Semiotic representations, “avoidable” and “unavoidable” misconceptions

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Summary. *Following D’Amore’s constructive interpretation for the term misconception, we present a distinction between “unavoidable” and “avoidable” misconceptions from a semiotic point of view, within the theoretical frameworks proposed by Raymond Duval and Luis Radford.*

1. Introduction

In this article we deal with one of the most used terms for decades in Mathematics Education research, the word “*misconception*”, interpreted according to a constructive perspective proposed by D’Amore (1999): «A misconception is a wrong concept and therefore it is an event to avoid; but it must not be seen as a totally and certainly negative situation: we cannot exclude that to reach the construction of a concept, it is *necessary* to go through a temporary misconception that is being arranged». According to this choice, misconceptions are considered as steps the students must go through, that must be controlled under a didactic point of view and that are not an obstacle for students’ future learning if they are bound to *weak and unstable* images of the concept; they represent an obstacle to learning if they are rooted in *strong and stable models* of a concept. For further investigation into this interpretation look D’Amore, Sbaragli (2005).

This semantic proposal is analogous with Brousseau’s use of the term

obstacle, starting from 1976 (Brousseau, 1976-1983), to which he gave a constructive role in Mathematics Education, interpreting it as knowledge that was successful in previous situations, but it does not "hold" in new situations.
Within this interpretation misconceptions have been divided into two big categories: *"avoidable"* and *"unavoidable"* (Sbaragli, 2005a); the first *do not depend directly on the teacher's didactic transposition,* whereas the second *depend exactly on the didactic choices performed by the teacher*.
We will analyze these categories within the theoretical frameworks upholded by Raymond Duval and Luis Radford.

2. Reference theoretical frameworks

According to Duval's formulation the use of signs, organized in semiotic registers, is constitutive of mathematical thinking since mathematical objects do not allow *ostensive* referrals; from this point of view he claims that there *isn't noetics without semiotics*. «The special epistemological situation of mathematics compared to other fields of knowledge leads to bestow upon semiotic representations a fundamental role. First of all they are the only way to access mathematical objects» (Duval, 2006).
This lack of ostensive referrals to concrete mathematical objects obliges also to face *Duval's cognitive paradox:* «(...) How learners could not confuse mathematical objects if they cannot have relationships but with semiotic representations? The impossibility of a direct access to mathematical objects, which can only take place through a semiotic representation leads to an *unavoidable* confusion» (Duval, 1993).
In particular, conceptual appropriation in mathematics requires to manage the following semiotic functions: the choice of the *distinguishing features* of the concept we represent, *treatment* i.e. transformation in the same register and *conversion* i.e. change of representation into another register. The very combination of these three "actions" on a concept represents the "construction of knowledge in mathematics"; but the coordination of these three actions is not spontaneous nor easily managed; this represents the cause for many difficulties in the learning of mathematics.
To better understand the learning processes it is suitable to integrate Duval's theoretical frame with the one proposed by Radford who

enlarges the notion of sign incorporating in the learning processes also the sensory and kinaesthetic activities of the body. Radford (2005) considers learning an *objectification* process that transforms *conceptual and cultural* objects into objects of our *consciousness*. This objectification process is possible only by turning to culturally constructed forms of mediation that Radford (2002) calls *semiotic means of objectification*; i.e. gestures, artifacts, semiotic registers, in general signs used to make an intention visible and to carry out an action.

Like Duval, also Radford (2005) underlines the importance of the *coordination* between representation systems, when he claims that conceptualization is forged out of the dialectical interplay of various semiotic systems, with their range of *possibilities and limitations*, mobilized by students and teachers in their *culturally mediated social practices*. In the continuation of the article we will read “avoidable” and “unavoidable” misconceptions according to these theoretical frameworks.

3. “Unavoidableness”

“Unavoidable” misconceptions, that do not derive from didactical transposition, can depend on the representations teachers are obliged to provide in order to explain a concept because of the intrinsic unapproachableness of mathematical objects. These representations, according to Duval’s paradox, can be confused with the object itself especially when a concept is proposed for the fist time. These representations can lead the student to consider valid “parasitical information” bound to the specific representation, in contrast with the generality of the concept. This “parasitical information” for example can stem from sensory, perceptive and motor factors of the specific representation since Radford (2003) claims that cognition is embodied in the subject’s spatial and temporal experience and therefore requires to mobilize semiotic means bound to the practical sensory-motor intelligence.

The “embodied” character of cognition and the use of semiotics makes these “*misconceptions unavoidable”* and interpretable as steps the student must go through in the construction of concepts.

As we will show in the following example, these particular misconceptions can also be put down to the *necessary gradualness of knowledge*. In fourth primary school one day the teacher shows how the request that highlights the “specific difference” between the “close genus” rectangles and the “subgenus” squares regards only the length of the sides (that must all be congruent). After drawing a square on the blackboard, the teacher claims that it is a particular rectangle. The possible misconception created in the mind of the student that the prototype image of a rectangle is a figure that must have consecutive sides with different lengths, may create at this stage a cognitive conflict with the new image proposed by the teacher. This example highlights that it is unthinkable to propose initially all the necessary considerations to characterize a concept from the mathematical point of view, not only for the necessary gradualness of knowledge, but also because in order to propose mathematical objects, they must be anchored to semiotic representations that often hide the totality and complexity of the concept. These examples of “avoidable” misconceptions seem to be bound to *ontogenetic* (that originate in the student) and *epistemological* (that depend on intrinsic facts to mathematics) obstacles (Brousseau, 1986); the last are considered by Luis Radford related to the social “practices” (D’Amore, Radford, Bagni, 2006).

4. “Avoidableness”

In the appropriation of a mathematical concept the pupil performs a desubjectification process, that leads him beyond the body spatial temporal dimension of his personal experience. The teacher has the delicate task of fostering a *cognitive rupture* to allow the pupil to incorporate his kinaesthetic experience in more complex and abstract semiotic means. The student thus goes beyond the embodied meaning of the object and endows it with its cultural interpersonal value (Radford, 2003). In this perspective, Duval (2006) offers important didactic indications to manage the rupture described above, when he highlights the importance of exposing the student, in a critical and aware manner, to many representations in different semiotic registers. Nevertheless didactic praxis is “undermined” by improper habits that expose pupils to univocal and inadequate semiotic representations. These habits cause misconceptions considered “avoidable”, since they are ascribable to the *didactic transposition.*

An emblematic example of inadequate choice of the distinguishing features that brings to improper and misleading information relative to the proposed concept, regards the habit of indicating the angle with a “little arc” between the two half-lines that determine it. Indeed, the limitedness of the “little arc” is in contrast with the boundlessness of the angle as a mathematical “object”. This implies that in a research involving students of the Faculty of Education, most of the persons interviewed claimed that the angle corresponds to the length of the little arc or to the limited part of the plane that it identifies.

An inadequate didactical transposition can in fact strengthen the confusion, lived by the student, between the symbolic representations and the mathematical object. The result is that «the student is unaware that he is learning signs that stand for concepts and that he should instead learn concepts; if the teacher has never thought over this issue, he will believe that the student is learning concepts, while in fact he is only “learning” to use signs» (D’Amore, 2003).

This misunderstanding derives also from the univocity of the representations that teachers usually provide students with, as is the case of geometry’s primitive entities. Researches aiming at detecting incorrect models built on image-misconceptions relative to these mathematical concepts show that as regards the mathematical point, some pupils and teachers ascribe to this mathematical entity a “roundish” shape (bidimensional o tridimensional) that derives from the univocal and conventional representations they have always encountered (Sbaragli, 2005b). Moreover, some students and teachers are led to associate with the wrong idea bound to the unique shape of mathematical points also a certain variable dimension.

From these results it emerges how often the choice of the representation, is not an aware didactical choice but it derives from teachers’ wrong models. And yet, to not create strong misunderstandings it is first required that the teacher knows the “institutional” meaning of the mathematical object that she wants her students to learn, secondly she must direct the didactical methods in a critical and aware manner. “Avoidable” misconceptions seem to be bound to the classical *didactic obstacles* (Brousseau, 1986) that originate in the didactic and methodological choices of the teacher.

From a didactical point of view, it is therefore absolutely necessary to overcome “unavoidable” misconceptions and prevent the “avoidable” ones, with particular attention to the semiotic means of objectification, providing a great variety of representations appropriately organized and integrated into a social system of meaning production, in which students experience shared mathematical practices.

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