# Local Branching: A Tutorial

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Joint work with Matteo Fischetti

Instances, codes, papers and slides at:
http://www.or.deis.unibo.it/research_pages/ORinstances/MIPs.html

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AIM: integrating local search and metaheuristic ideas within Mixed Integer Programming

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- Constraint (1) defines a neighborhood of $\bar{x}$, and, since the constraint is linear, the neighborhood can be explored using a generic MIP solver.

# A general MIP with 0-1 variables

- We consider a generic MIP with 0-1 variables of the form:

$$
\begin{aligned}
(P) \qquad & \min c^T x && && (2)\\
& Ax \geq b && && (3)\\
& x_j \in \{0,1\} && \forall j \in \mathcal{B} \neq \emptyset && (4)\\
& x_j \geq 0, \text{ integer} && \forall j \in \mathcal{G} && (5)\\
& x_j \geq 0 && \forall j \in \mathcal{C} && (6)
\end{aligned}
$$

- We consider the case in which $\mathcal{B} \neq \emptyset$ and more precisely, we assume that

> fixing the binary variables strongly simplifies the problem

- Moreover, we assume to have an initial solution, $\bar{x}$ at hand, so-called reference solution, and let $\overline{S} := \{j \in \mathcal{B} : \bar{x}_j = 1\}$ denote the binary support of $\bar{x}$.

# The local branching framework

- For a given positive integer parameter $k$, we define the *$k$-OPT neighborhood* $\mathcal{N}(\bar{x}, k)$ of $\bar{x}$ as the set of the feasible solutions of $(P)$ satisfying the additional *local branching constraint*:

$$\Delta(x, \bar{x}) := \sum_{j \in \overline{S}} (1 - x_j) + \sum_{j \in \mathcal{B} \setminus \overline{S}} x_j \leq k \tag{7}$$

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- When the cardinality of $\overline{S}$ of any feasible solution of $(P)$ is a constant, the local branching constraint assumes the asymmetric form:

$$\sum_{j \in \overline{S}} (1 - x_j) \leq k' \ (= k/2) \tag{8}$$

  which is the classical $k'$-OPT neighborhood for the *Symmetric Traveling Salesman Problem*.

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- The neighborhoods defined by the local branching constraints can be explored by using, as a black-box, a MIP solver, i.e., a standard tactical branching criterion such as, e.g., branching on fractional variables.

# The basic local branching scheme

① initial solution $\bar{x}^1$

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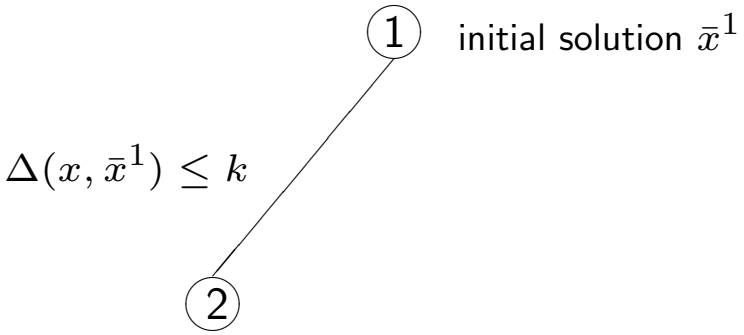

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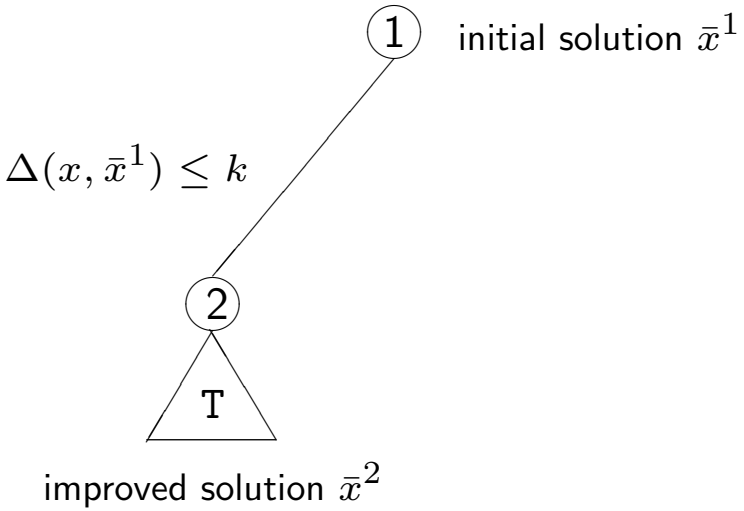

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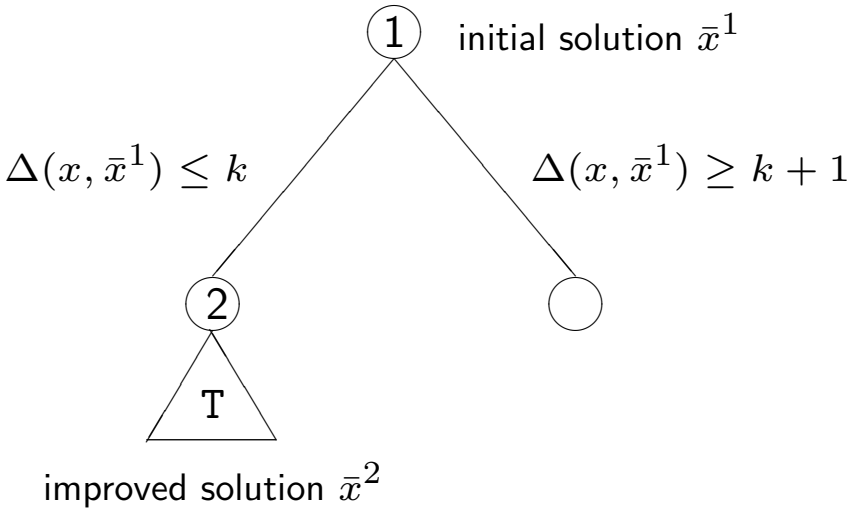

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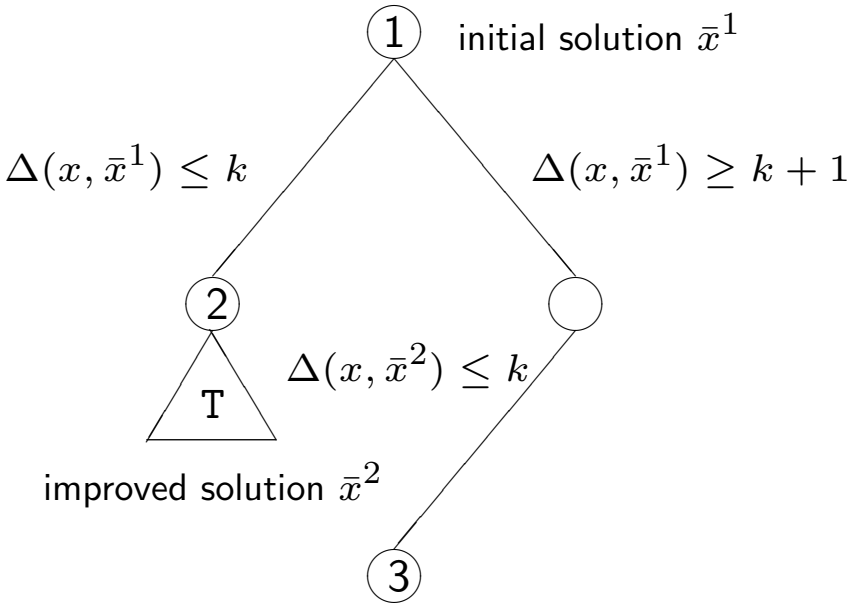

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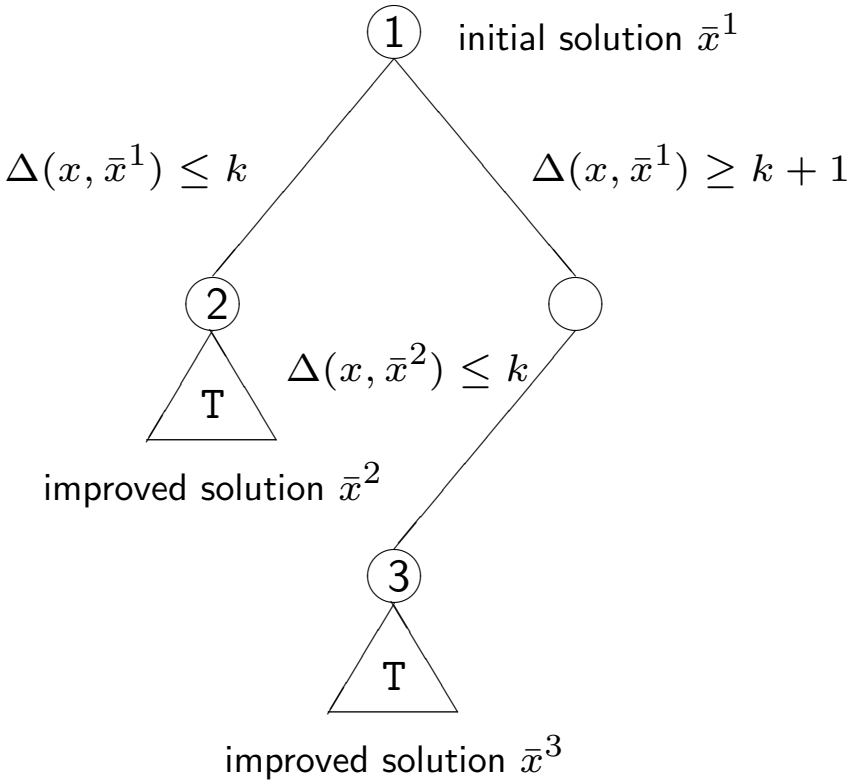

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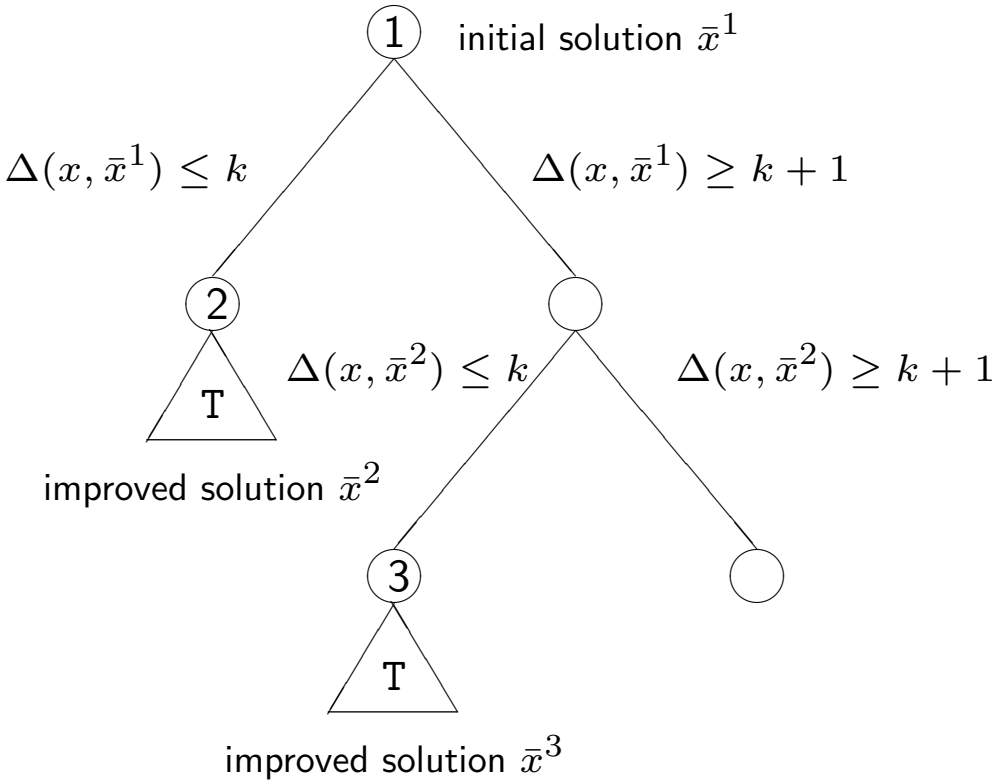

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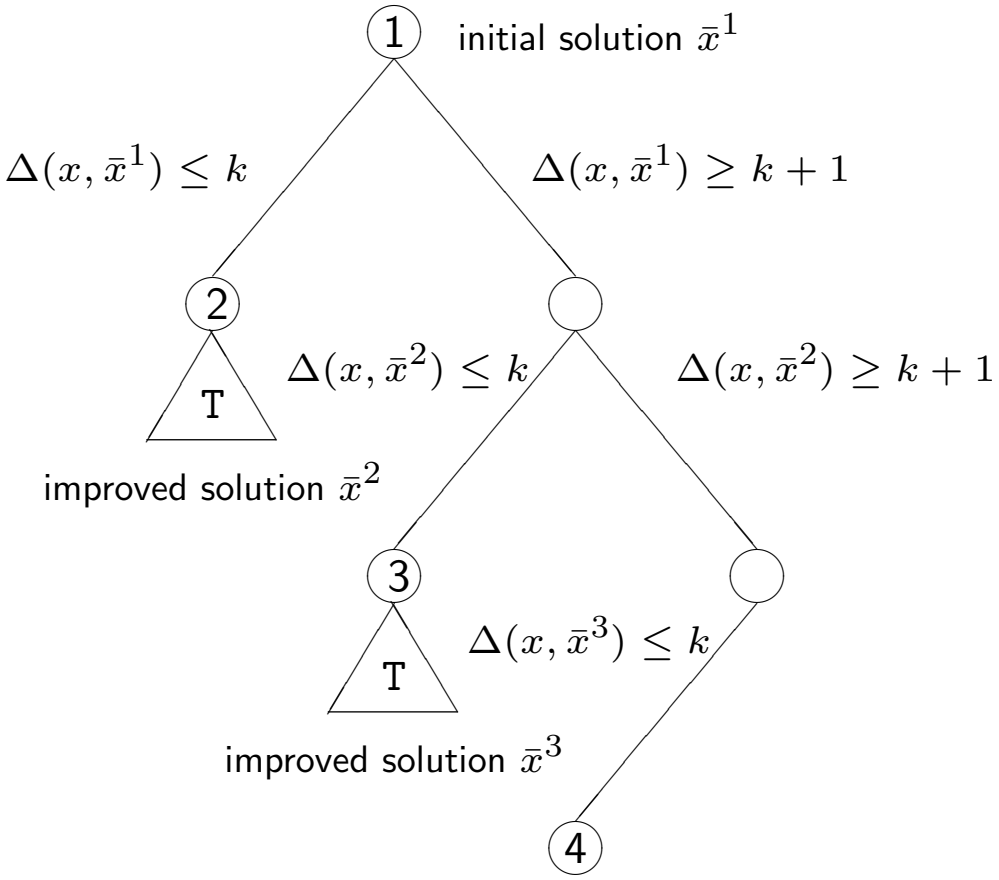

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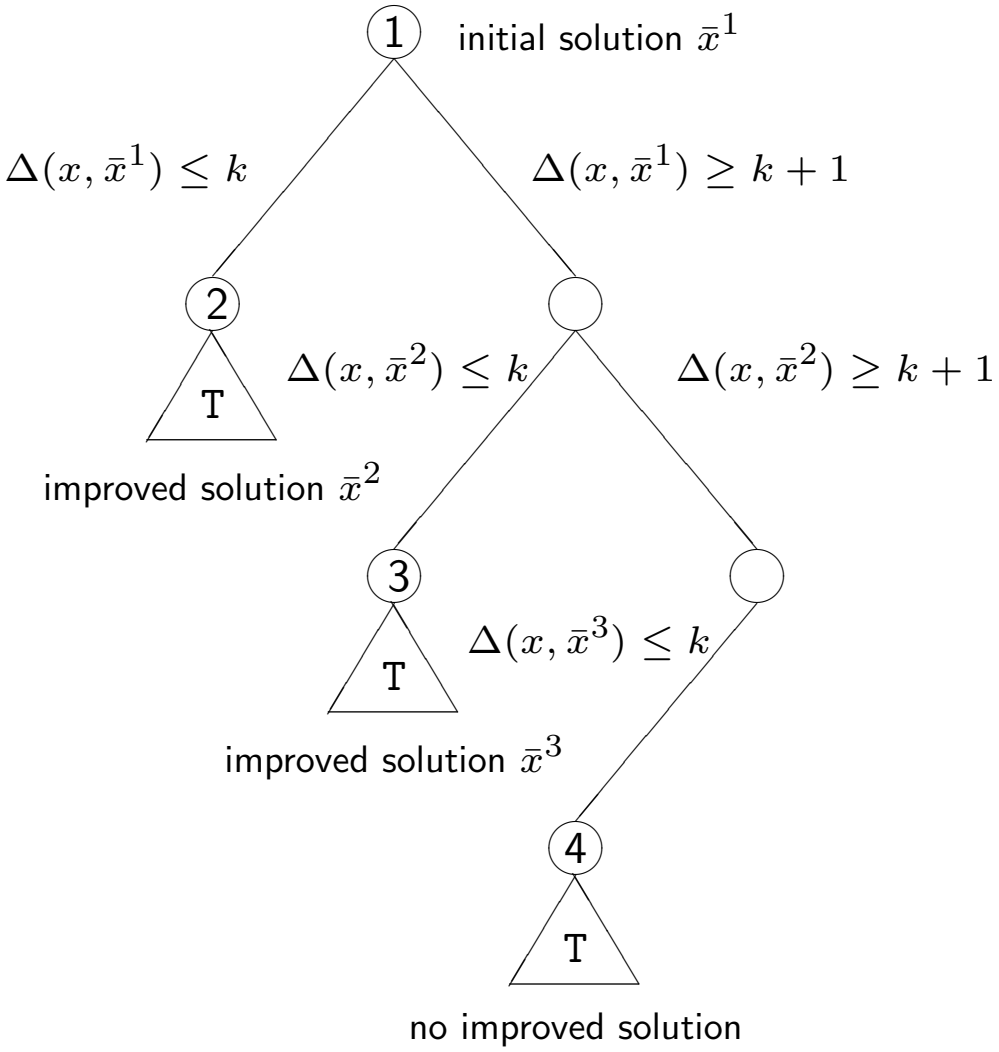

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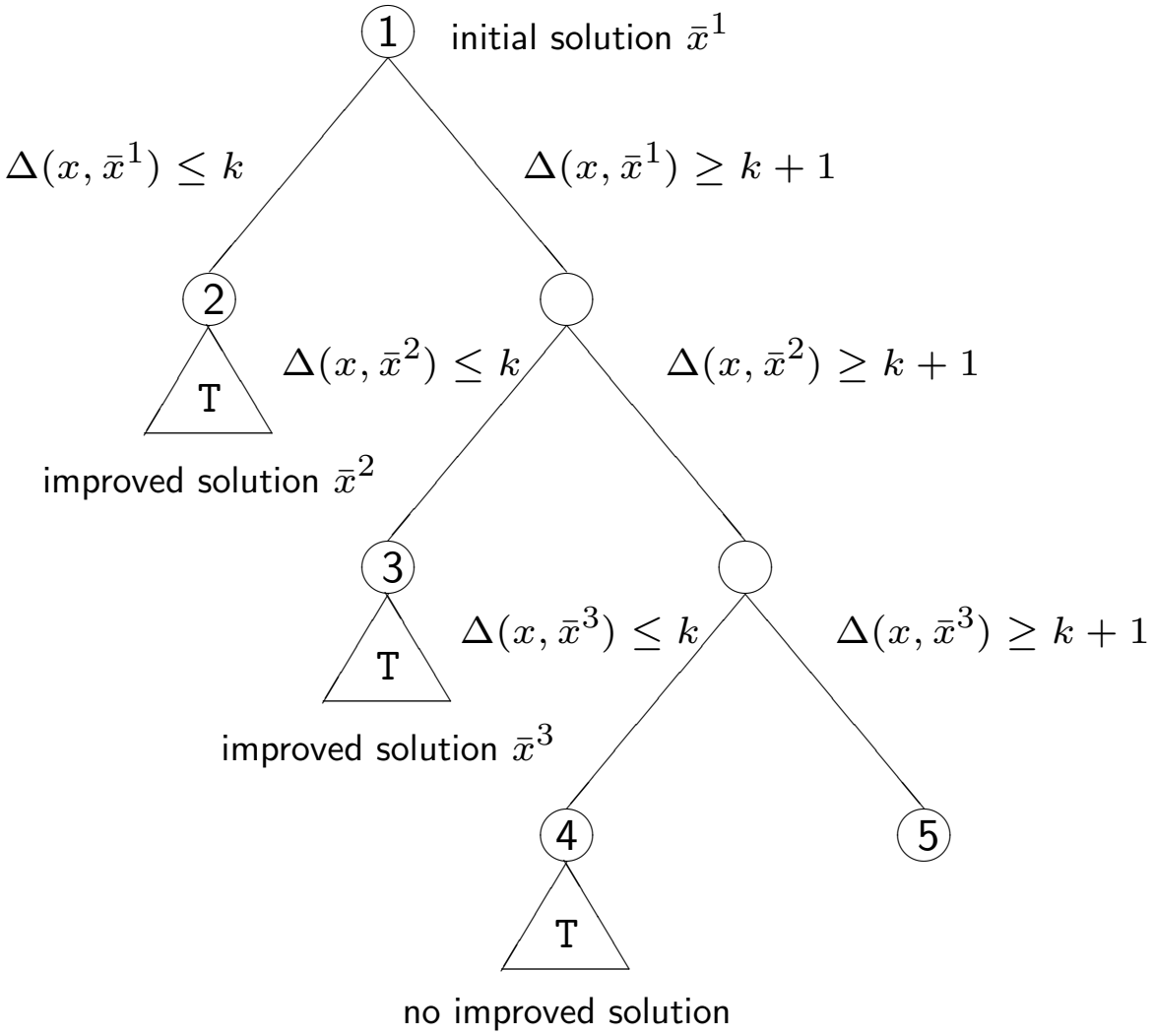

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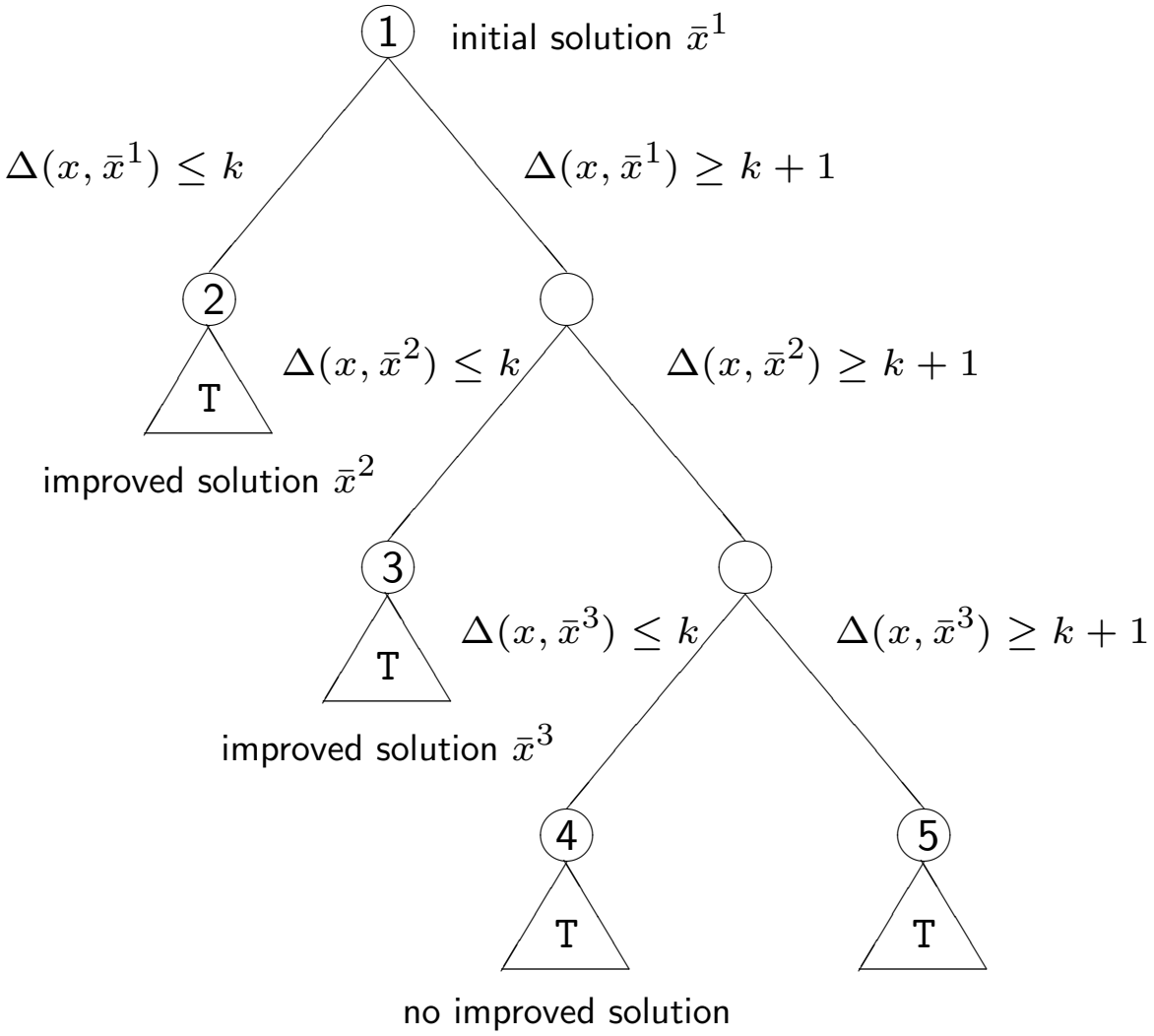

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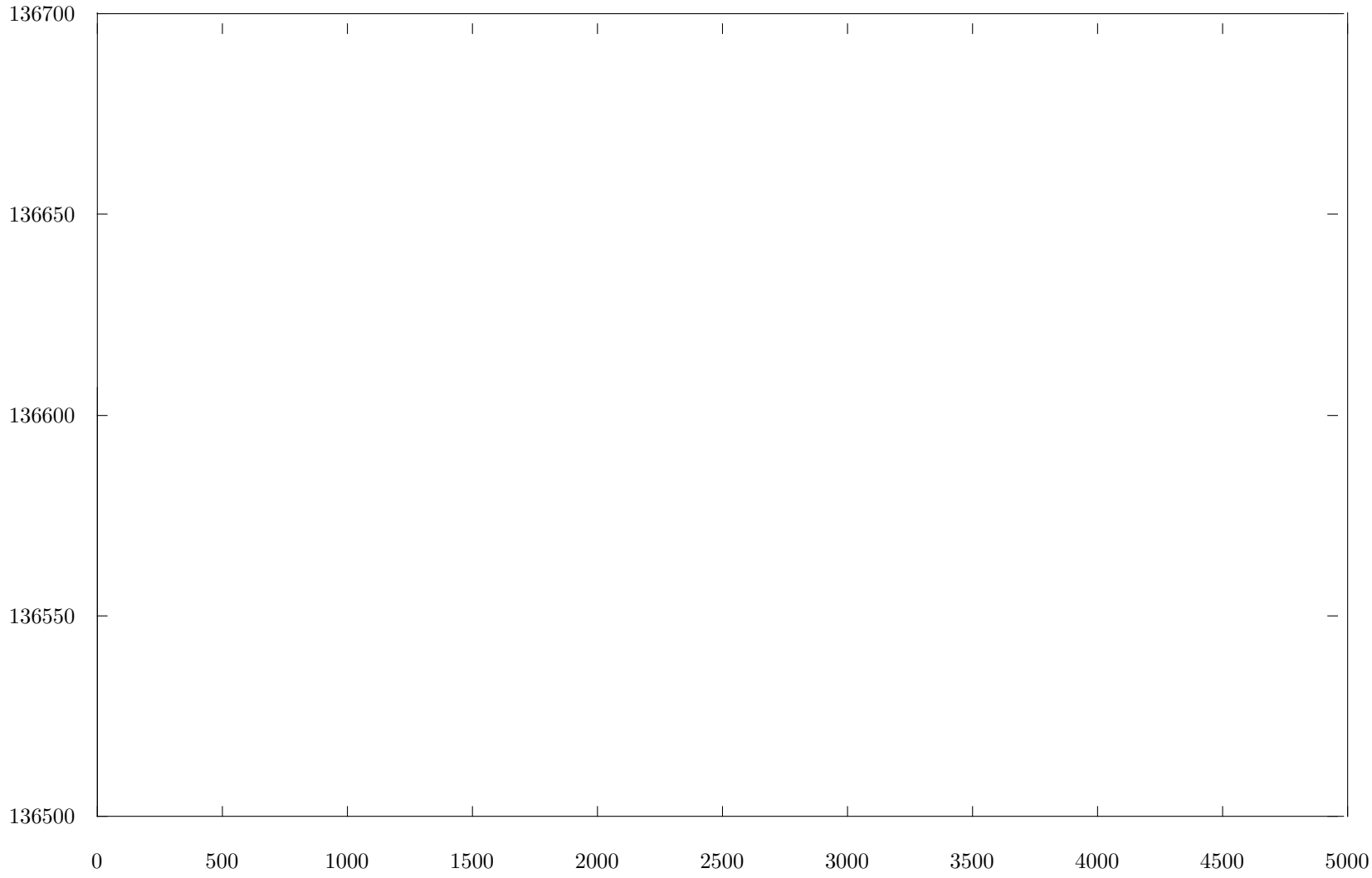

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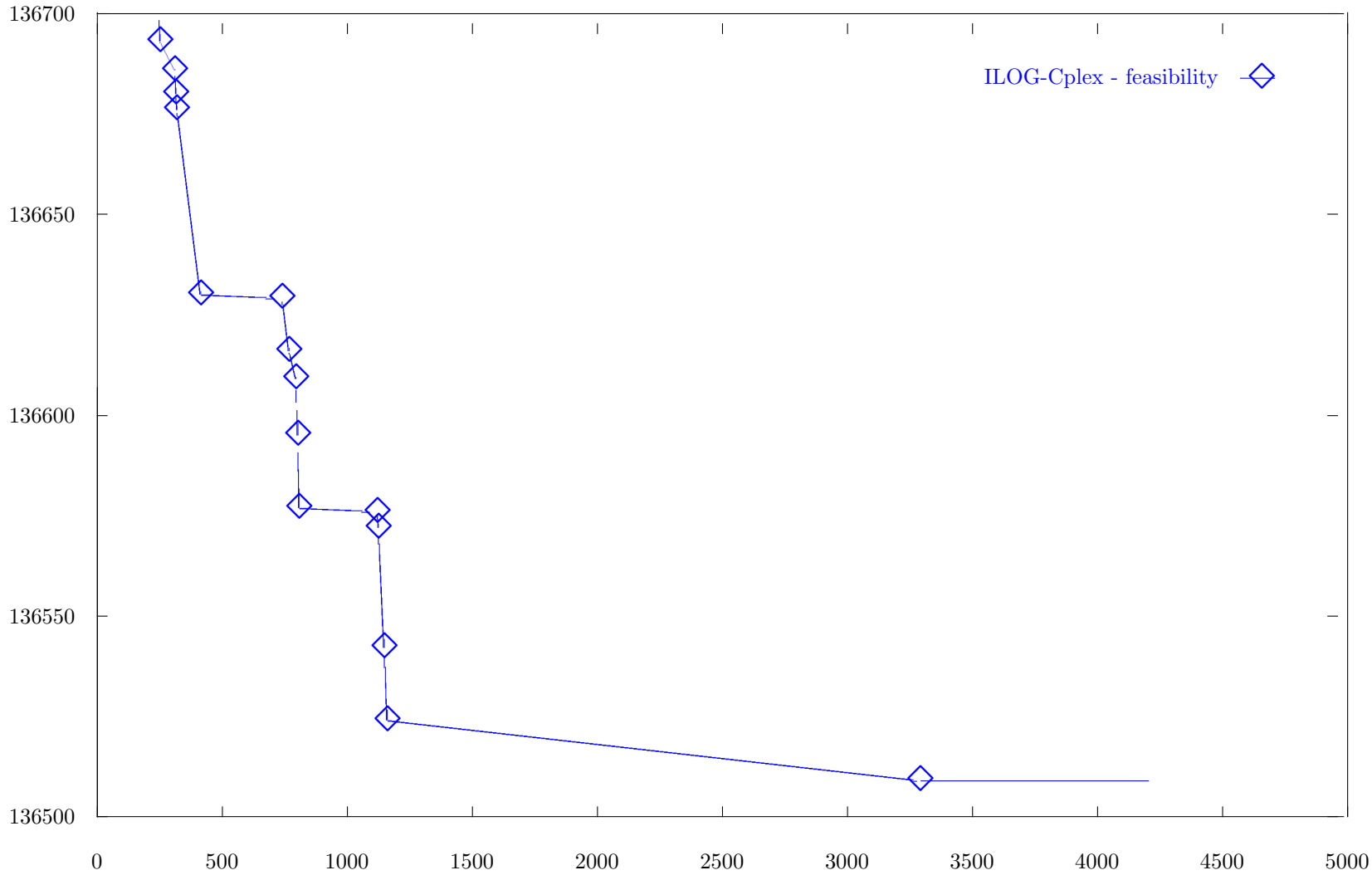

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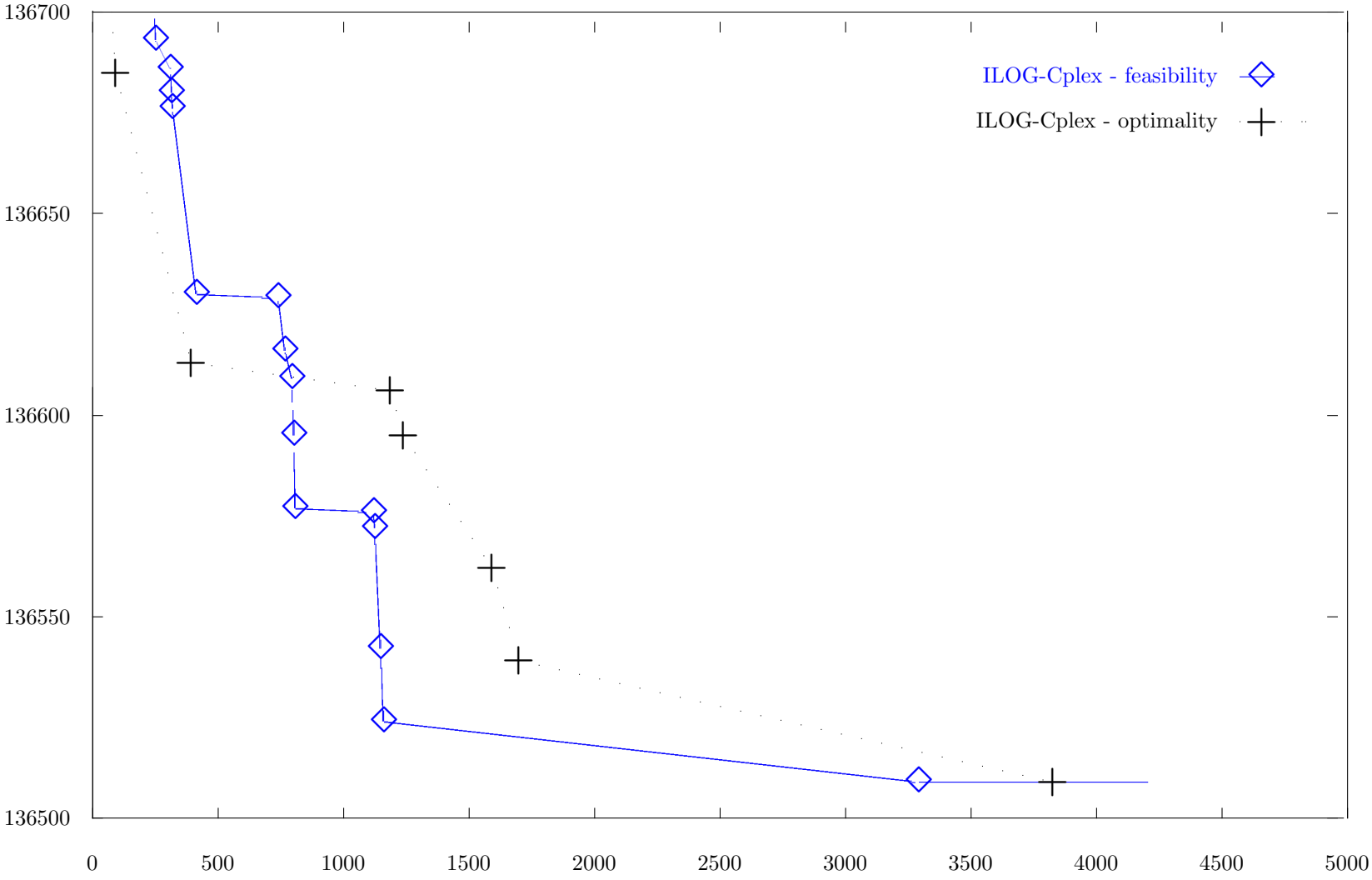

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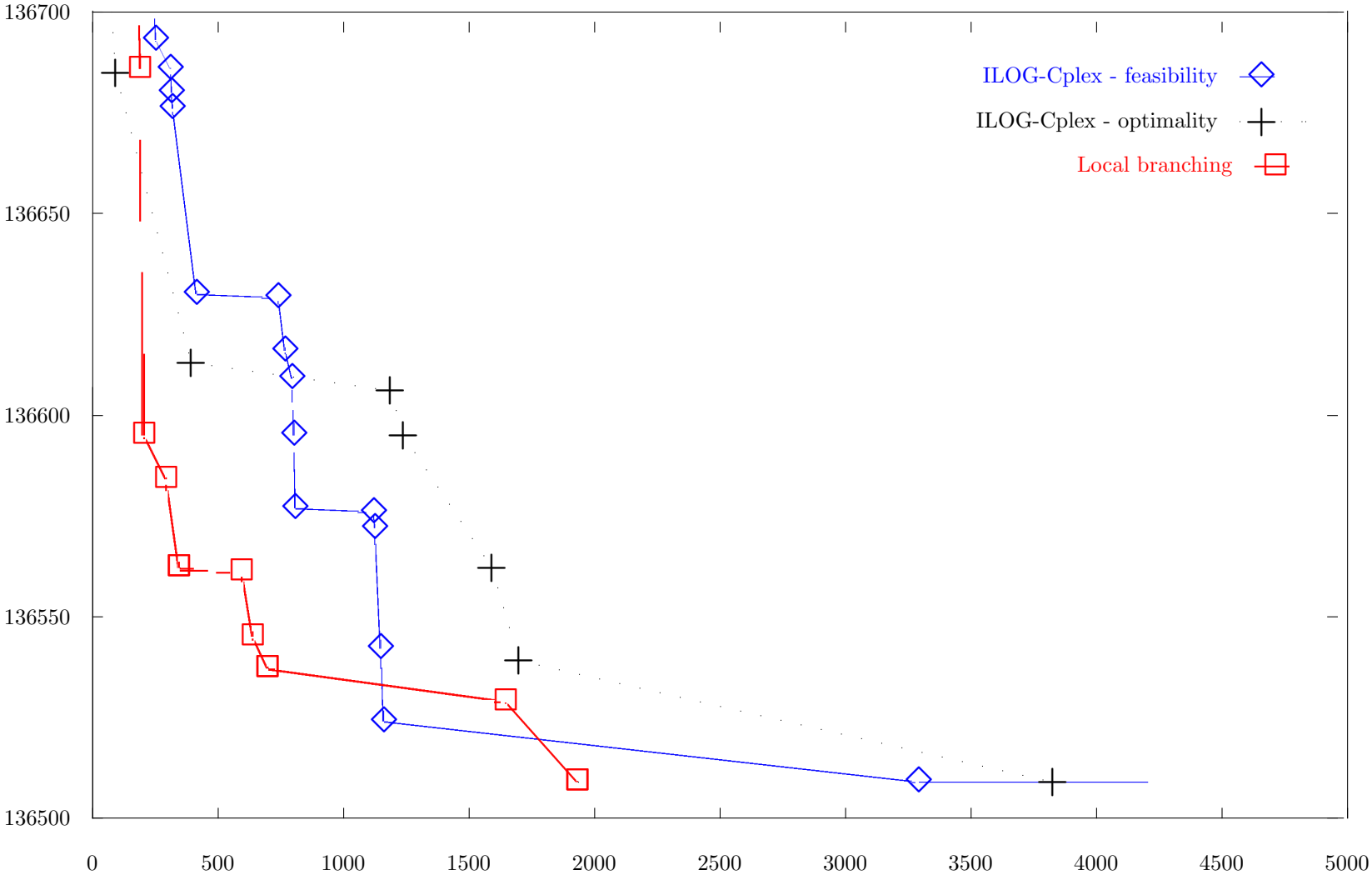

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- In the nodes of the scheme which are explored through tactical branching (T-nodes), a large number of branch-and-bound nodes are enumerated but the information associated with them is in some sense "lost" in the following.
- The enhanced convergence behavior of the local branching scheme in proving optimality cannot be guaranteed in all cases: we are currently working to a project devoted to this specific matter.

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- **Diversification**:

  A further improvement of the heuristic performance of the method can be obtained by exploiting well-known diversification mechanisms borrowed from local search metaheuristics.

  In our scheme, diversification is worth applying whenever the current left-node is proved to contain no improving solutions. Div

# Working with a node time limit

In case the time limit is exceeded, we have two cases:

- **Case (a)**:
  If the incumbent solution has been improved, we backtrack to the father node and create a new left-branch node associated with the new incumbent solution, without modifying the value of parameter $k$. case (a)

- **Case (b)**:
  If the time limit is reached with no improved solution, instead, we reduce the size of the neighborhood in an attempt to speed-up its exploration.
  This is obtained by reducing the right-hand side term by, e.g., $\lceil k/2 \rceil$. case (b)

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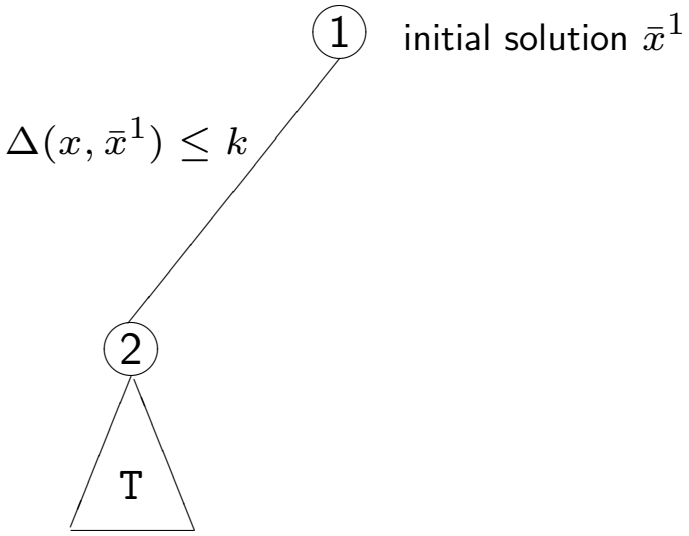

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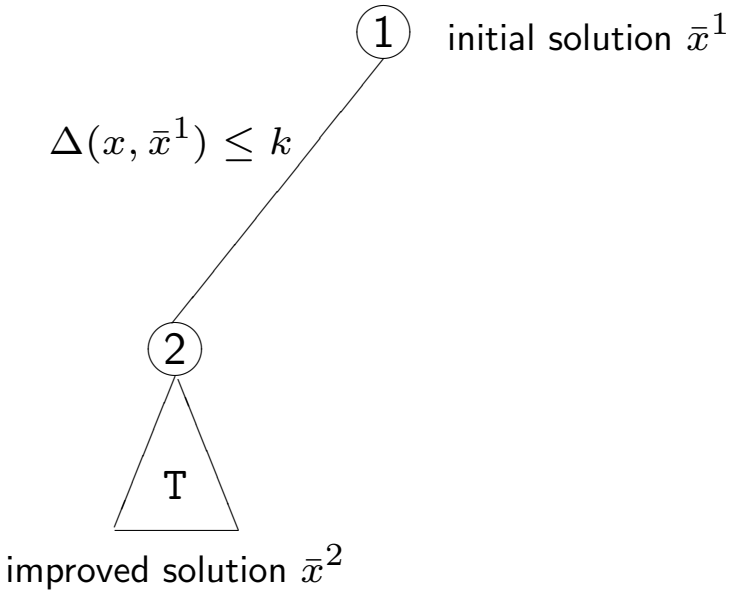

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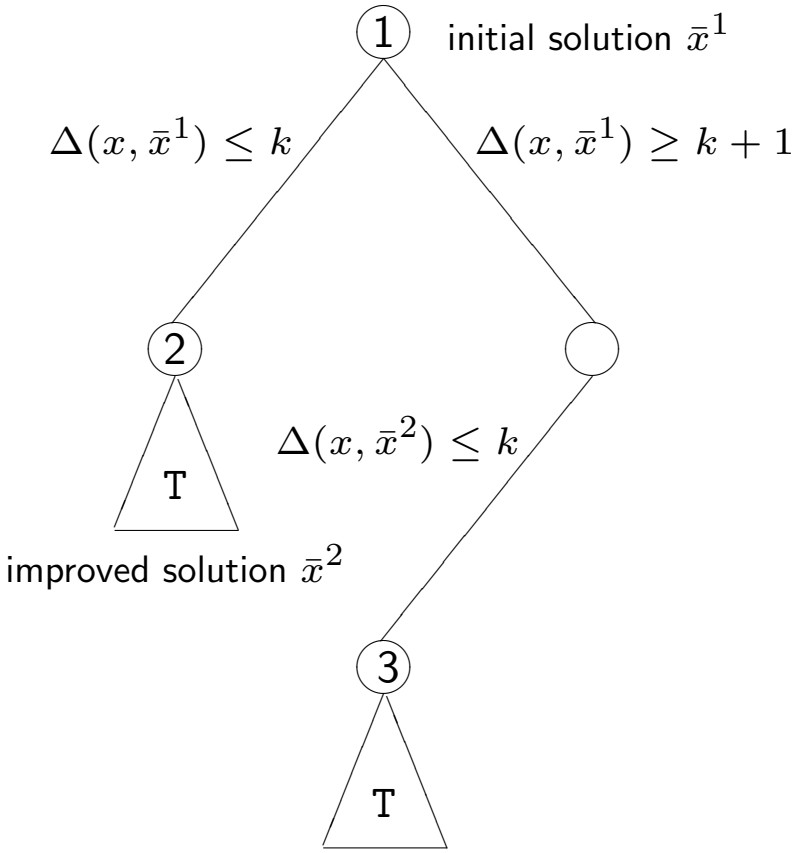

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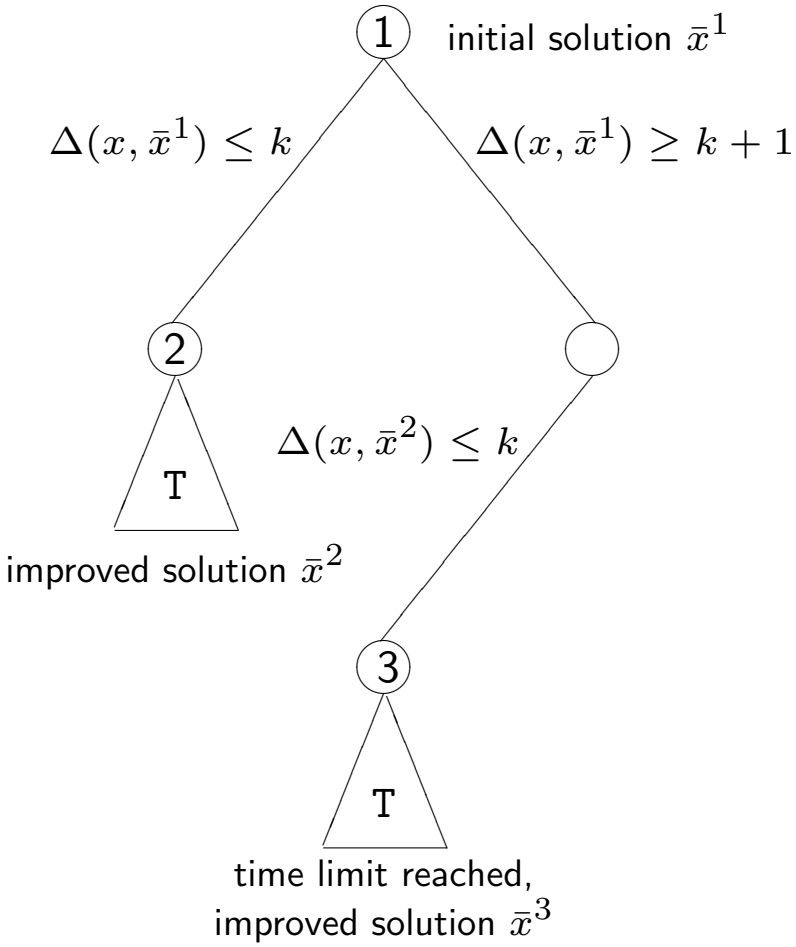

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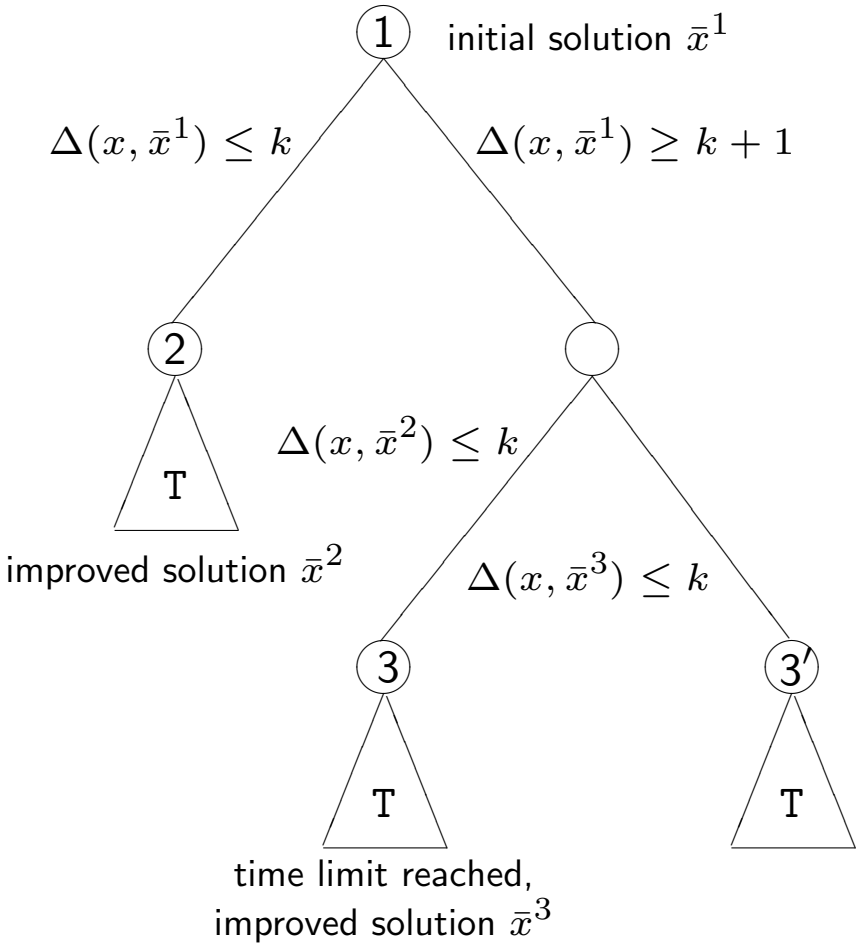

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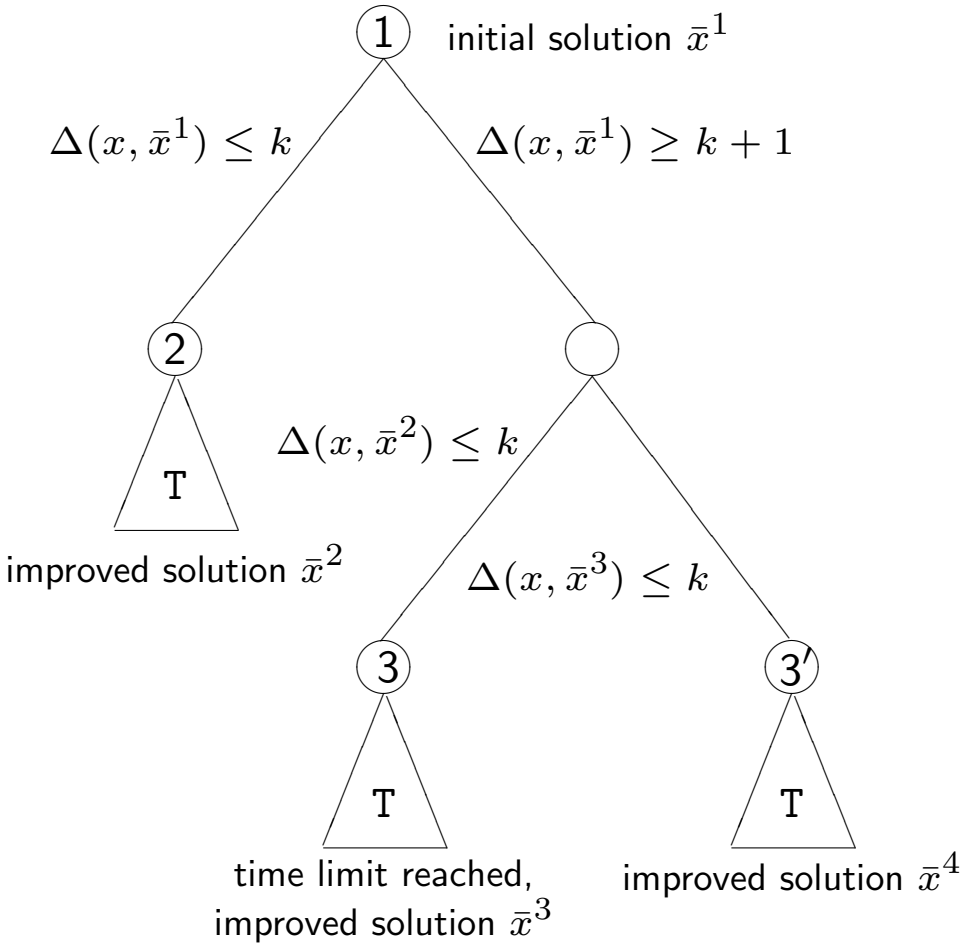

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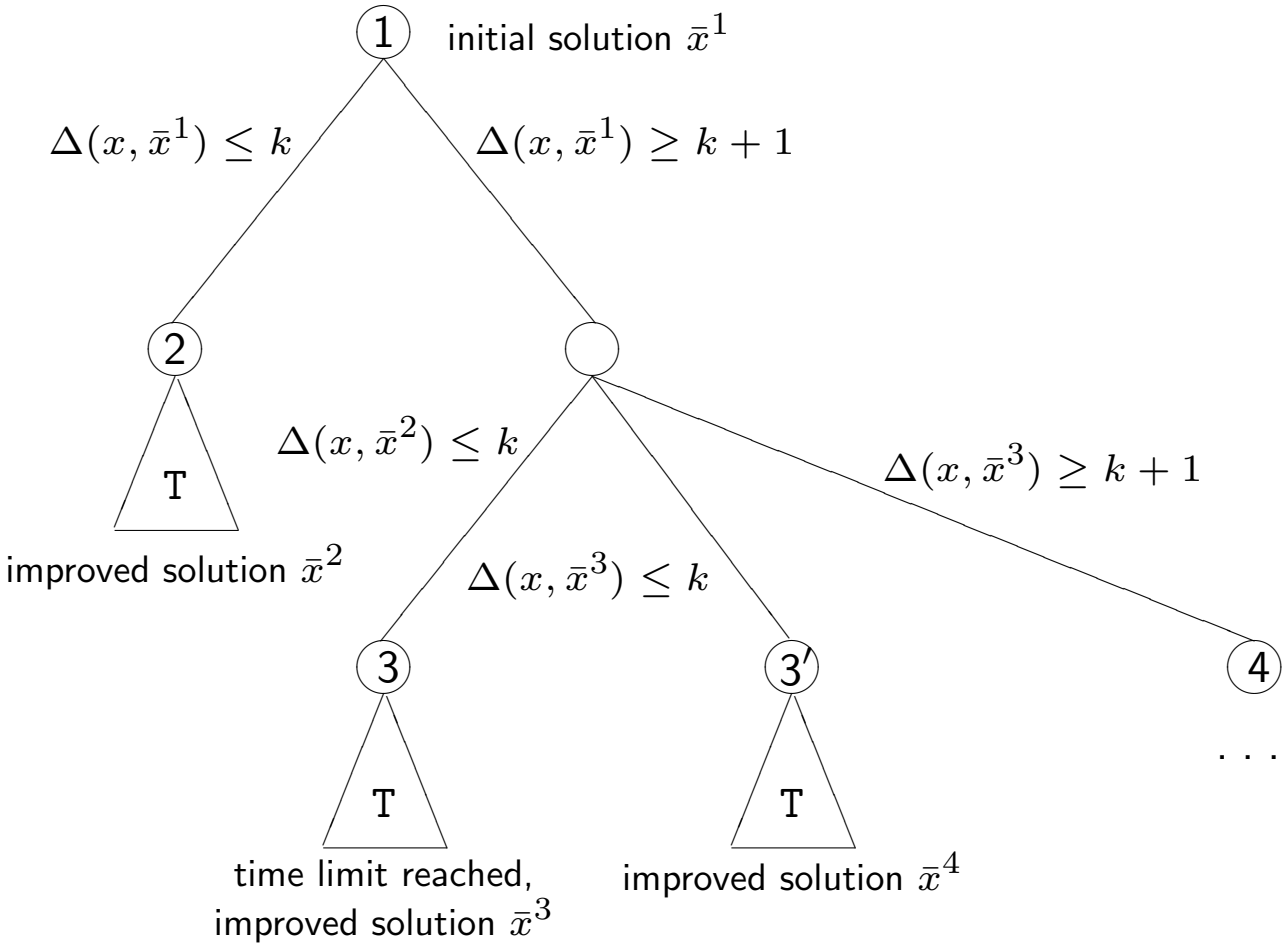
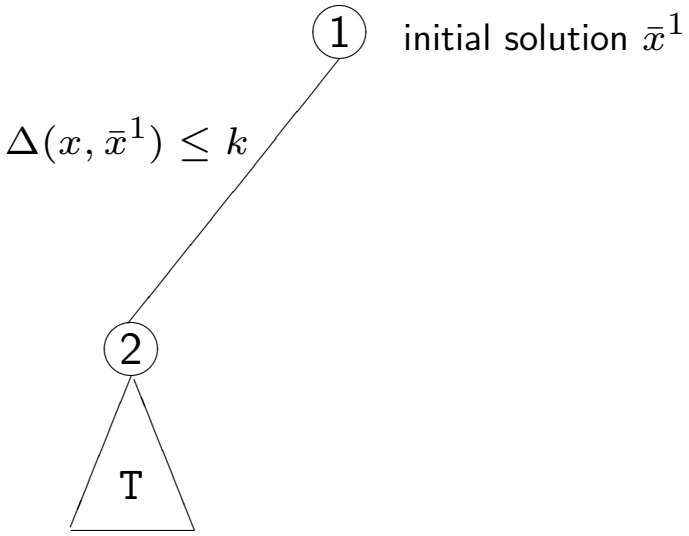



go back

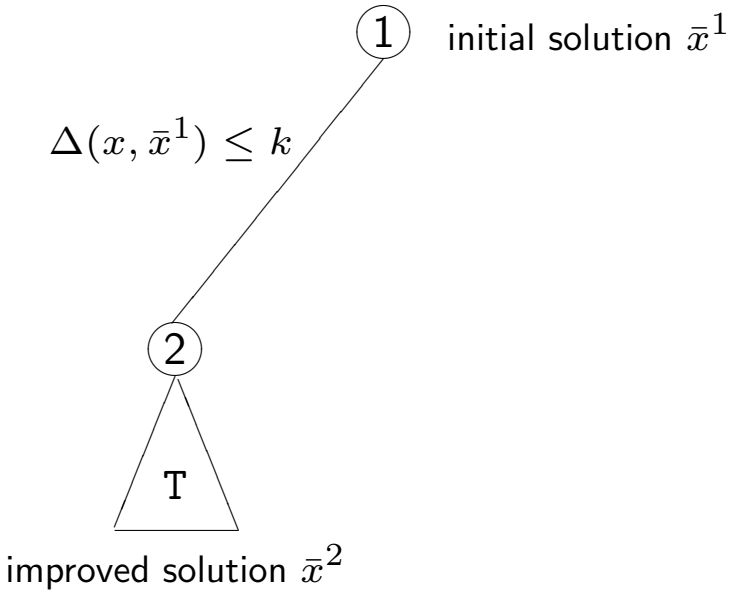
# Working with a node time limit: case (b)

① initial solution $\bar{x}^1$

# Working with a node time limit: case (b)



# Working with a node time limit: case (b)



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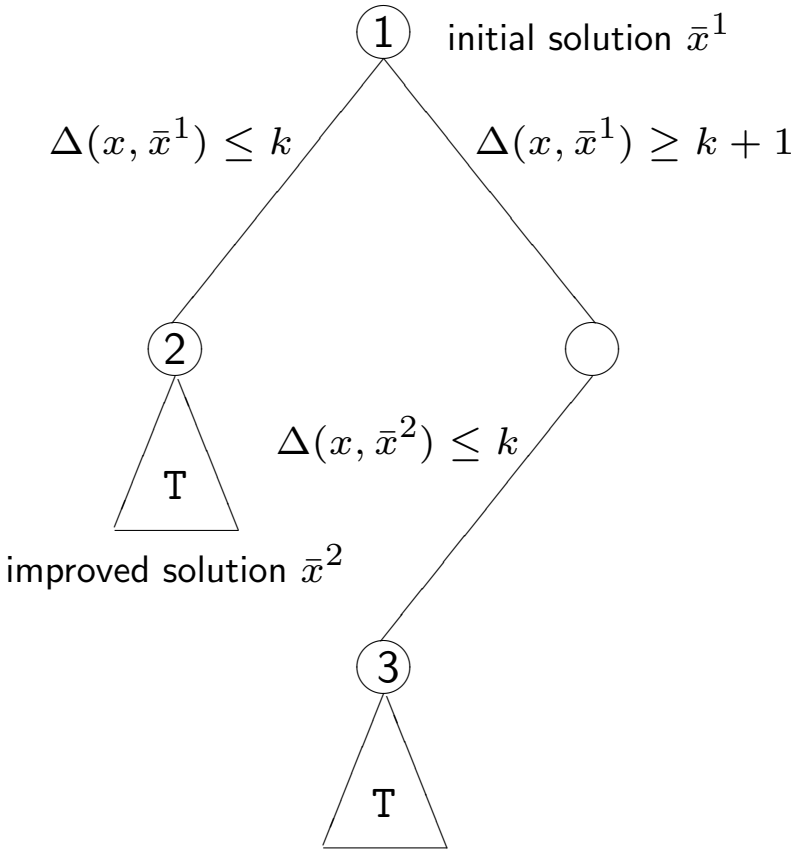

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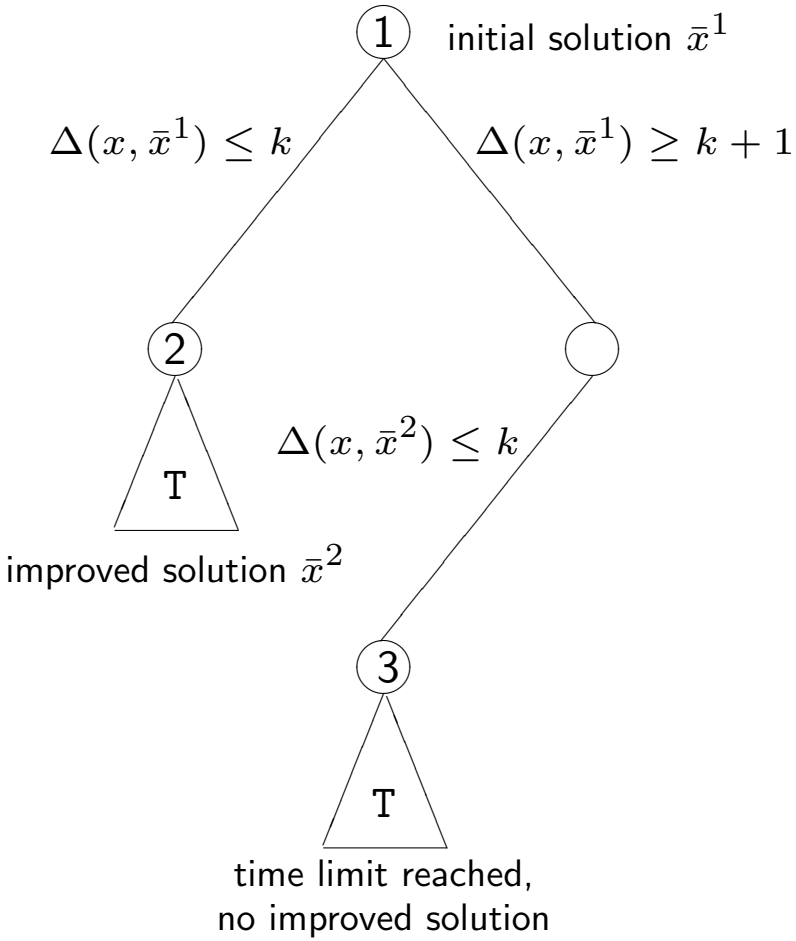

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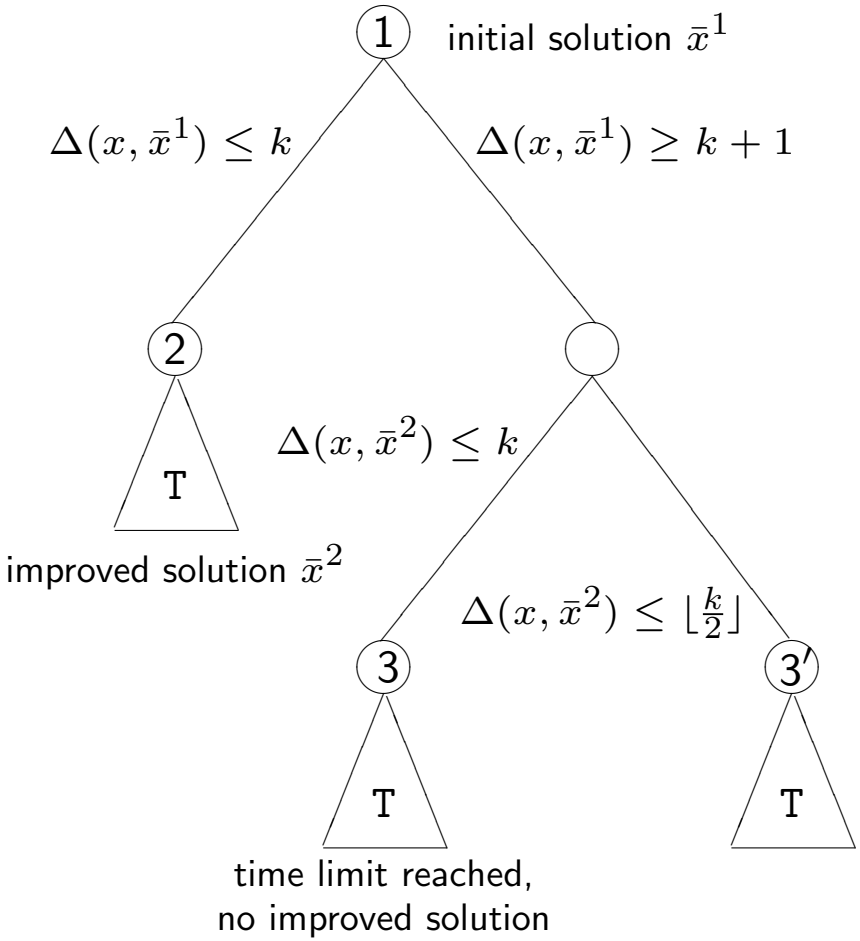

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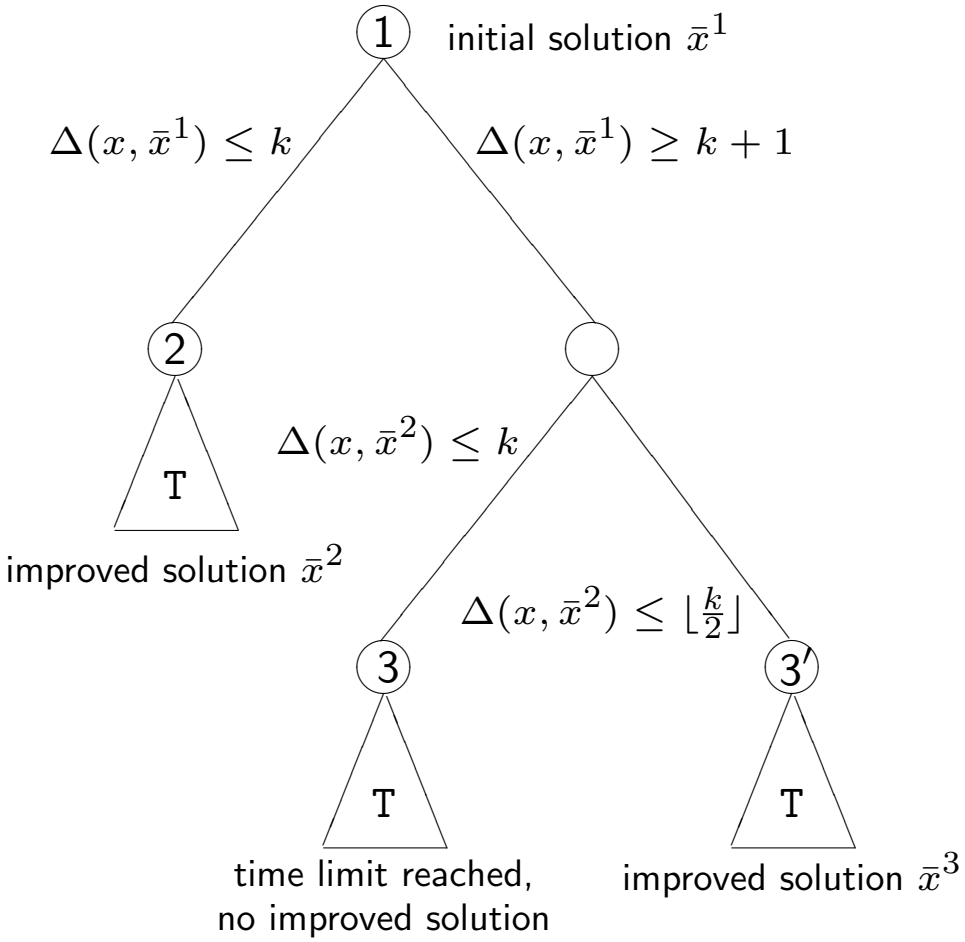

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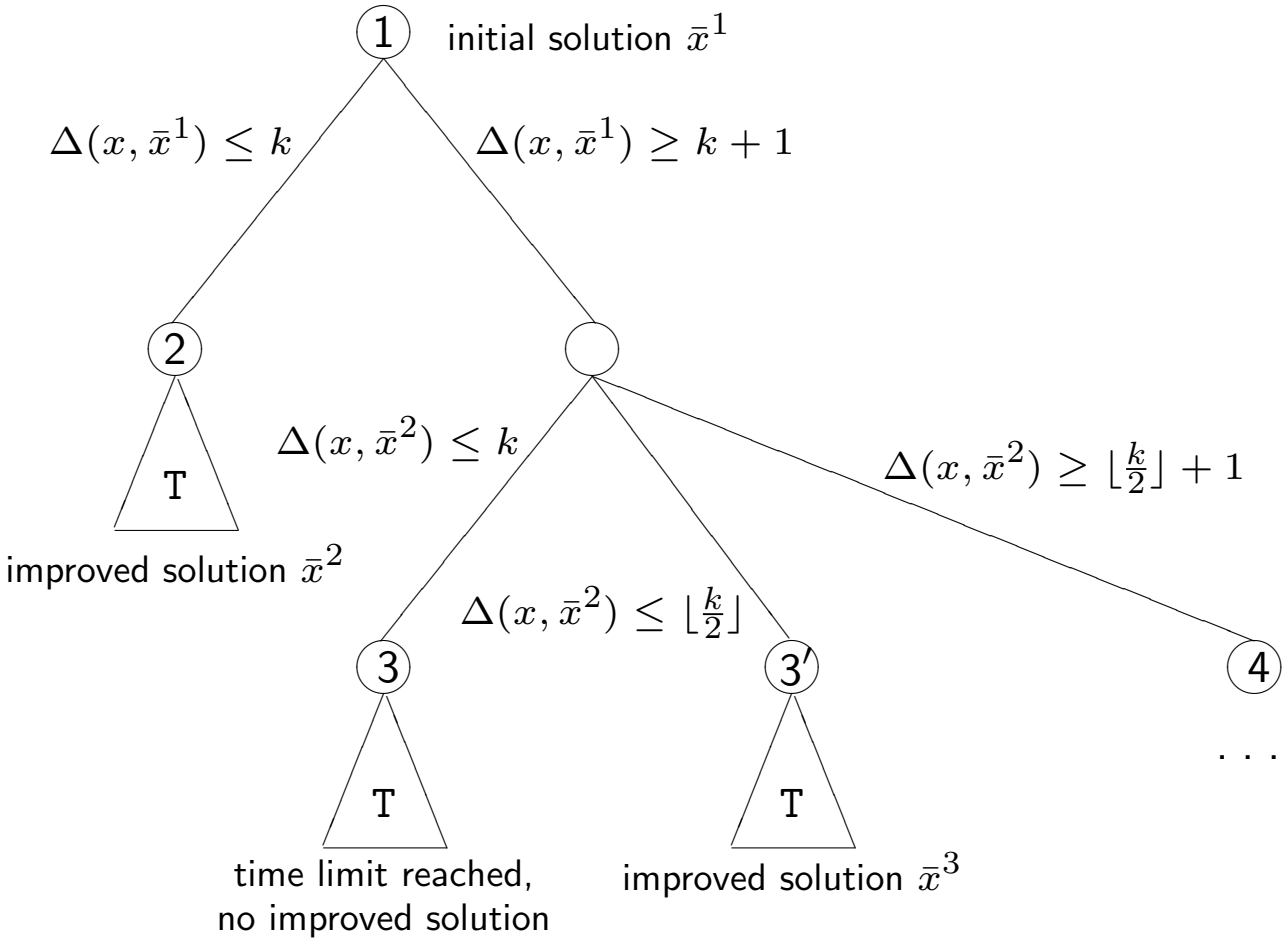
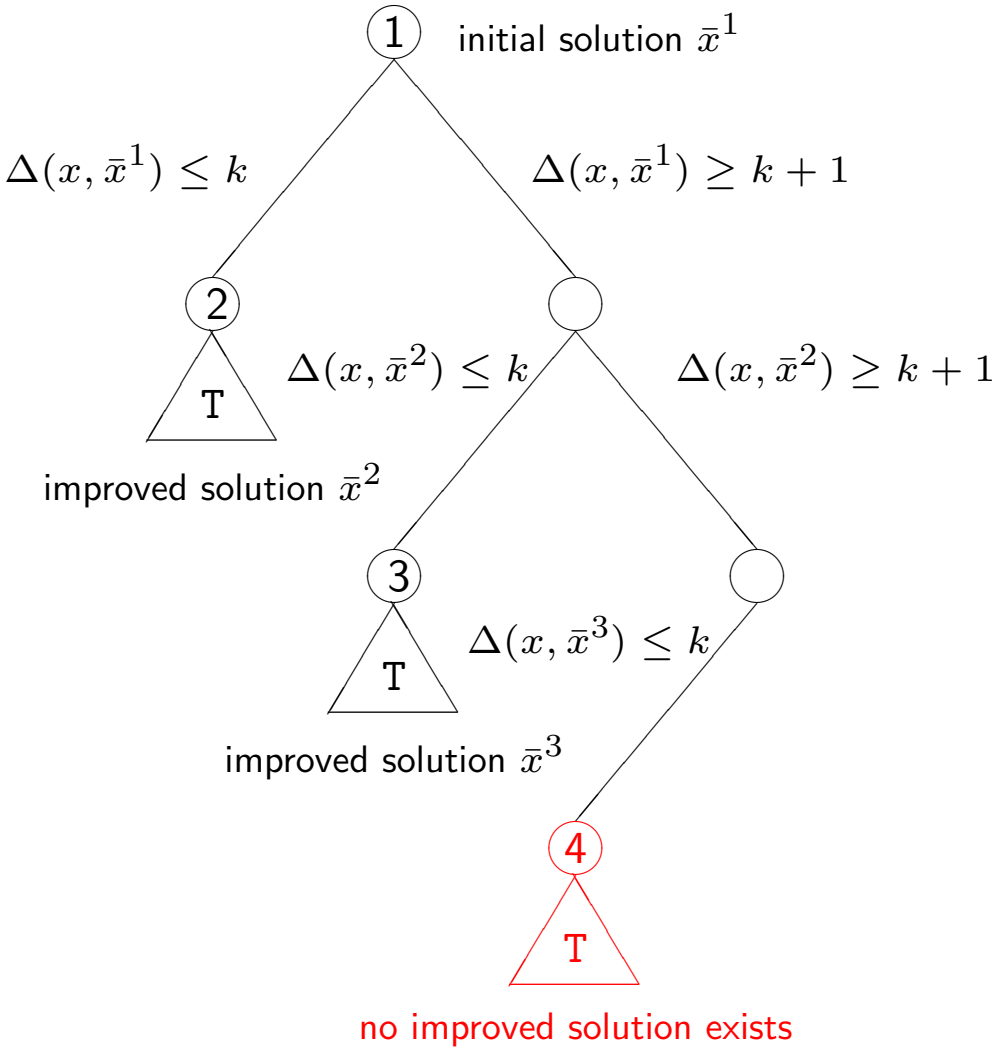



go back

# Diversification



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- **Soft** diversification:

  We first apply a "soft" action consisting in *enlarging* the current neighborhood by increasing its size by, e.g., $\lceil k/2 \rceil$.

  A new "left-branch" is then explored and in case no improved solution is found even in the enlarged neighborhood (within the time limit), we apply a stronger action in the spirit of *Variable Neighborhood Search*. [Mladenovíc & Hansen, 1997]

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- **Strong** diversification:

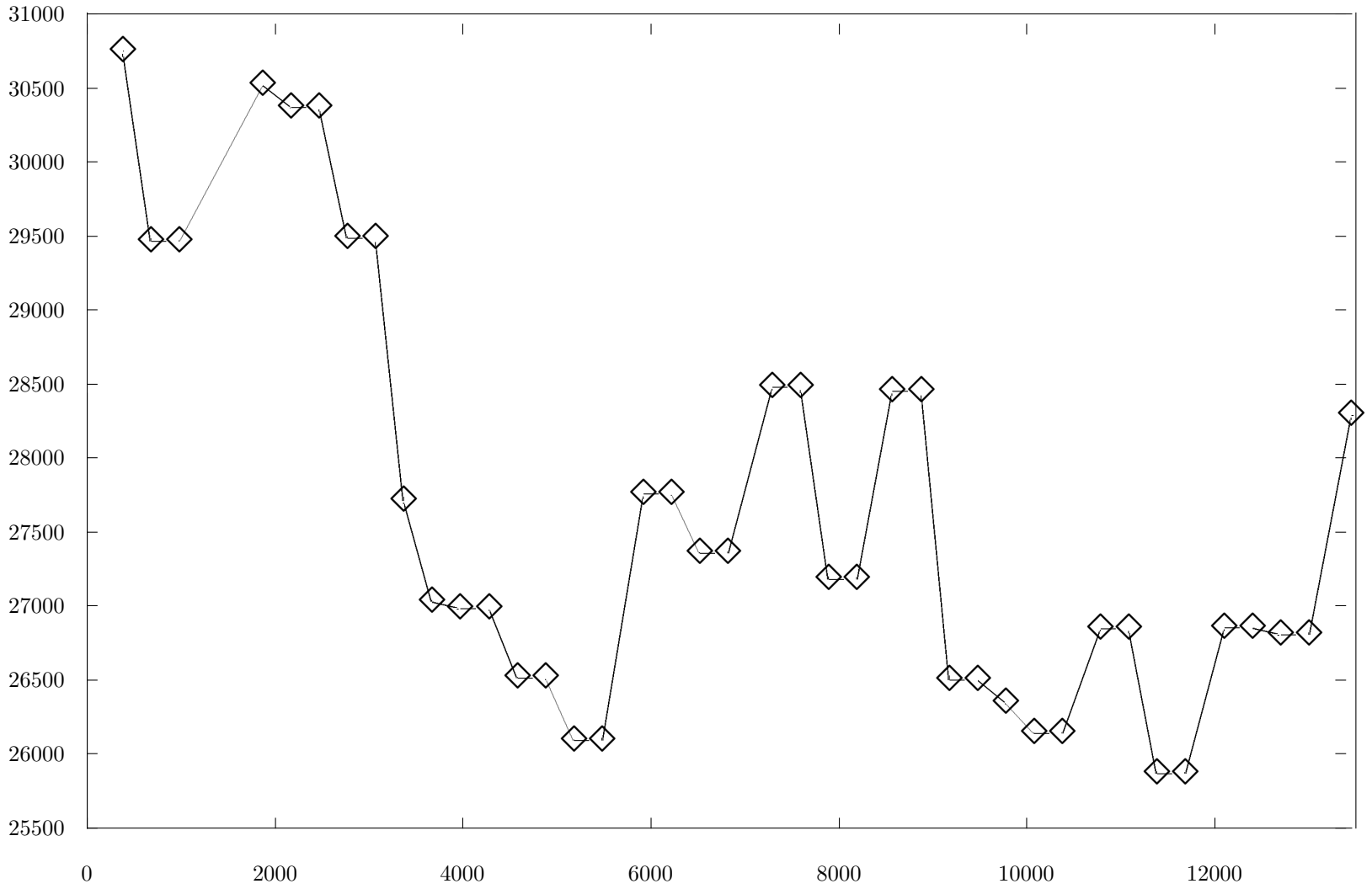
  We look for a solution (typically worse than the incumbent one) which is not "too far" from the current reference solution.

  We apply tactical branching to the current problem amended by $\Delta(x, \bar{x}^3) \leq k + 2\lceil k/2 \rceil$, but without imposing any upper bound on the optimal solution value.

  The exploration is aborted as soon as the first feasible solution is found.

  This solution (typically worse than the current best one) is then used as the new reference solution.

# LocBra as a heuristic for instance B1C1S1 (solution value vs. CPU seconds)



# Computational results (1)

| | | 1 hour | | | 3 hours | | | 5 hours | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Name | Gap | cpx-O | cpx-F | LocBra | cpx-O | cpx-F | LocBra | cpx-O | cpx-F | LocBra |
| mkc | % | 3.765 | 2.399 | 2.281 | 3.765 | 1.294 | 0.404 | 3.765 | 1.294 | **0** |
| swath | % | 94.504 | 2.507 | 1.599 | 26.374 | 1.153 | **0** | 26.374 | 1.153 | **0** |
| danoint | % | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** |
| markshare1 | Abs. | 1 | 5 | 10 | 1 | **0** | 2 | 1 | **0** | 2 |
| markshare2 | Abs. | 9 | 1 | 34 | 9 | **0** | 11 | 9 | **0** | 11 |
| arki001 | % | 0.024 | 0.028 | **0** | 0.017 | 0.016 | **0** | 0.013 | 0.013 | **0** |
| seymour | Abs. | 11 | 5 | 2 | 11 | 3 | **0** | 11 | 3 | **0** |
| net12 | Abs. | – | – | – | 41 | – | 41 | 41 | **0** | 41 |
| biella1 | % | 0.256 | 31.313 | 0.241 | 0.256 | 18.272 | 0.172 | 0.256 | 11.475 | **0** |
| NSR8K* | % | – | – | – | 1843.8 | 1526.4 | 109.3 | 1426.8 | 1526.4 | **0** |
| rail507 | Abs. | 2 | 6 | 1 | 2 | 5 | **0** | 1 | 5 | **0** |
| rail2536c | Abs. | 5 | 3 | **0** | 1 | 3 | **0** | **0** | 3 | **0** |
| rail2586c | Abs. | 54 | 34 | 7 | 14 | 34 | 2 | 14 | 34 | **0** |
| rail4284c | Abs. | 51 | 40 | 10 | 42 | 40 | 2 | 38 | 40 | **0** |
| rail4872c | Abs. | 73 | 69 | 27 | 70 | 49 | 9 | 45 | 49 | **0** |

*Gaps for NSR8K refer to 1 hour, 5 hours, and 10 hours of CPU time, respectively.

# Computational results (2)

| | | 1 hour | | | 3 hours | | | 5 hours | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Name | Gap | cpx-O | cpx-F | LocBra | cpx-O | cpx-F | LocBra | cpx-O | cpx-F | LocBra |
| glass4 | % | 22.835 | 7.087 | 19.685 | 14.226 | 7.087 | 10.236 | 13.386 | 7.087 | **0** |
| UMTS | % | 6.403 | **0** | 2.413 | 6.403 | **0** | 1.216 | 6.403 | **0** | 0.126 |
| van | % | – | – | – | 30.845 | **0** | **0** | 5.108 | **0** | **0** |
| roll3000 | % | 2.763 | 3.804 | 3.131 | 2.763 | 3.804 | **0** | 2.763 | 3.375 | **0** |
| nsrand_ipx | % | 0.932 | 0.932 | 0.621 | 0.932 | 0.932 | **0** | 0.932 | 0.932 | **0** |
| A1C1S1 | % | 7.297 | 5.438 | 4.464 | 5.569 | 5.438 | 2.361 | 5.429 | 3.423 | **0** |
| A2C1S1 | % | 7.615 | 7.261 | 0.995 | 6.379 | 5.123 | **0** | 5.846 | 5.079 | **0** |
| B1C1S1 | % | 11.672 | 13.689 | 4.495 | 11.672 | 7.749 | 0.863 | 11.672 | 7.132 | **0** |
| B2C1S1 | % | 18.196 | 0.268 | 11.642 | 18.196 | **0** | 5.037 | 14.734 | **0** | 5.037 |
| tr12-30 | % | 0.036 | 0.573 | 0.622 | 0.007 | 0.410 | 0.332 | **0** | 0.389 | 0.332 |
| sp97ar | % | 2.494 | 0.842 | 1.171 | 2.494 | 0.428 | **0** | 2.376 | 0.124 | **0** |
| sp97ic | % | 5.453 | 0.622 | 3.675 | 3.834 | 0.622 | 0.761 | 3.834 | 0.622 | **0** |
| sp98ar | % | 1.724 | 2.715 | 0.602 | 1.724 | 1.409 | **0** | 1.724 | 0.282 | **0** |
| sp98ic | % | 1.350 | 0.872 | 0.247 | 1.350 | 0.872 | **0** | 1.350 | 0.872 | **0** |

# Computational results (3), flexibility

Improved solution values for set covering instances.

| elapsed | LocBra with local branching constraint $\sum_{j \in \overline{S}} (1 - x_j) \leq k' = 10$ | | | | | |
|---|---|---|---|---|---|---|
| Time | seymour | rail507 | rail2536c | rail2586c | rail4284c | rail4872c |
| 1:00 | * 423 | * 174 | 691 | 964 | 1081 | 1588 |
| 3:00 | * 423 | * 174 | 690 | * 954 | * 1076 | 1561 |
| 5:00 | * 423 | * 174 | * 690 | * 954 | * 1071 | * 1552 |

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| 5:00 | * 423 | * 174 | * 690 | * 954 | * 1071 | * 1552 |

Alternative LocBra runs.

| | LocBra emphasizing feasibility at the tactical level | | | | | |
|---|---|---|---|---|---|---|
| elapsed | Abs. Gap | Abs. Gap | Abs. Gap | % Gap | % Gap | % Gap |
| Time | markshare1 | markshare2 | net12 | UMTS° | B2C1S1 | tr12-30 |
| 1:00 | 4 | * 5 | – | * -0.048 | 8.317 | 2.597 |
| 3:00 | 3 | * 5 | – | * -0.066 | 8.317 | 2.129 |
| 5:00 | 2 | * 3 | * **0** | * -0.069 | 7.299 | 2.018 |

°The negative gaps for instance UMTS indicate an improvement of the best known

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- **Tighter integration within the MIP solver.**

  The described approach uses the MIP solver as a black-box for performing the tactical branchings.

  This is remarkably simple to implement, but has the disadvantage of wasting part of the computational effort devoted, e.g., to the exploration the nodes where no improved solution could be found within the node time limit.

  Therefore, a more integrated (and flexible) framework where the two branching rules work in tight cooperation is expected to produce an enhanced performance.

  [Andreello, Fischetti & Lodi, Work in progress]

# Local branching extensions (cont.d)

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- **Use of local branching constraints within special-purpose codes.**

  Despite the overall discussion, there is no need of using local branching constraints within a general-purpose MIP solvers.

  These constraints can be integrated within special-purpose codes, both exacts and heuristics, (black-boxes) designed for specific problems so as to enhance their heuristic capability.

  Obviously, the only requirement is that the code is able to cope with linear inequalities.

  Good results in this context. [Hernández-Pérez & Salazar-González, 2003]

# Local branching extensions (cont.d)

- **Dealing with general-integer variables.**

# Local branching extensions (cont.d)

- **Dealing with general-integer variables.**

  Local branching is based on the assumption that $\mathcal{B} \neq \emptyset$, i.e., there is a set of binary variables, and moreover, this set is of relevant importance.

  According to our computational experience, this is true even in case of MIPs involving general integer variables, in that the 0-1 variables (which are often associated with big-M terms) are likely to be largely responsible for the difficulty of the model.

  However, in the general case of integer variables $x_j \mid l_j \leq x_j \leq u_j$, the local branching constraint can be written as:

$$\Delta_1(x, \bar{x}) := \sum_{j \in \mathcal{I}: \bar{x}_j = l_j} \mu_j (x_j - l_j) + \sum_{j \in \mathcal{I}: \bar{x}_j = u_j} \mu_j (u_j - x_j) + \sum_{j \in \mathcal{I}: l_j < \bar{x}_j < u_j} \mu_j (x_j^+ + x_j^-) \leq k$$

  where weights $\mu_j$ are defined, e.g., as $\mu_j = 1/(u_j - l_j)\ \forall\ j \in \mathcal{I}$, while the variation terms $x_j^+$ and $x_j^-$ require the introduction into the MIP model of additional constraints of the form:

$$x_j = \bar{x}_j + x_j^+ - x_j^-, \quad x_j^+ \geq 0,\ x_j^- \geq 0, \quad \forall j \in \mathcal{I} : l_j < \bar{x}_j < u_j.$$

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  As stated, the local branching framework requires a starting (feasible) reference solution $\bar{x}^1$.

  However, for difficult MIPs (such as, e.g., hard set partitioning models) even the definition of this solution may require an excessive computing time.

  In this case, one should consider the possibility of working with infeasible reference solutions by adding slack variables to (some of) the constraints, while penalizing them in the objective function.

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  Moreover, another interesting point is using local branching ideas to devise a method to converge to an initial feasible solution without using the branch-and-bound framework.

  This means using the concept of neighborhood to define a distance between a **feasible continuous** solution and an **infeasible integer** one, and then solve a sequence of LPs by minimizing this distance.

  [Fischetti, Glover & Lodi, Work in progress]

# Local branching dissemination

- **Relaxation induced neighborhood search.**

[Danna, Le Pape & Rothberg, 2003]

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A similar concept of neighborhood was recently introduced by taking into account simultaneously both the incumbent solution, $\bar{x}$, and the the solution of the continuous relaxation, say $x^*$, at a given node of the branch-and-bound tree.

Then, $\bar{x}$ and $x^*$ are compared and all the binary variables which assume the same value are hard-fixed in an associated MIP.

This associated MIP is then solved by using the MIP solver as a black-box, and in case the incumbent solution is improved, $\bar{x}$ is updated in the rest of the tree.

This method turns out to give very competitive results on general MIPs and it is particularly suitable in the scheduling context where sometimes the problem is very constrained and a non-trivial value of $k$ would be too large (thus defining too large neighborhoods).

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this is a big challenge for this community