Atmospheric radio occultations using ray-tracing techniques

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1. Introduction

Numerical ray-tracing (RT) has proven to be a powerful tool for processing data of radio occultations by oblate axisymmetric atmospheres with zonal winds [1]. If this technique is nowadays the most generic and the most accurate, it presents two disadvantages. Firstly, it does not provide a comprehensive description of how the atmosphere oblateness nor the zonal winds influence the light path and the light time. Secondly, because the initial pointing of the S/C's antenna has to be determined iteratively, the computation time is usually important.

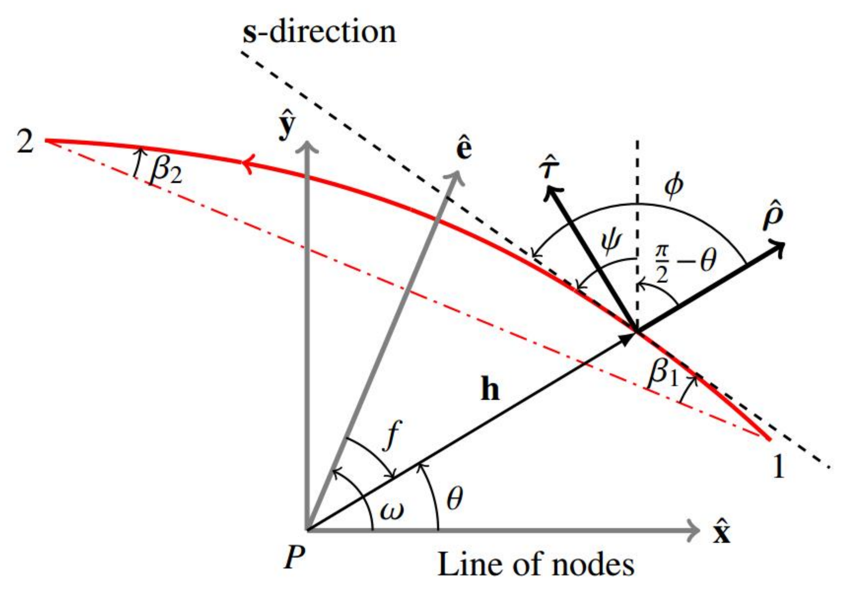
To overpass these two difficulties, a purely analytical approach (analytical RT) has been recently proposed [2]. This method is based on a full reformulation of the fundamental equations of geometrical optics into a set of osculating equations which are similar to perturbation equations of celestial mechanics. This set of equations describes how the constants of integration of a reference solution (also called "*hyperbolic solution*") change according to variations in the refractivity profile of the planetary atmosphere.

2. Geometrical optics

Both numerical and analytical RT techniques are based under the assumption that geometrical optics (GO) is valid.

GO assumes that the phase of an electromagnetic signal varies more rapidly than its amplitude (short wavelength). In this approximation, the concept of rays is introduced as curves whose tangents at each point coincide with the direction of propagation of the wave [3].

Then, the direction of a light ray changes according to Eq. (2) (see Fig. 1), where $n(\mathbf{h})$ is the index of refraction of the medium. Consequences following GO approximation are summarized in Fig.1.



- Separation vector : $\mathbf{h}$,
- Tangent vector : $\mathbf{s}$,

$$\mathbf{s} \equiv \frac{\mathrm{d}\mathbf{h}}{\mathrm{d}s}, \tag{1}$$

- Infinitesimal length : $\mathrm{d}s = \mathrm{d}\mathbf{h}\cdot\mathrm{d}\mathbf{h}$,
- Ray vector : $\mathbf{n} \equiv n\mathbf{s}$,
- Vector Eikonal equation :

$$\frac{\mathrm{d}\mathbf{n}}{\mathrm{d}s} = \boldsymbol{\nabla} n, \tag{2}$$

- Elapsed time :

$$\frac{\mathrm{d}t}{\mathrm{d}s} = \frac{n}{c}. \tag{3}$$

Figure 1: Geometrical optics approximation. *Left panel*. Trajectory of a light ray between a transmitter (labelled 1) and a receiver (labelled 2). *Right panel*. Fundamental equations.

3. General assumptions

Assuming that the atmosphere is in hydrostatic equilibrium, it can be shown [1,2] that the index of refraction is a function of a generalized potential, that is $n = n(\Phi)$ where Φ is the sum of the Newtonian (including non-spherical contribution), the centrifugal, and tidal potentials.

4. Numerical ray-tracing

The basis of the numerical RT are presented e.g. in [1]. It consists in solving numerically Eqs. (1-3) assuming that $n=n(\Phi)$ where we recall that Φ is a generalized potential. The observable is the received frequency on ground while the S/C oscillator frequency in its rest frame is considered as constant. The planetary atmosphere is usually built up of layers of constant refractivity gradient from the outside in (see Fig. 2). The complete refractivity profile is determined by tracing ray through the previously known layers, and then the refractivity gradient of the unknown layer is adjusted so that the ray reaches the DSN antenna with the actual received frequency.

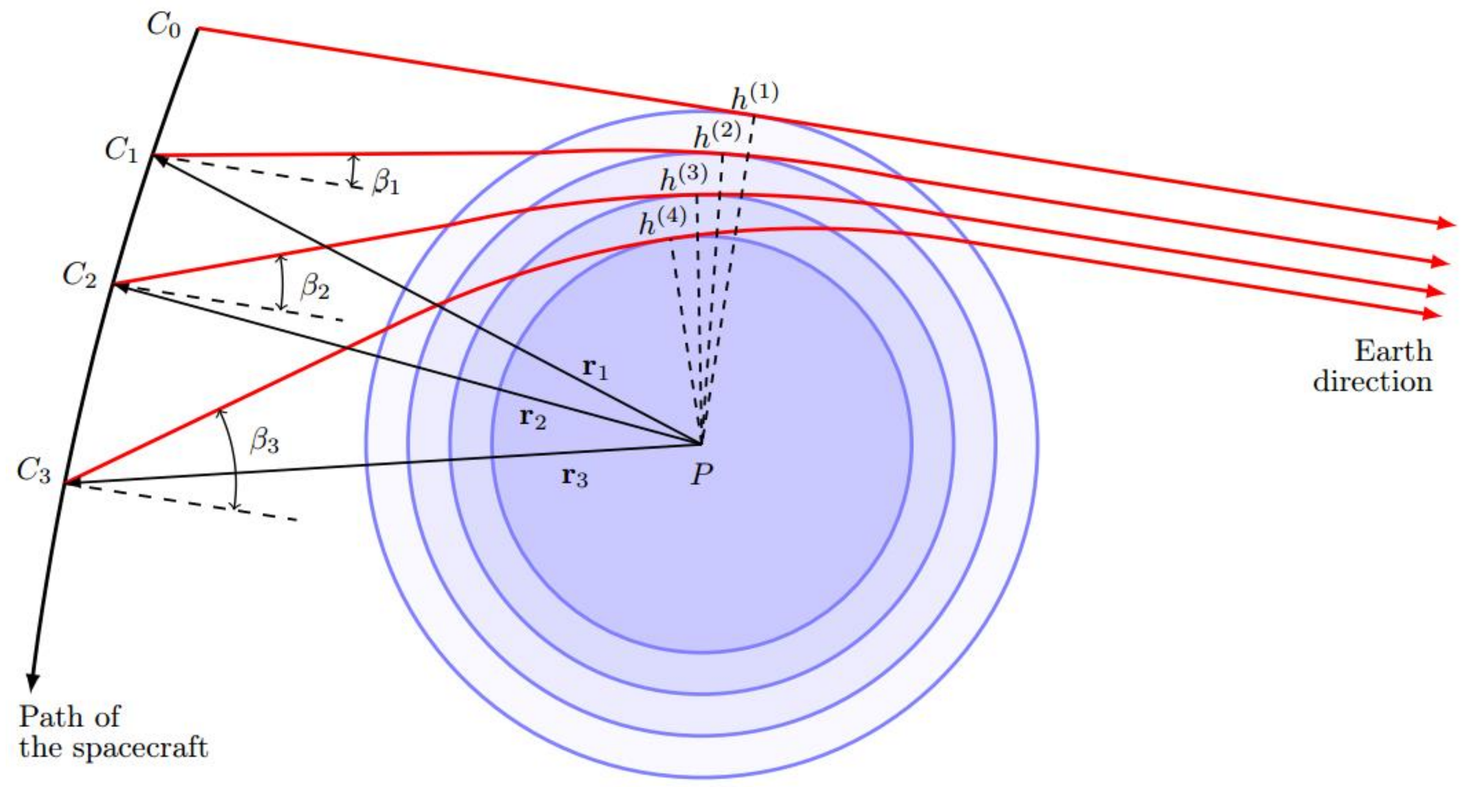


Figure 2: Sketch of the RT techniques applied to a multi-layered atmosphere. The initial pointing and the refractivity gradient are determined from the Doppler shift and from the condition that the ray reaches the Earth.

From the refractivity profile, the temperature and pressure profiles can be determined assuming an atmospheric chemical composition given *a priori*. Indeed, from the refractivity profile and the refractivity per mollecules, we can deduce the mean density in the atmosphere, and then by solving the hydrostatic equilibrium equation towards the profile, we get the temperature and pressure evolution with altitude [4]. In Fig. 3 is an example of Titan's atmospheric temperature profile computed from 3D RT technique.

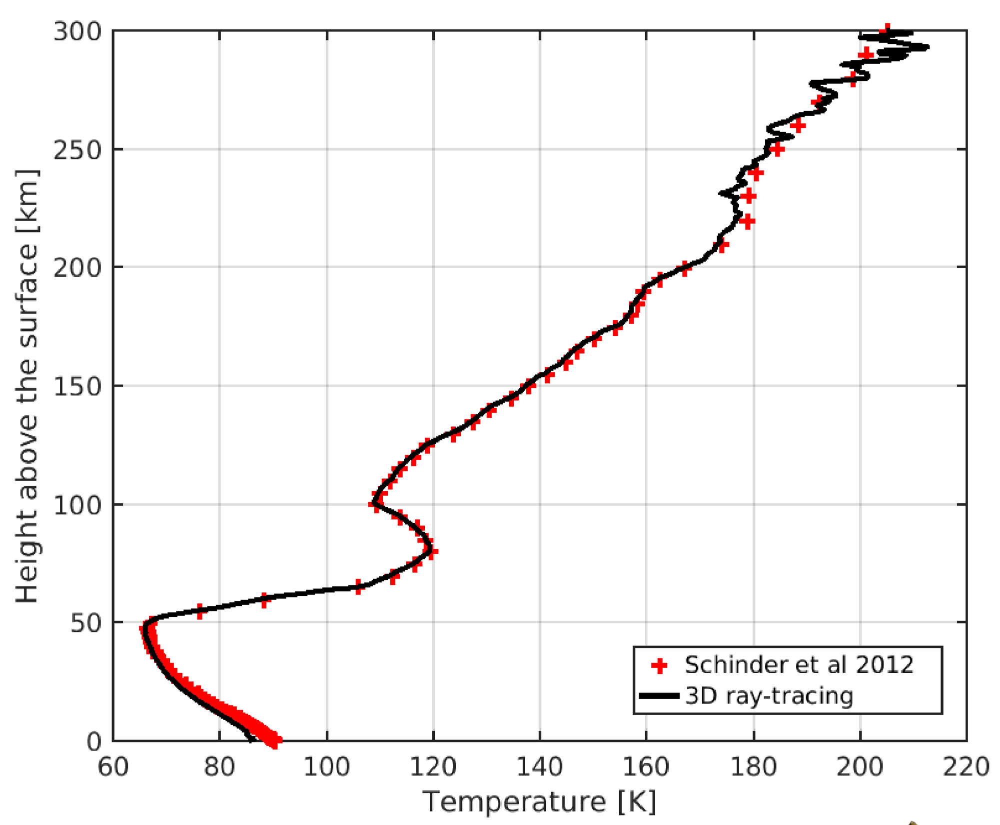
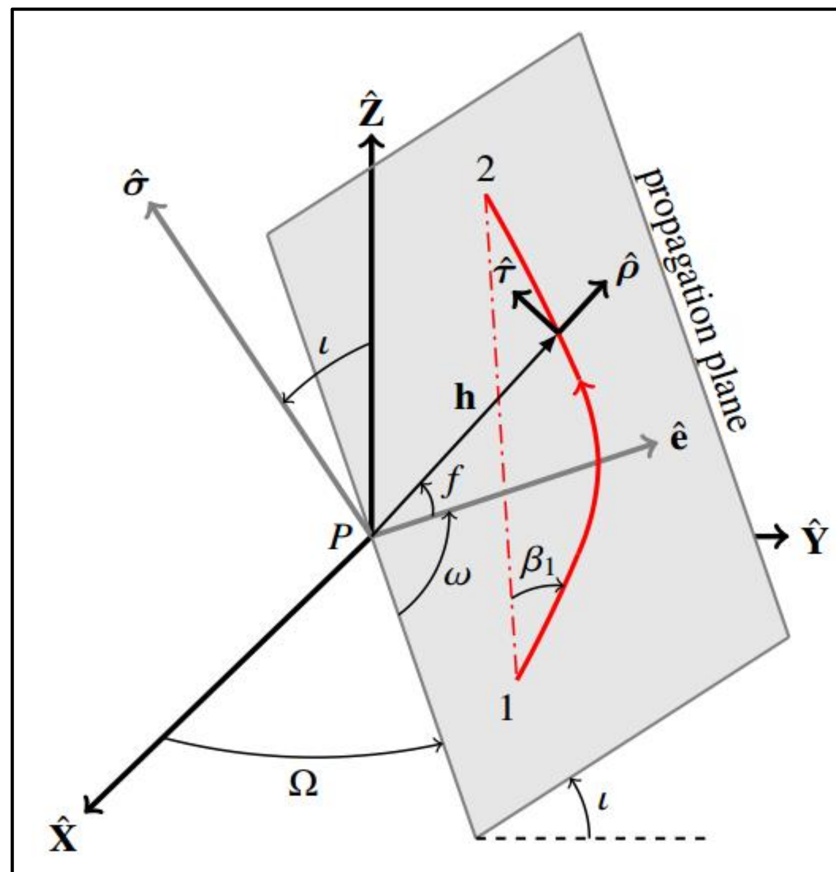


Figure 3: Titan atmospheric temperature profile computed with a 3D ray-tracing technique. The data used for the computation are X-band radiometric observables and have been acquired during the Cassini occultation performed on June 22nd, 2009 (T57). Schinder's results are taken from [5].

5. Analytical ray-tracing

Assuming that the generalized potential reduces to the Newtonian potential U_0, that is $n=n(U_0)$, it has been shown in [2] that Eqs. (1-3) admit a simple solution of reference named the "*hyperbolic solution*" (cf. Fig. 4). This solution is constrained by seven constants of integration called "*hyperbolic elements*" (the constants are collected into the vector **C**, see Fig. 4).



- Solution in the plane : (p, e, ω),
- Solution in space : (ι, Ω),
- Solution along the light path : (S, T),
- Hyperbolic solution :

$$\mathbf{h} = \mathbf{h}_0(s, \mathbf{C}), \tag{4}$$
$$\mathbf{n} = \mathbf{n}_0(s, \mathbf{C}), \tag{5}$$
$$t = t_0(s, \mathbf{C}), \tag{6}$$

- Generic solution :

$$\mathbf{h} = \mathbf{h}_0(s, \mathbf{C}(s)), \tag{7}$$
$$\mathbf{n} = \mathbf{n}_0(s, \mathbf{C}(s)), \tag{8}$$
$$t = t_0(s, \mathbf{C}(s)), \tag{9}$$

with $\mathbf{C} = (p, e, \iota, \Omega, \omega, S, T)$.

Figure 4: Hyperbolic solution. *Left panel*. Light ray trajectory in space. *Right panel*. Solution described with the help of constant hyperbolic elements.

In general, Φ cannot be reduced to the Newtonian potential alone and then the reference solution in Eqs. (4-6) cannot be employed. However, applying the method of variation of arbitrary constants, the hyperbolic solution allows us to rearrange Eqs. (1-2) into a set of six first order differential equations describing the evolution of the hyperbolic elements following the change $n(U_0) \rightarrow n(\Phi)$ (cf. Eqs. (54) of [2]). This set of equations, called "*perturbation equations*", is perfectly equivalent to Eqs. (1-2), however, at first order in the refractive perturbation they can be analytically solved more easily.

In Fig. 5, we show a comparison between results from analytical and numerical integrations of a light ray across an oblate atmosphere.

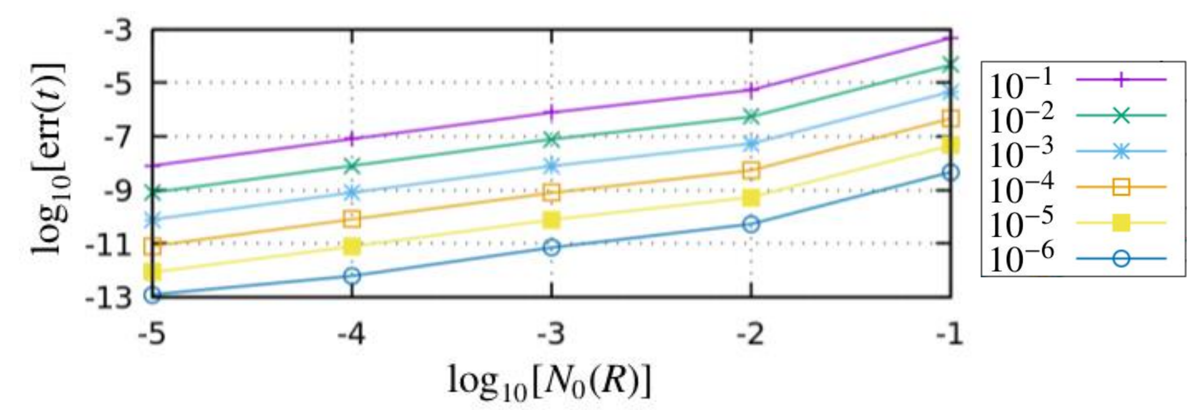


Figure 5: Relative error on the change in light-time after crossing an oblate atmosphere for different values of the refractivity at a given altitude R (x-axis), and for different values of J_2 (see values in the inset).

5. Future Work

The analytical RT could be used either to improve the determination of the pointing in the numerical RT (this would spare computation time), or either to replace the numerical RT considering the achievable accuracy.

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