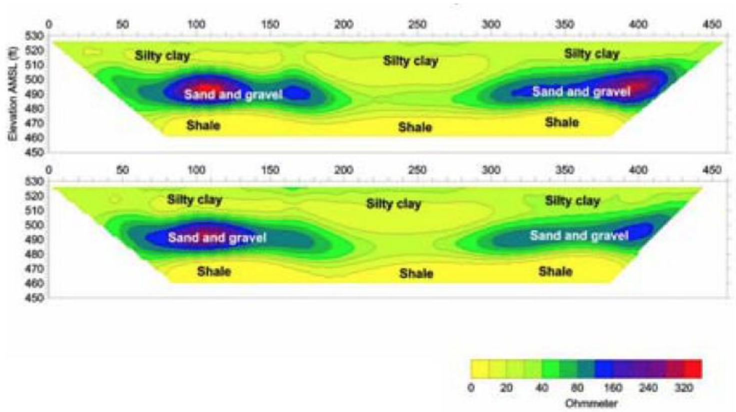
Different techniques to investigate the subsurface:

- Seismic waves
- Electrical resistivity

REFLECTION & REFRACTION SEISMOLOGY



GEOELECTRIC PROSPECTING



GEOPHYSICS: A BRIEF OVERVIEW

GEOPHYSICAL EXPLORATION: APPLICATIONS TO DAILY LIFE

Plan for a building foundation excavation

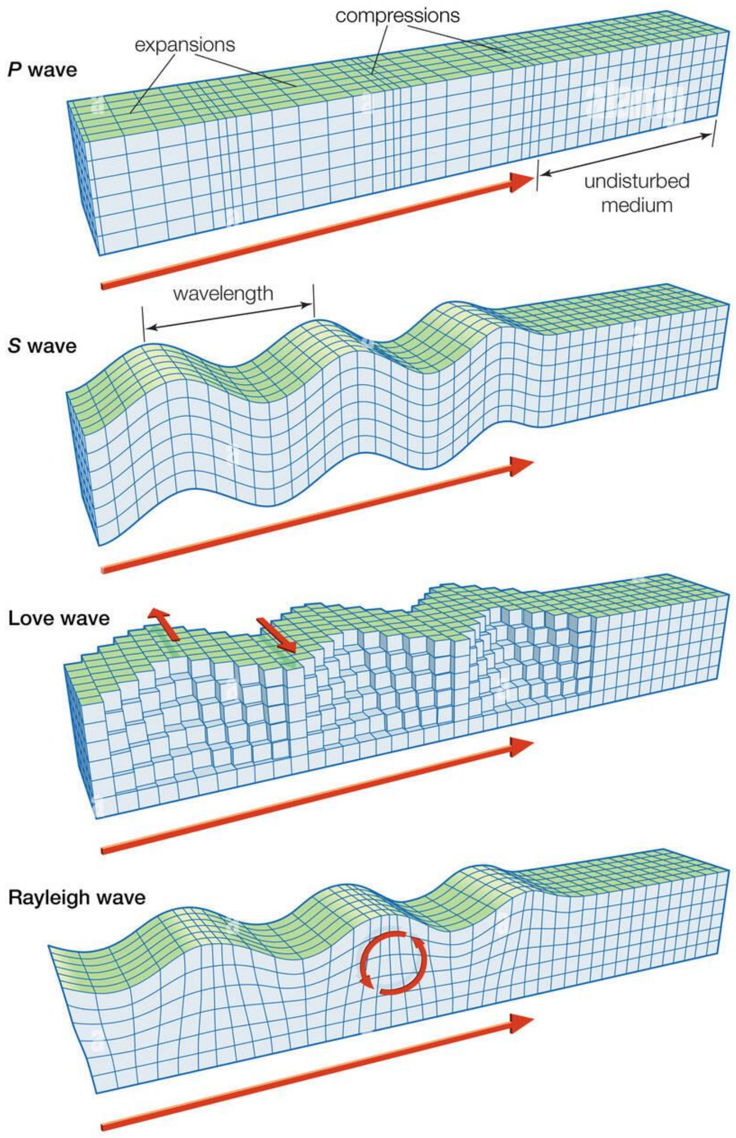
Planning to bore new subway tunnel

Locate buried utilities

Leaky storage tank remediation

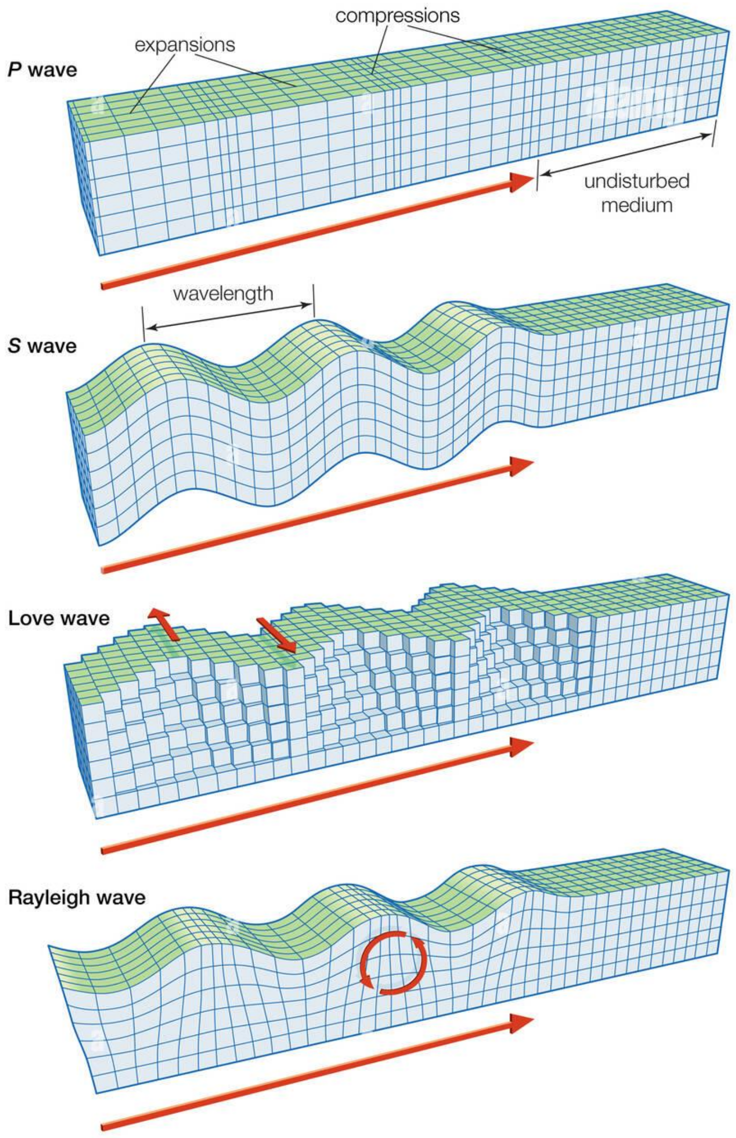
SEISMIC WAVES AND INTERACTION WITH A DISCONTINUITY

SEISMIC WAVES



P WAVE	**Body wave**	LONGITUDINAL compression and dilatation	Faster waves, propagating in all materials
S WAVE		TRANSVERSAL (VERTICAL) oscillations	SECONDARY arrival ($v_P/v_S \approx \sqrt{3}$) Producing most of the damages on buildings
LOVE WAVE	**Superficial wave**	Transversal, horizontal deformation (SH). Amplitude DECREASES WITH DEPTH	$v_{S1} < c_L < v_{S2}$ DISPERSIVE waves
RAYLEIGH WAVE		Elliptic retrograde trajectory (P+SV). Amplitude DECREASES WITH DEPTH	$v_R < 0.92\ v_S$ DISPERSIVE waves

PROPAGATION VELOCITY: P vs S WAVES



Each type of soil propagates the perturbation with a characteristic VELOCITY.

Assessing wave velocity can then provide insights on the nature of the subsurface.

Soil type	V_P (m/s)	V_s (m/s)
Saturated Clay	1500	100 - 250
Sand	300 - 500	120 - 200
Dense Sand	400 - 600	200 - 400
Gravel	500 - 750	300 - 600
Sandstone	1500 - 4500	700 - 1500
Marlstone	1500 - 4500	600 - 1500
Other materials		
Water	1500	
Air	330	

WAVE INTERACTION WITH A DISCONTINUITY

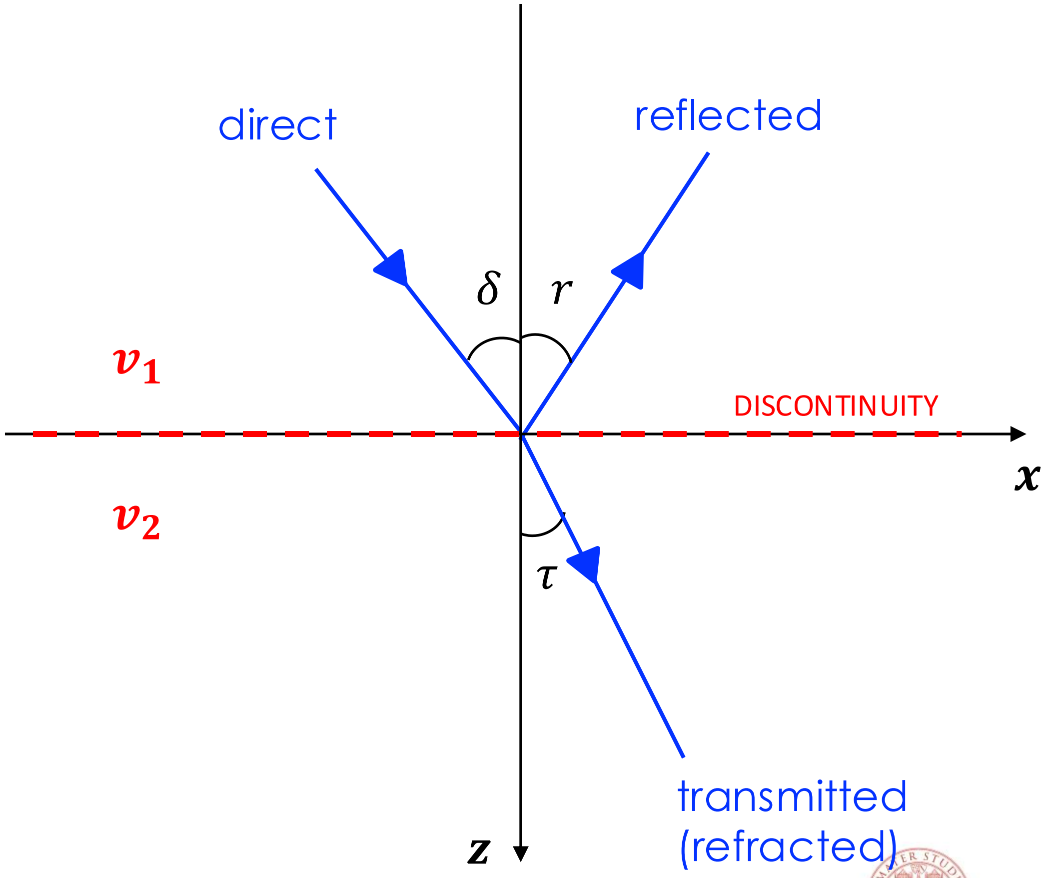
The interaction of a seismic wave with a DISCONTINUITY in the subsurface can be described through the **SNELL'S LAWS**:

REFLECTION (upper half-space) $\boldsymbol{\sin\delta = \sin r}$

TRANSMISSION (lower half-space) $\boldsymbol{\dfrac{\sin\delta}{v_1} = \dfrac{\sin\tau}{v_2}}$

Using the Snells' laws we can write:

$$\sin\tau = \frac{v_2}{v_1}\sin\delta$$



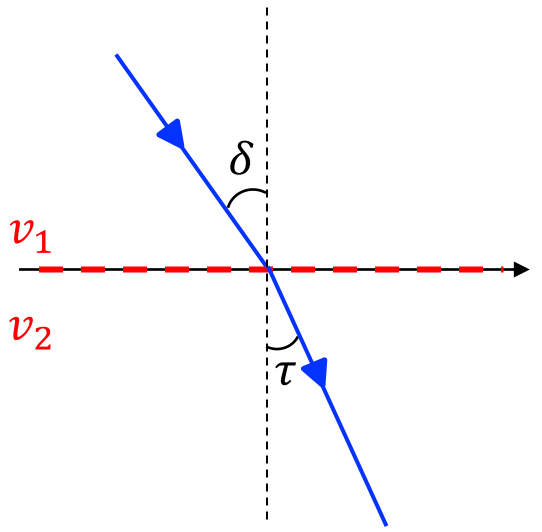
WAVE INTERACTION WITH A DISCONTINUITY

The refracted wave direction depends then on the incident angle δ and on the ratio of the wave velocities of the two layers:

$$\boldsymbol{\sin \tau = \frac{v_2}{v_1} \sin \delta}$$

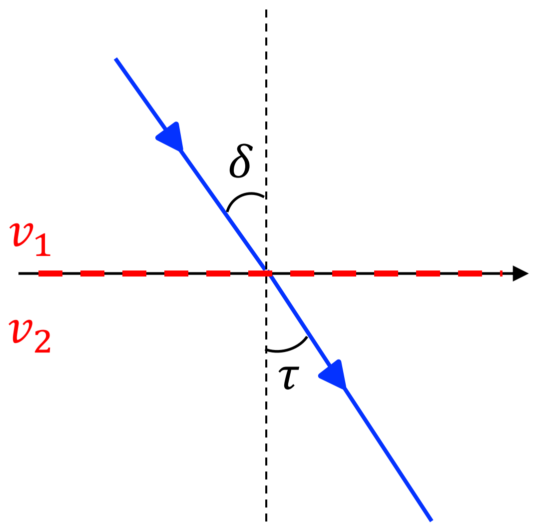
CASE 1

$v_1 > v_2 \implies \boldsymbol{\tau < \delta}$



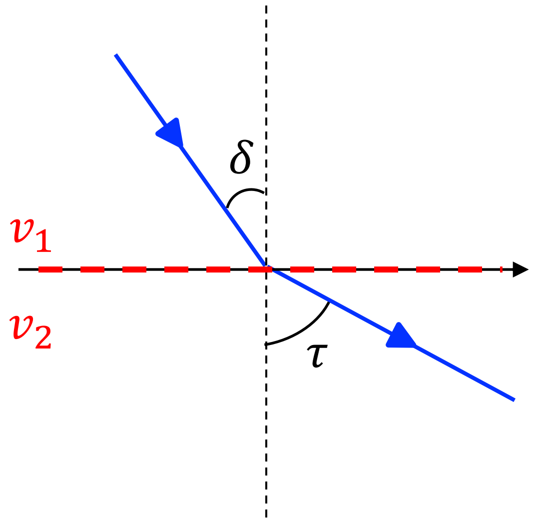
CASE 2

$v_1 = v_2 \implies \boldsymbol{\tau = \delta}$



CASE 3

$v_1 < v_2 \implies \boldsymbol{\tau > \delta}$



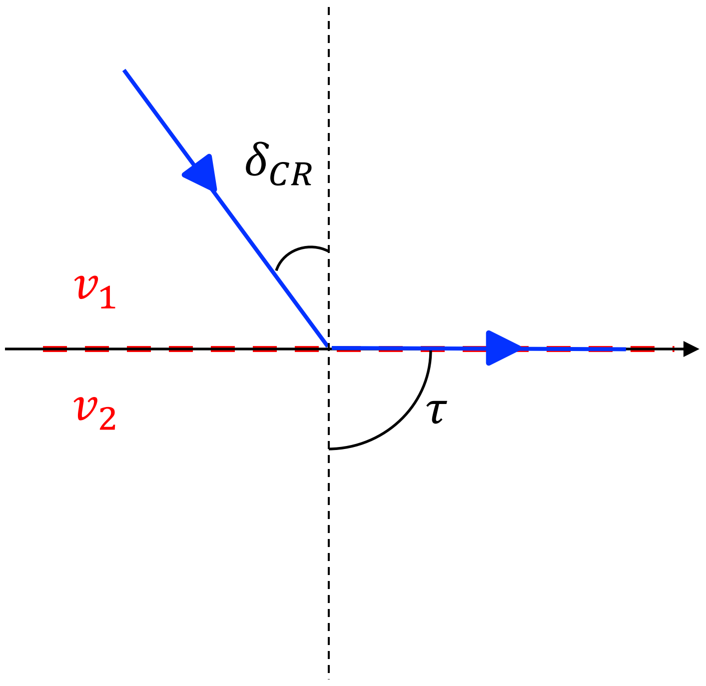
THE CRITICAL ANGLE

As the ratio v_2/v_1 grows, the limit case:

$$\tau = \pi/2$$

can be reached.

For incidence angles bigger than δ, there is not energy transfer to the lower layer



This value is known as **CRITICAL ANGLE, $\boldsymbol{\delta_{CR}}$**:

$$\boldsymbol{\sin \delta_{CR} = \frac{v_1}{v_2}}$$

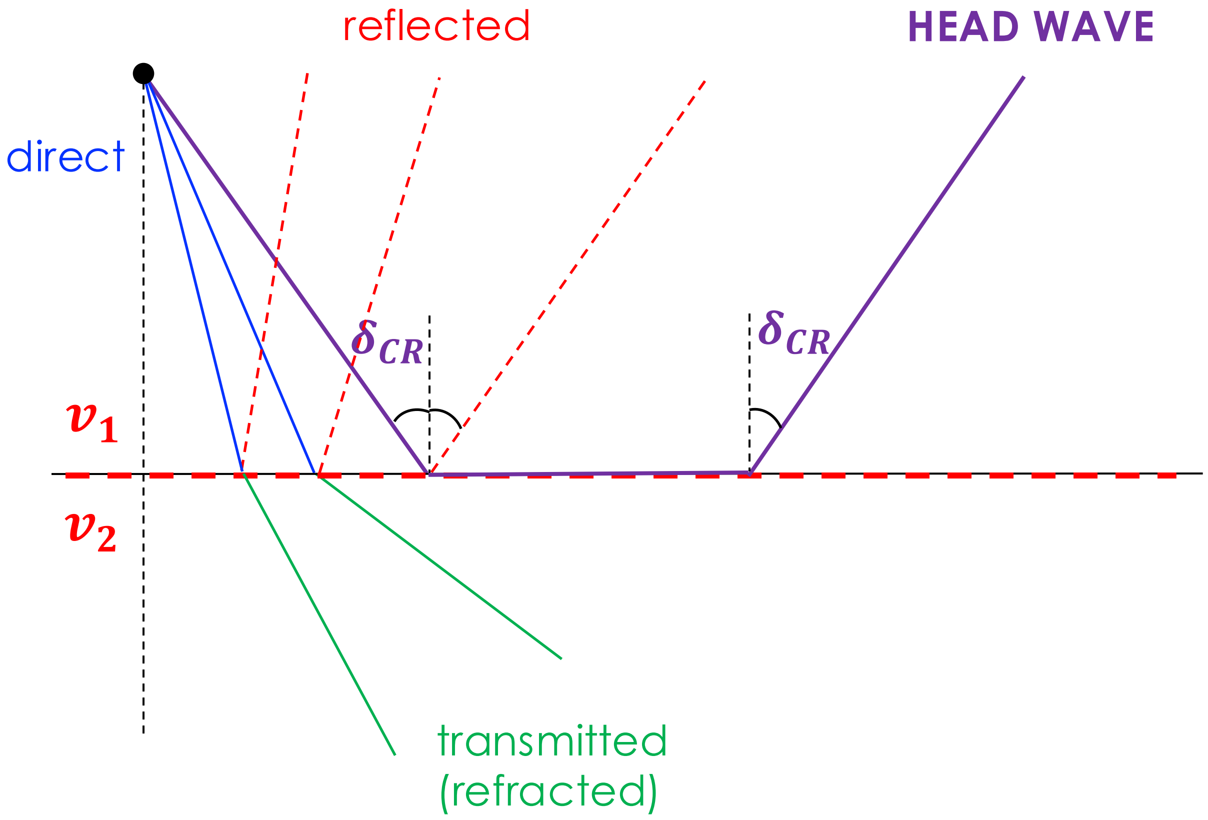
HEAD WAVES

For a given interface, a wave with inclination $> \delta_{CR}$ do not propagate to the lower layer.

The wave energy remains “trapped” in the upper layer, running along the interface and then EMERGING FROM THIS WITH THE SAME ANGLE δ_{CR}.

These are called **HEAD WAVES** and are the basis of the SEISMIC REFRACTION METHOD.

Since $\delta_{CR} = \mathrm{asin}(v_1/v_2)$, head waves transport information about the layer below the discontinuity.

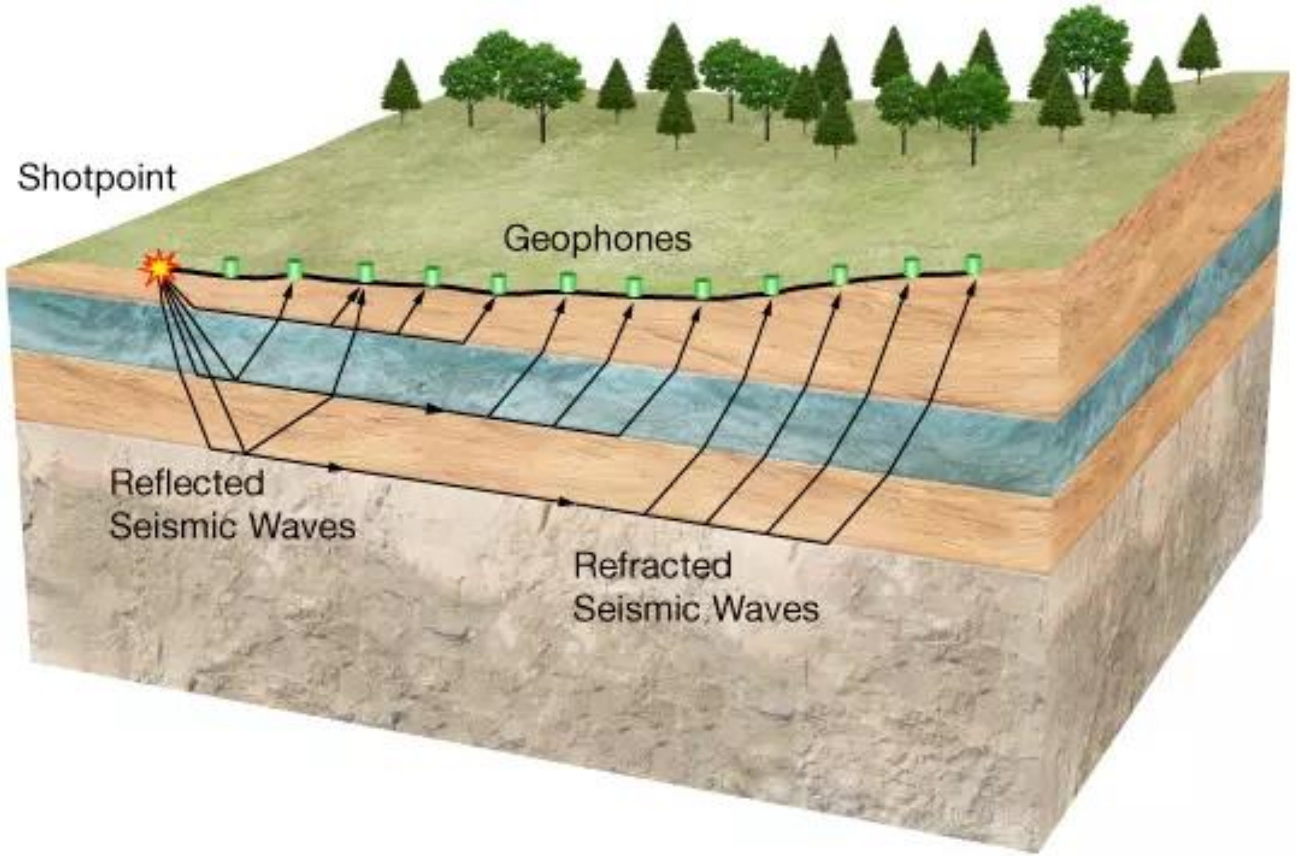


SEISMIC PROSPECTION: REFRACTION

SEISMIC PROSPECTION

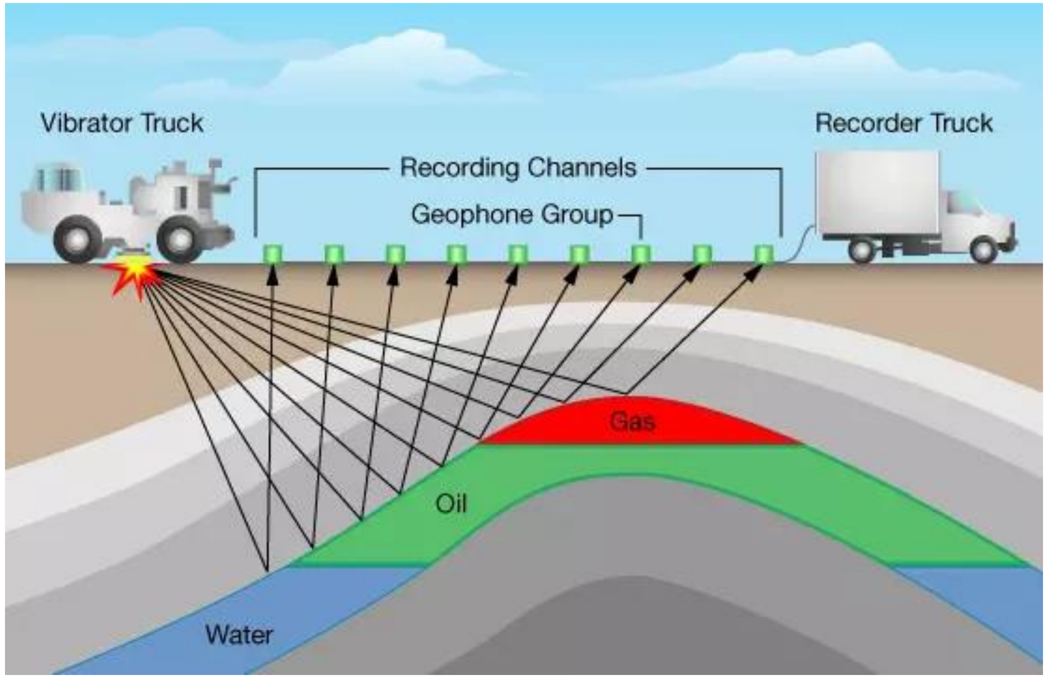
The exploration of the underground with **ACTIVE SEISMIC PROSPECTION** is based on the energization of the ground with an artificial source (hammer, explosive,…) and on the measure of the seismic waves reaching the receivers (geophones).

Their interaction with the underground structures transport information about its structure, i.e. the presence of interfaces or discontinuities.

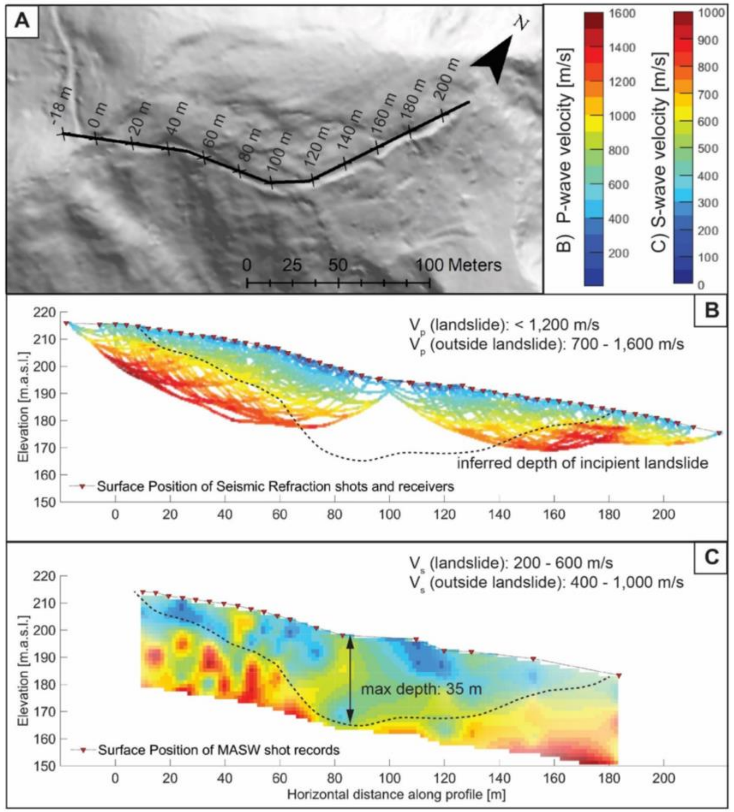


SEISMIC PROSPECTION: EXAMPLES OF APPLICATION

Seismic prospection can be used for the detection of GEOLOGICAL DEPOSITS, such as oil, gas or groundwater reservoirs.



The presence of a discontinuity in the underground can indicate the presence of a RUPTURE SURFACE of a LANDSLIDE.

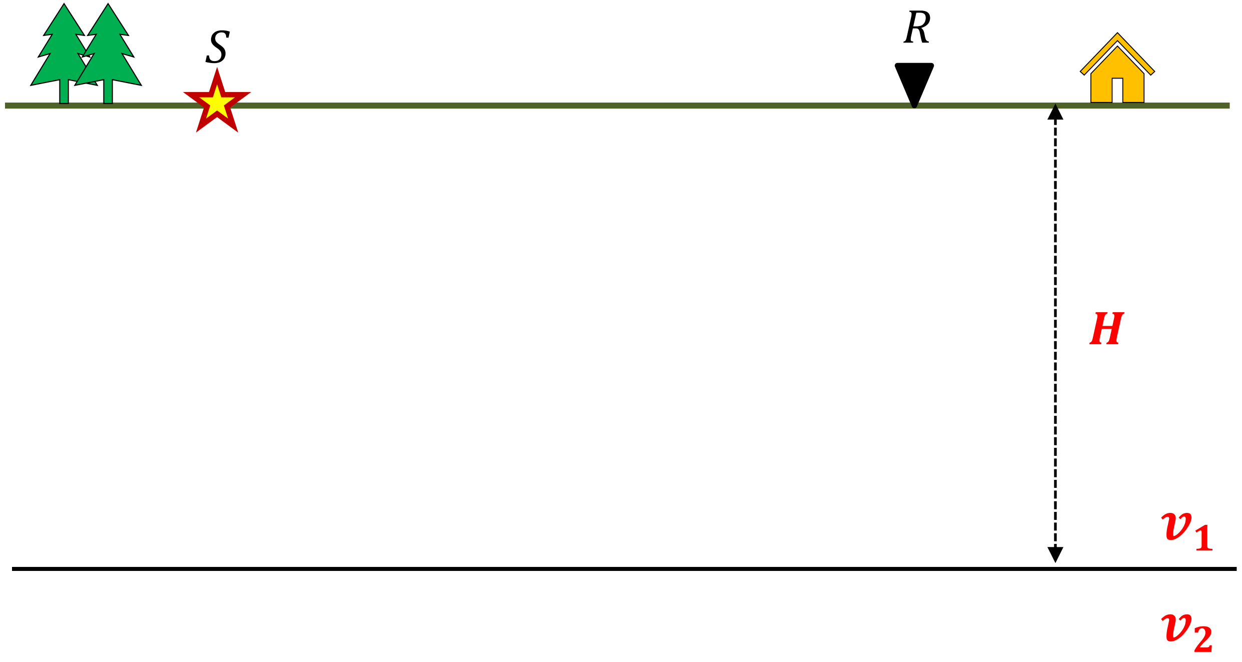


HORIZONTAL DISCONTINUITY

The simplest subsoil model is a HORIZONTAL INTERFACE, placed at a constant depth H.

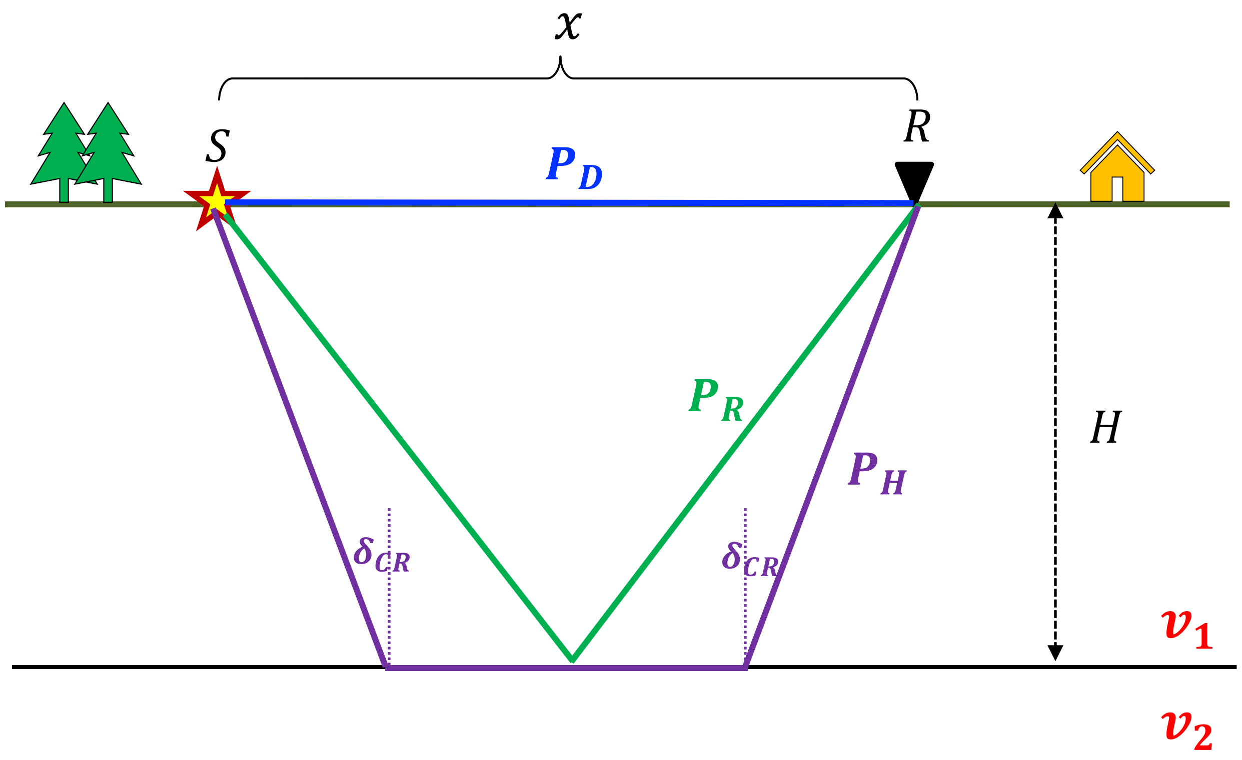
The structure is well described when H and the P-waves velocities v_1 and v_2 are determined.

To do this, we rely on the WAVE TRAVEL TIME.



HORIZONTAL DISCONTINUITY

Calling x the distance source-receiver, the travel times of the three types of wave reaching R are:



DIRECT WAVE $$T_D = \frac{x}{v_1}$$

REFLECTED WAVE $$T_R = \frac{\sqrt{x^2+4H^2}}{v_1}$$

HEAD WAVE $$T_H = \frac{x}{v_2} + 2H\sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}}$$

HORIZONTAL DISCONTINUITY

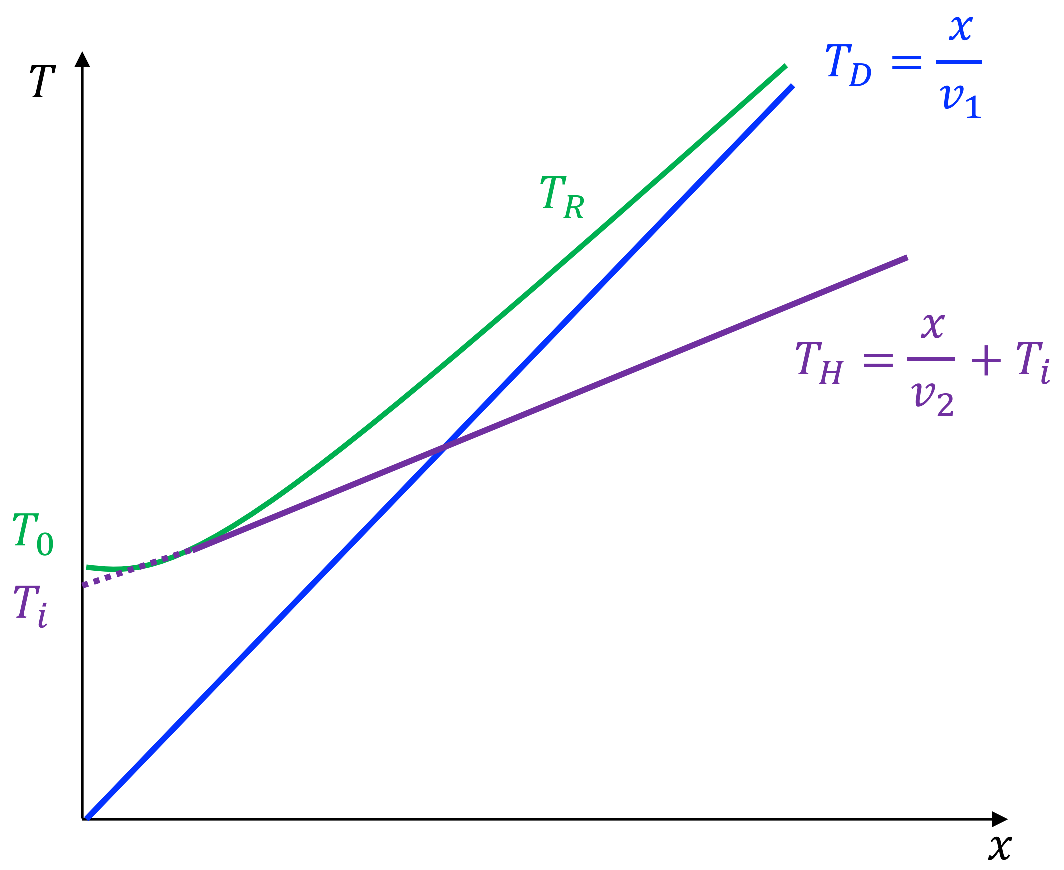
Representing the three functions in a time-distance plot, one gets the DROMOCHRONES:

DIRECT WAVE $T_D = \frac{x}{v_1}$ Line

REFLECTED WAVE $T_R = \frac{\sqrt{x^2+4H^2}}{v_1}$ Hyperbola

HEAD WAVE $T_H = \frac{x}{v_2} + T_i$ Line

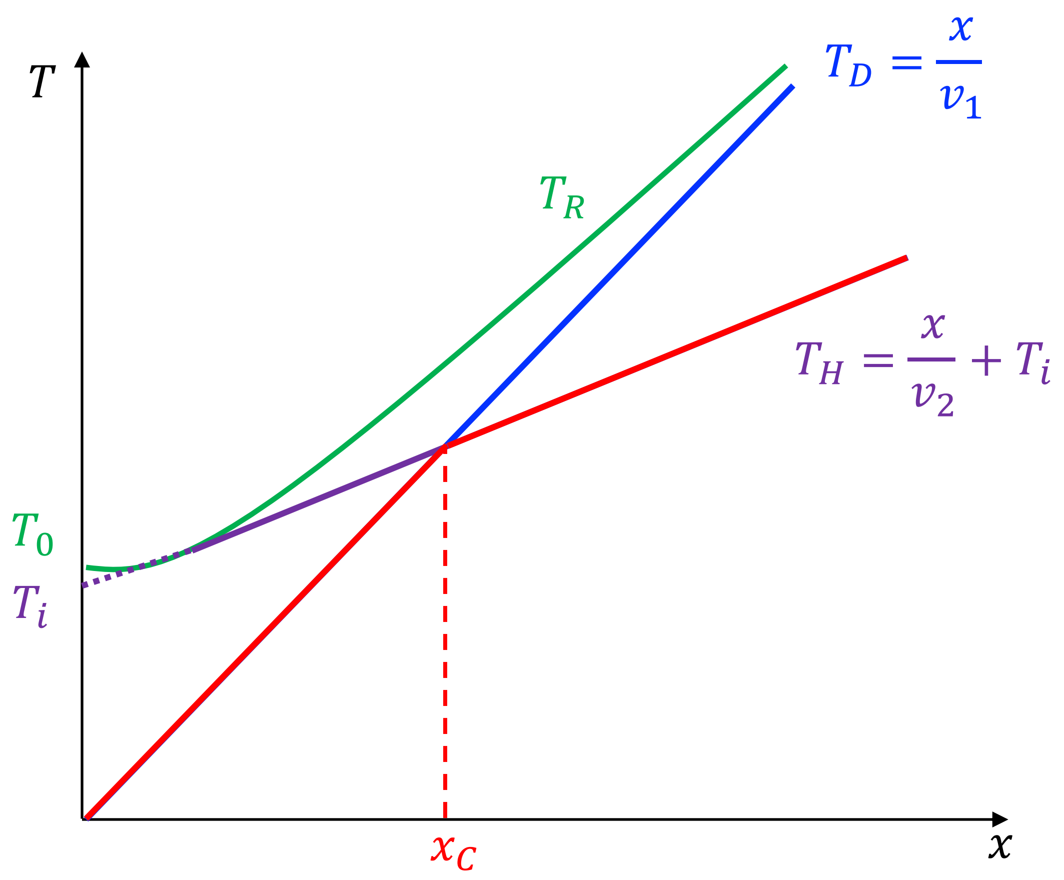
$$\left(T_i = 2H\sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}}\right)$$



HORIZONTAL DISCONTINUITY

Considering the FIRST WAVE ARRIVAL for each point along the x axis, one can notice that:

- the respective dromochrone is a POLYLINE with slopes $\mathbf{1/v_1}$ e $\mathbf{1/v_2}$.
- While $x < x_C$ (x_C is called CROSSOVER DISTANCE) the first waves reaching the receiver are DIRECT ones.
- For $x > x_C$, HEAD WAVES are the fastest.
- The REFLECTED wave never reaches the receiver as the first signal.

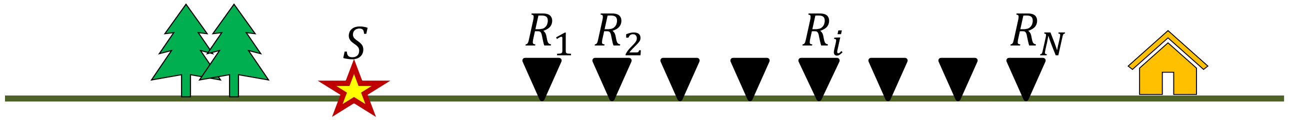


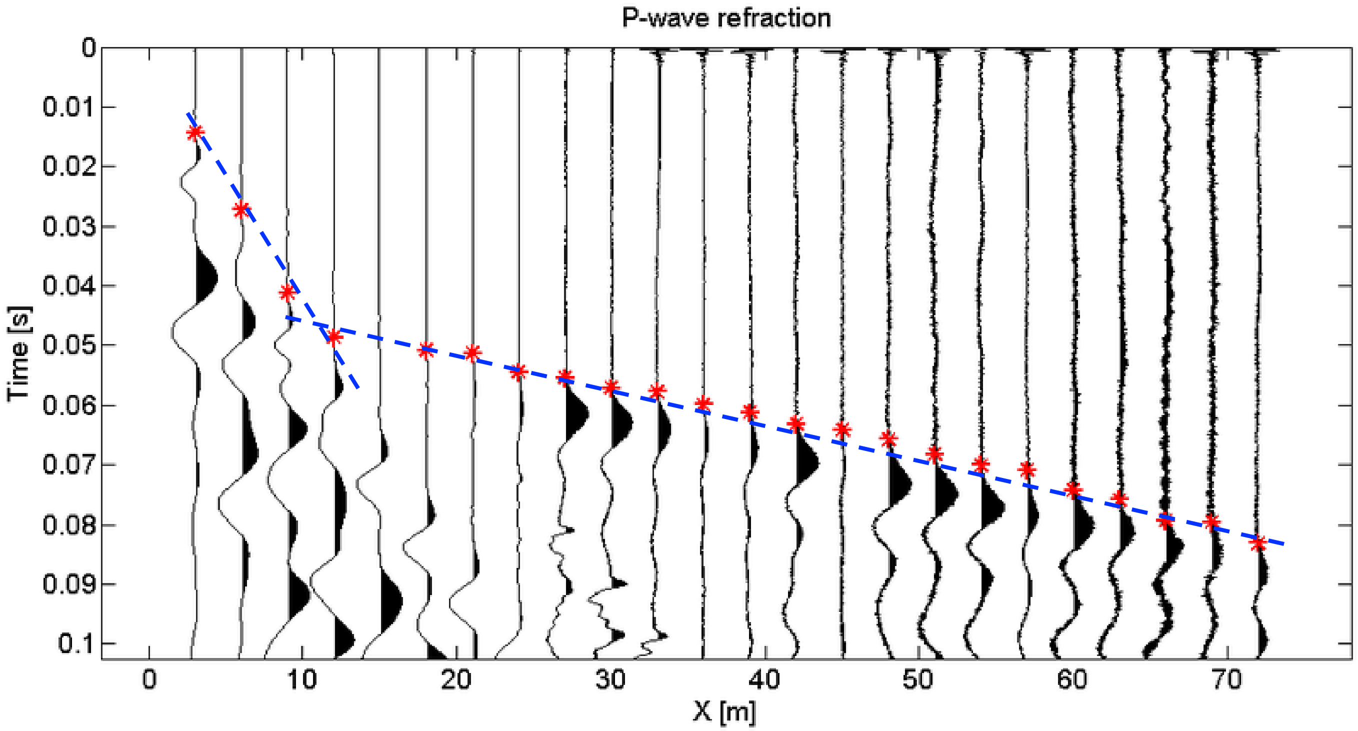
HORIZONTAL DISCONTINUITY

Seismic prospection with refraction requires a set of N geophones (ARRAY) registering the arrival of the waves (MULTI-CHANNEL survey).

The array configuration (number of sensors, inter-geophone distance) depends on the purpose of the survey.

For each sensor, a seismogram is obtained: refraction technique bases on the recognition of the FIRST ARRIVAL for each record (PICKING).





HORIZONTAL DISCONTINUITY

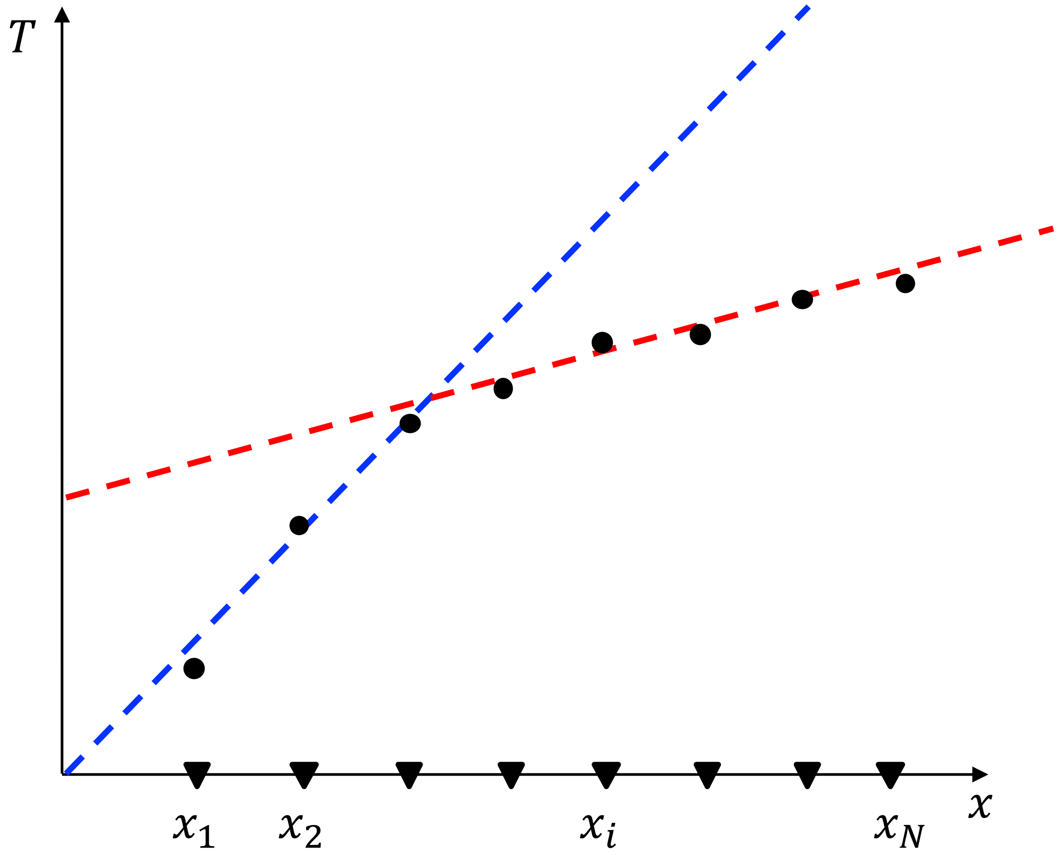
Picking provides the arrival times for each sensor position.

Two lines can be identified, with different slopes, corresponding to the arrival of the DIRECT and HEAD waves.

Through a best-fit procedure, the parameters characterizing the lines can be determined:

$$t = ax$$

$$t = bx + c$$



HORIZONTAL DISCONTINUITY

Comparing the lines equation to the respective dromochrones:

$$T_D = \frac{x}{v_1} \qquad t = ax$$

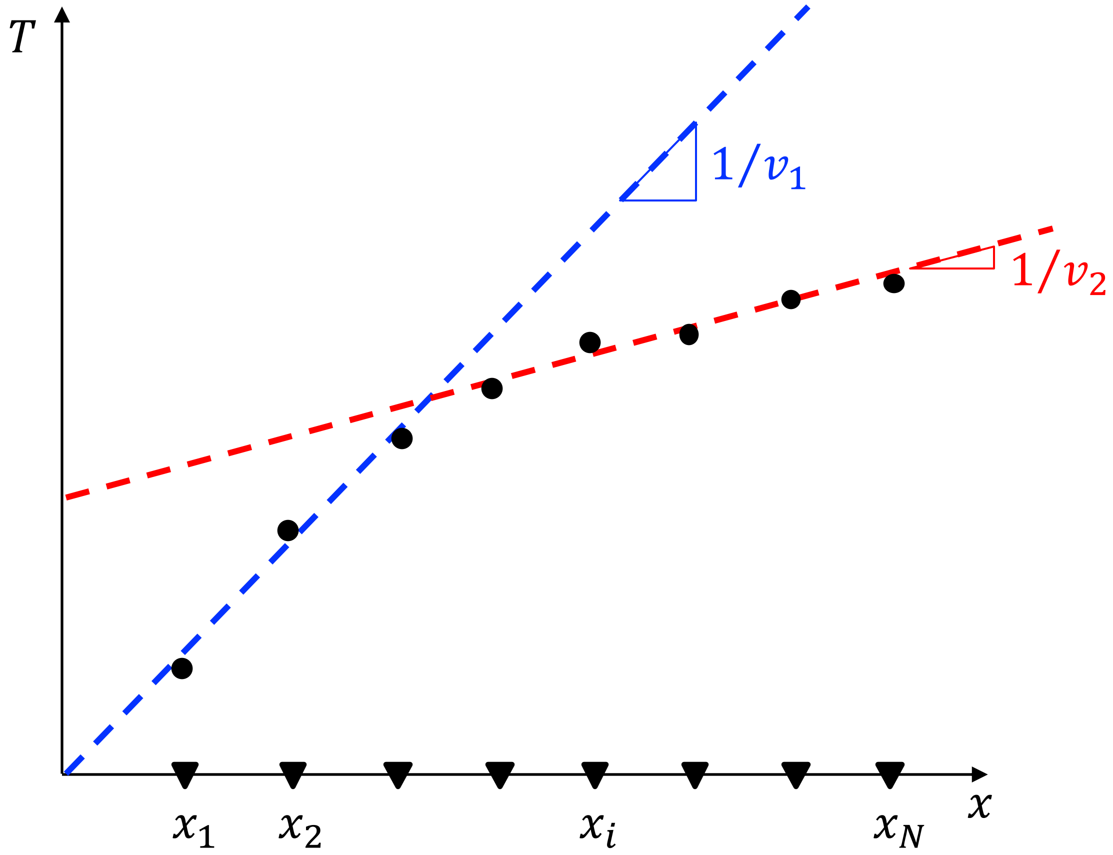
$$T_H = \frac{x}{v_2} + 2H\sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}} \qquad t = bx + c$$

It is possible to obtain the subsurface model for a **SINGLE HORIZONTAL DISCONTINUITY** basing on the lines' coefficients:

$$\boldsymbol{v_1 = \frac{1}{a}}$$

$$\boldsymbol{v_2 = \frac{1}{b}}$$

$$\boldsymbol{H = \frac{c}{2\sqrt{a^2 - b^2}}}$$



INCLINED DISCONTINUITY

Let's consider now an inclined interface, with slope θ.

The HEAD WAVES travel time can be computed through the expression:

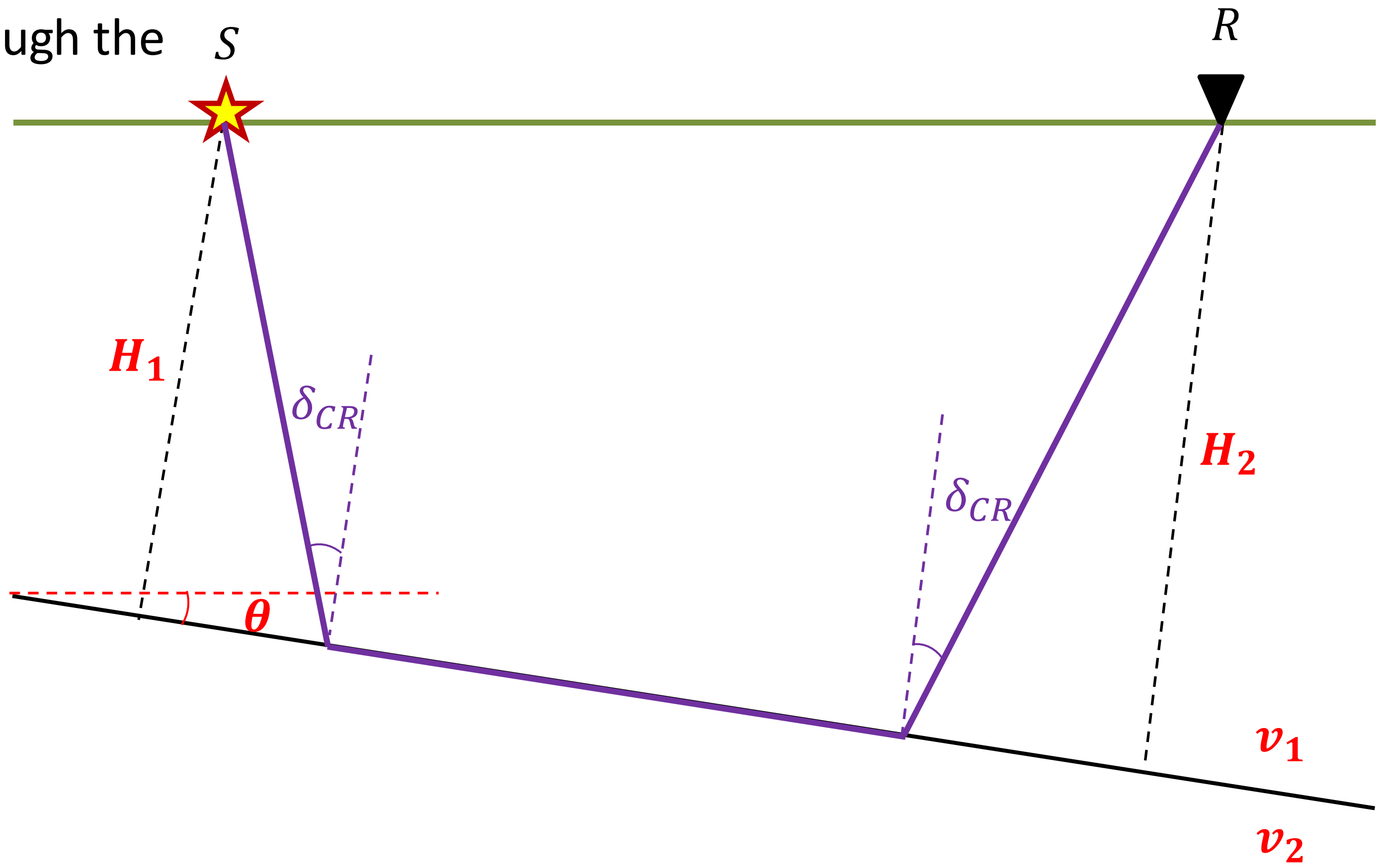
$$\boldsymbol{T_H = \frac{x}{v_1}\sin(\delta_{CR}+\theta) + \frac{2H_1}{v_1}\cos\delta_{CR}}$$

Which is again a LINE with slope

$$\frac{dT}{dx} = \frac{1}{v_D}$$

where

$\boldsymbol{v_D = \frac{v_1}{sin(\delta_{CR}+\theta)}}$ is the DOWNDIP VELOCITY.

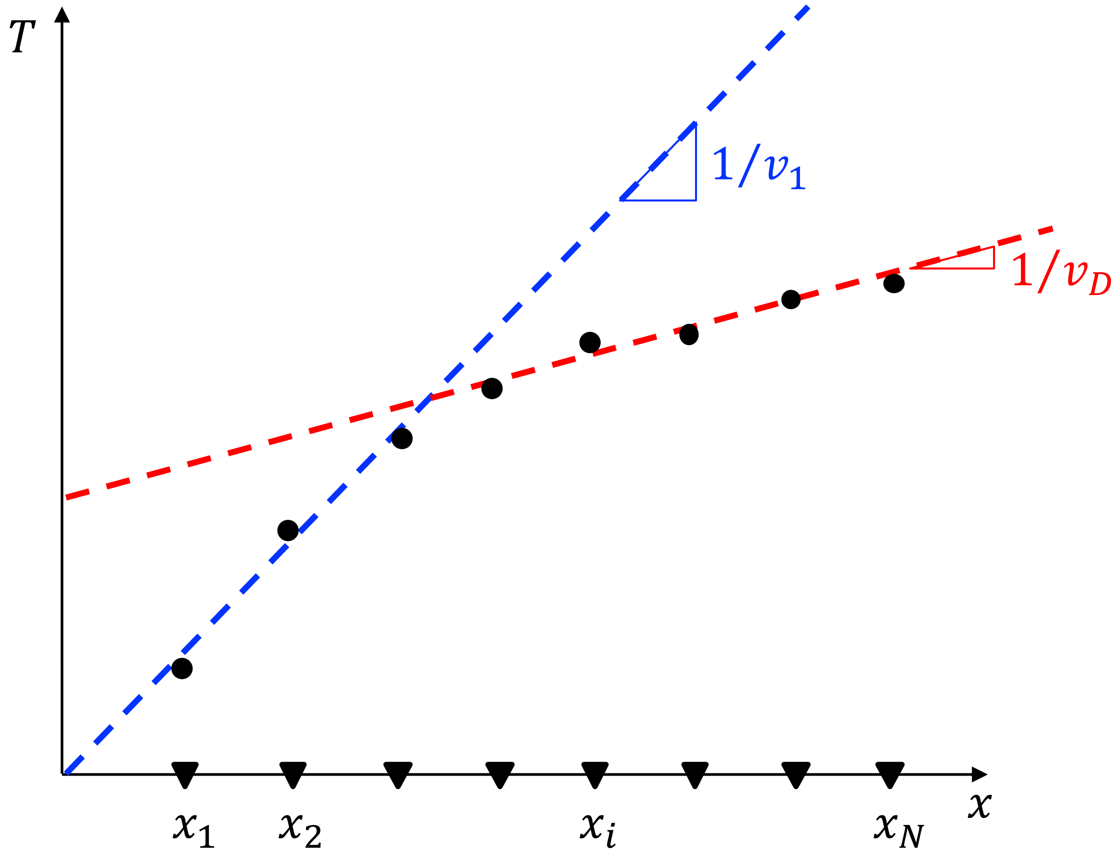


INCLINED DISCONTINUITY

The field survey provides again a polyline, with slopes $1/v_1$ and, in this case, $1/v_D$.

The linear fit produces then three coefficients: a, b and c, while the unknowns of the problems are five: H_1, H_2, v_1, v_2 and θ.

How can we solve this indetermination?



CONJUGATE PATH

Inverting the positions of the source and the receiver (CONJUGATE PATH), it is possible to write the respective travel time as:

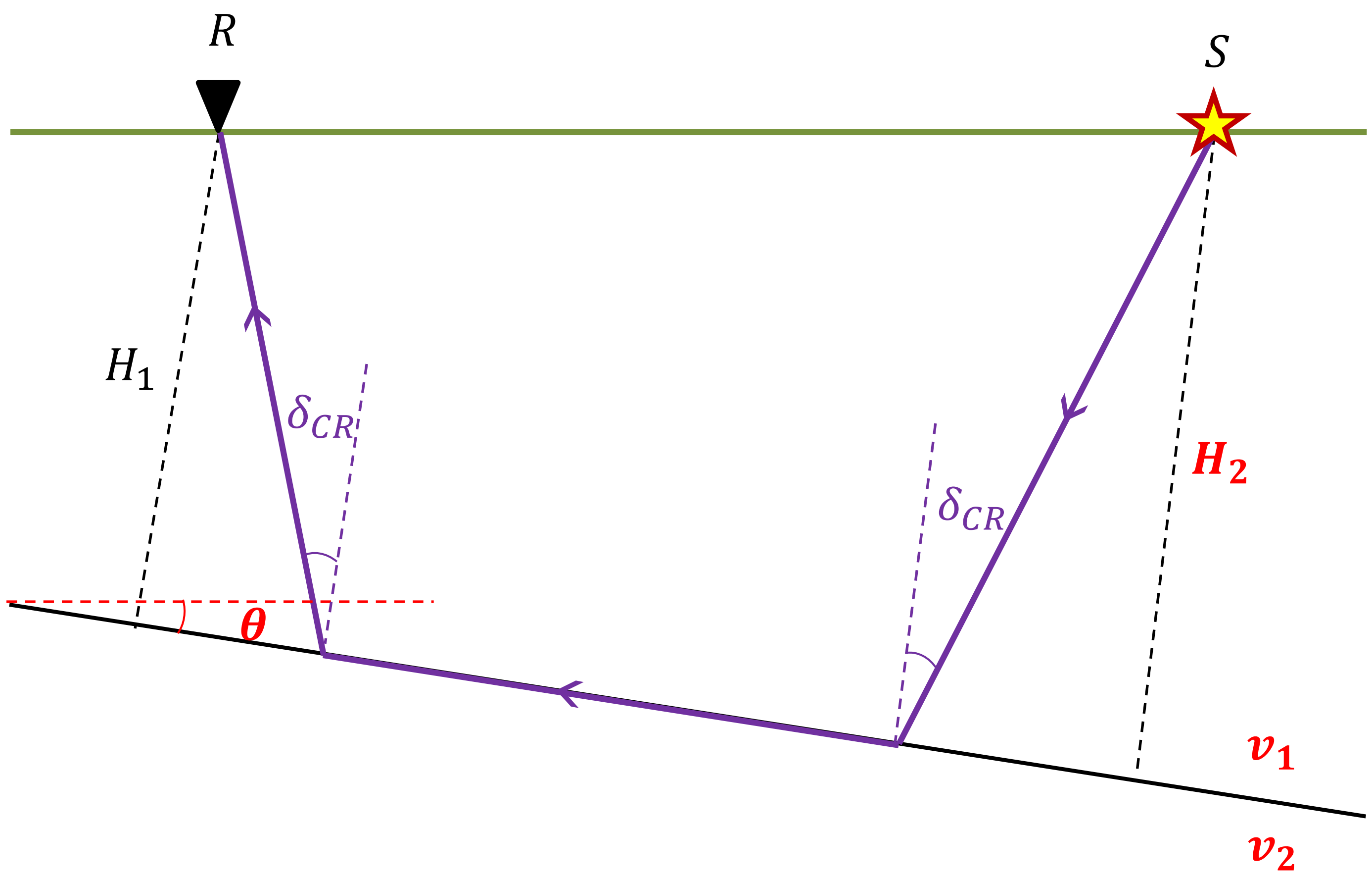
$$\boldsymbol{T_H = \frac{x}{v_1}\sin(\delta_{CR} - \theta) + \frac{2H_2}{v_2}\cos\delta_{CR}}$$

Similarly to the previous case, the slope is:

$$\frac{dT}{dx} = \frac{1}{v_U}$$

where

$$\boldsymbol{v_U = \frac{v_1}{\sin(\delta_{CR} - \theta)}}$$ is the UPDIP VELOCITY.



CONJUGATE PATH

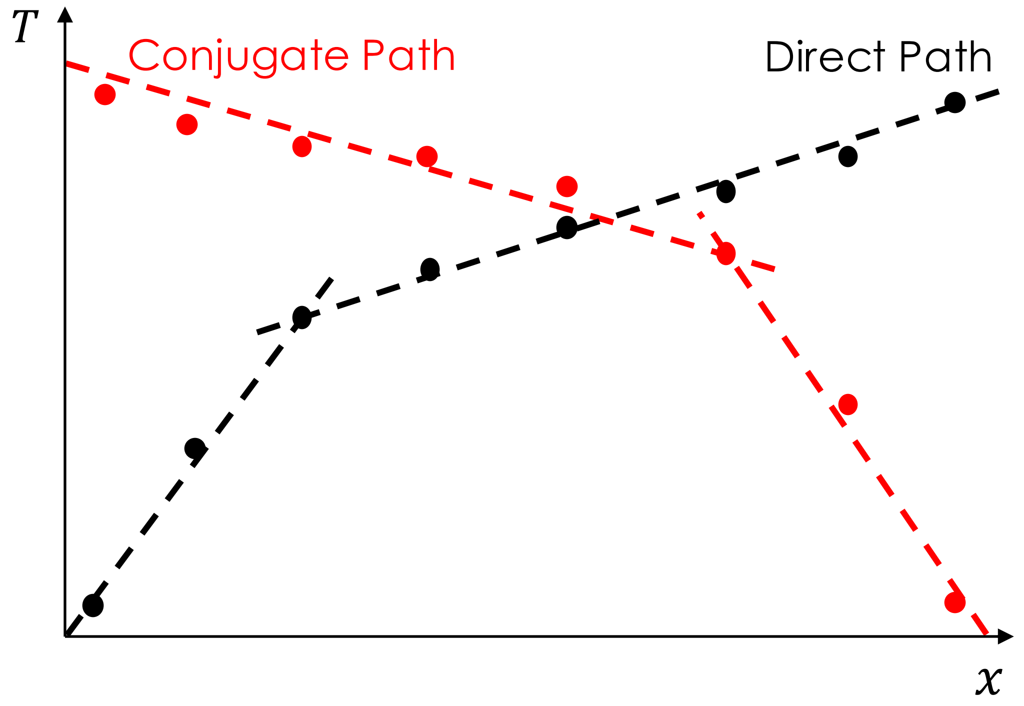
We have then, from the analysis of the survey results:

DIRECT PATH

$$\begin{cases} t = a_D x \\ t = b_D x + c_D \end{cases} \qquad \begin{cases} T_{DD} = \dfrac{x}{v_1} \\ T_{HD} = \dfrac{x}{v_1}\sin(\delta_{CR} + \theta) + \dfrac{2H_1}{v_1} cos\, \delta_{CR} \end{cases}$$

CONJUGATE PATH

$$\begin{cases} t = a_U x \\ t = b_U x + c_U \end{cases} \qquad \begin{cases} T_{DU} = \dfrac{x}{v_1} \\ T_{HU} = \dfrac{x}{v_1}\sin(\delta_{CR} - \theta) + \dfrac{2H_2}{v_1} cos\, \delta_{CR} \end{cases}$$



The best-fit procedure provides 6 coefficients ($a_D, b_D, c_D, a_U, b_U, c_U$) which can be used to determine the subsurface model: $\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{H_1}, \boldsymbol{H_2}$ and $\boldsymbol{\theta}$.

INCLINED DISCONTINUITY

Formulas for the determination of the subsurface model for an inclined discontinuity:

$$v_1 = \frac{1}{2}\left(\frac{1}{a_D} + \frac{1}{a_U}\right)$$

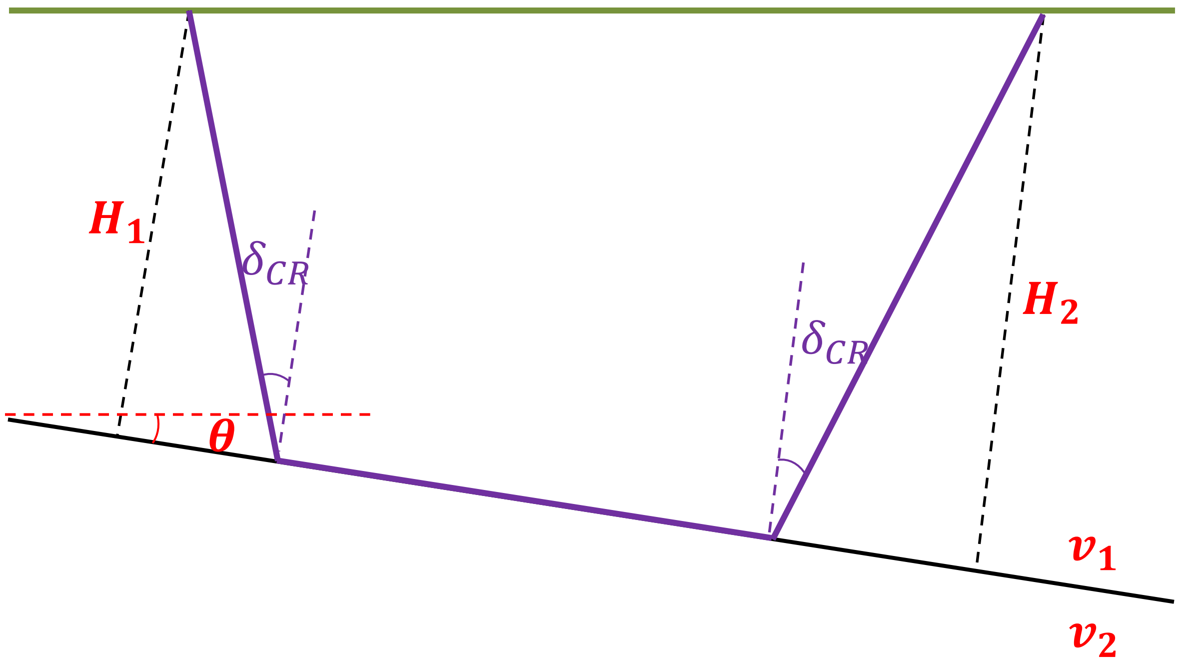
$$\theta = \frac{1}{2}\left[\arcsin(b_D v_1) - \arcsin(b_U v_1)\right]$$

$$v_2 = \frac{2\cos\theta}{b_U + b_D}$$

$$\delta_{CR} = asin\left(\frac{v_1}{v_2}\right)$$

$$H_1 = \frac{v_1 c_D}{2\cos\delta_{CR}}$$

$$H_2 = \frac{v_1 c_U}{2\cos\delta_{CR}}$$



THANKS FOR YOUR KIND ATTENTION

and…

LET’S GO ON THE FIELD!