Pre-course, part II:
black-box machine learning for image denoising

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A statistical learning perspective on image denoising

Statistical learning - a supervised regression problem

On a completely different note...

[Cucker, Smale, *On the mathematical foundations of learning*, 2002]

A supervised regression problem

Given two random variables x on X and y on Y

find a function $f\colon Y \to X$ s.t. $f(y) \approx x$

- (ideal setting) knowing the joint distribution $\pi_{y,x} \sim (y, x)$;
- (real setting) knowing a sample $\{(\mathbf{y}_i, \mathbf{x}_i)\}_{i=1}^{N} \sim_{i.i.d} \pi_{y,x}$;

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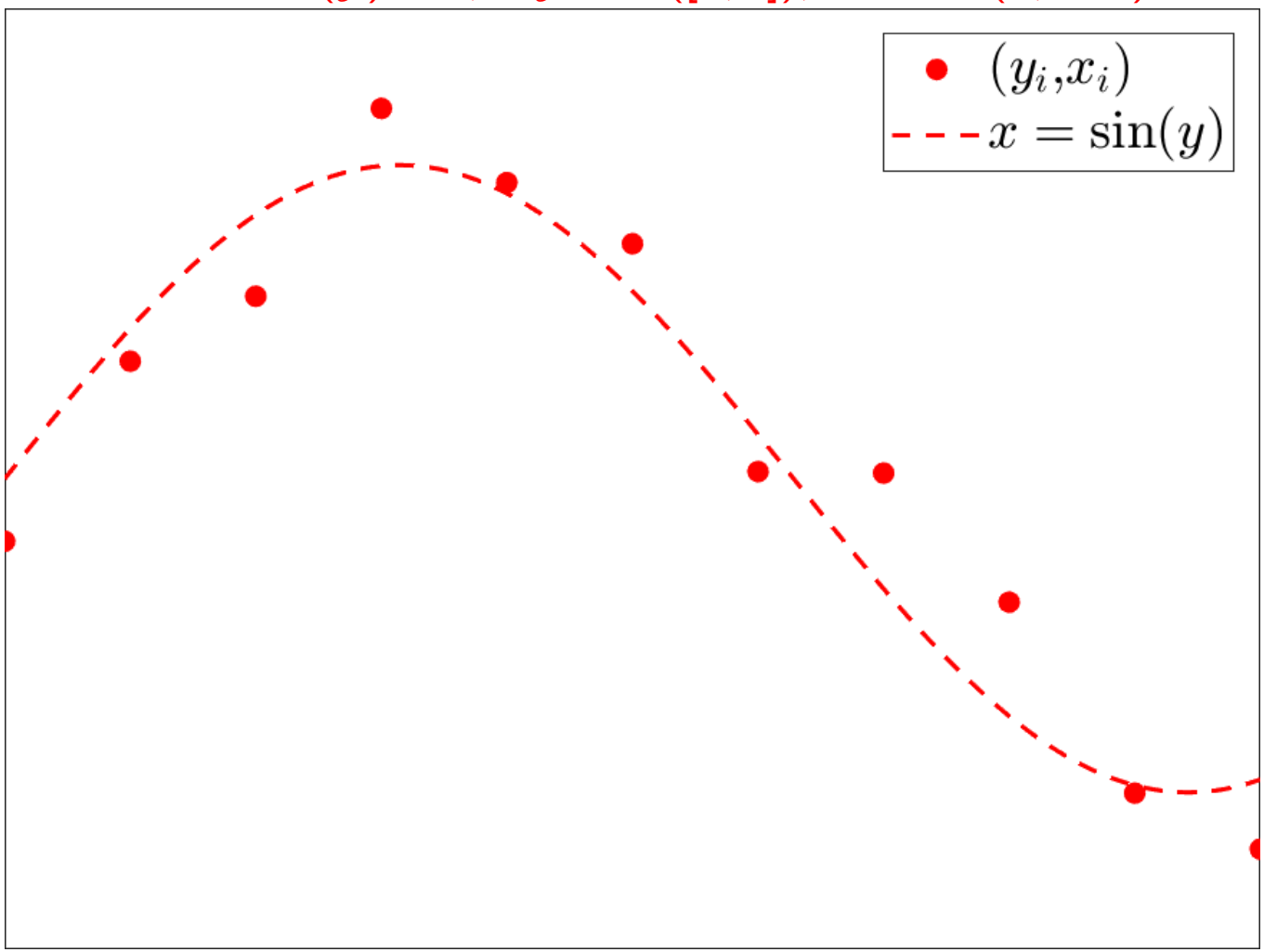
1) Consider a loss function, e.g. $\ell(\mathbf{x}', \mathbf{x}) = \|\mathbf{x}' - \mathbf{x}\|_X^2$.
2) Introduce $\mathcal{F}$: hypothesis space, a set of functions from Y to X.
3.id) Minimize the **Expected Loss**: $\rightsquigarrow \min_{f \in \mathcal{F}} \mathbb{E}_{(x,y)\sim\pi_{xy}} \Big[\ell(f(y), x)\Big]$
3.re) Minimize the **Empirical Risk**: $\rightsquigarrow \min_{f \in \mathcal{F}} \frac{1}{N} \sum_{i=1}^{N} \ell(f(\mathbf{y}_i), \mathbf{x}_i)$

How we usually see regression...

$$X = \mathbb{R}, \quad Y = \mathbb{R},$$

$$x = \sin(y) + \varepsilon, \quad y \sim \mathcal{U}([0, 5]), \quad \varepsilon \sim \mathcal{N}(0, 0.2^2)$$



How we usually see regression...

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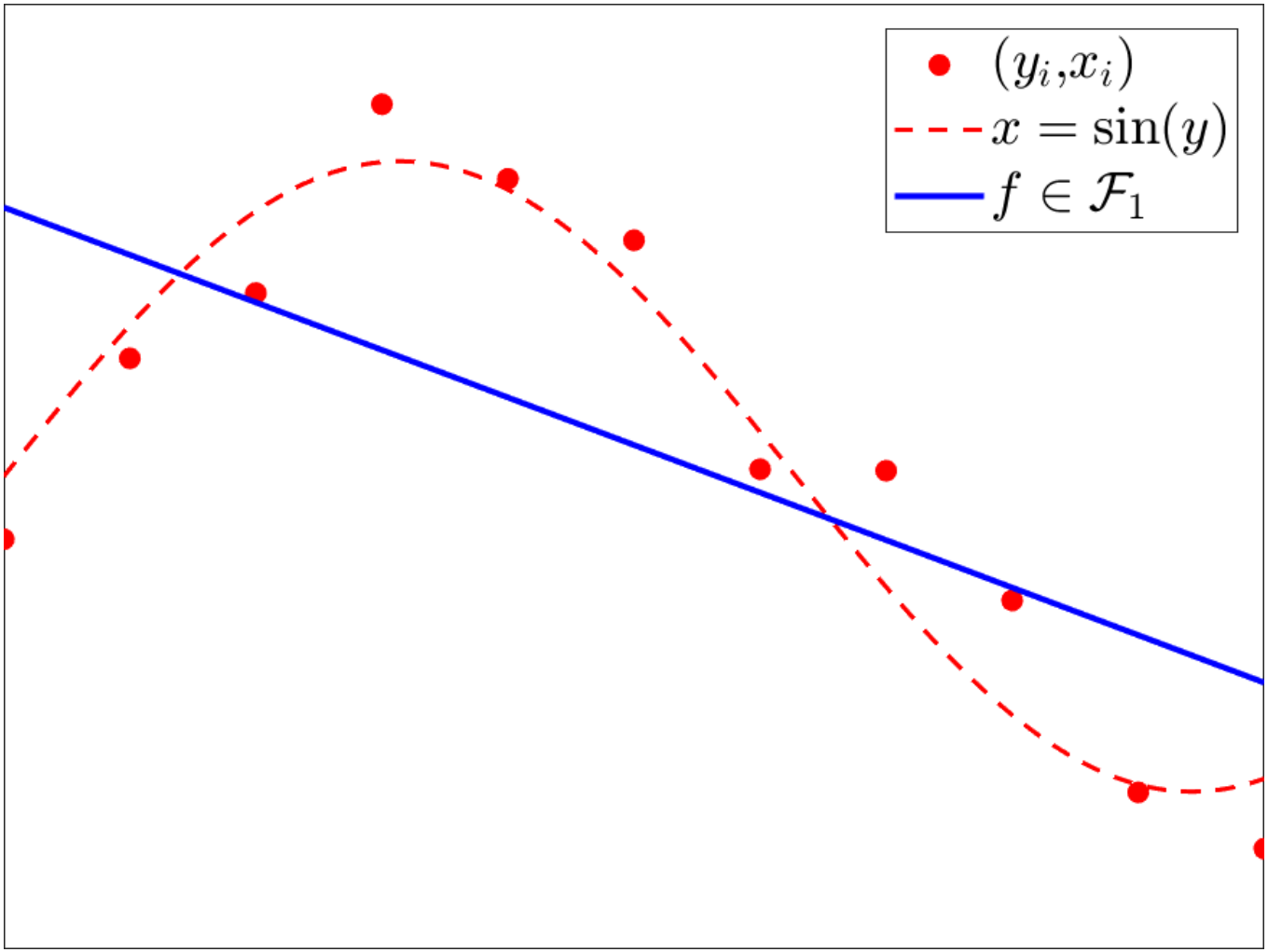
$\mathcal{F}$: polynomials of degree 1



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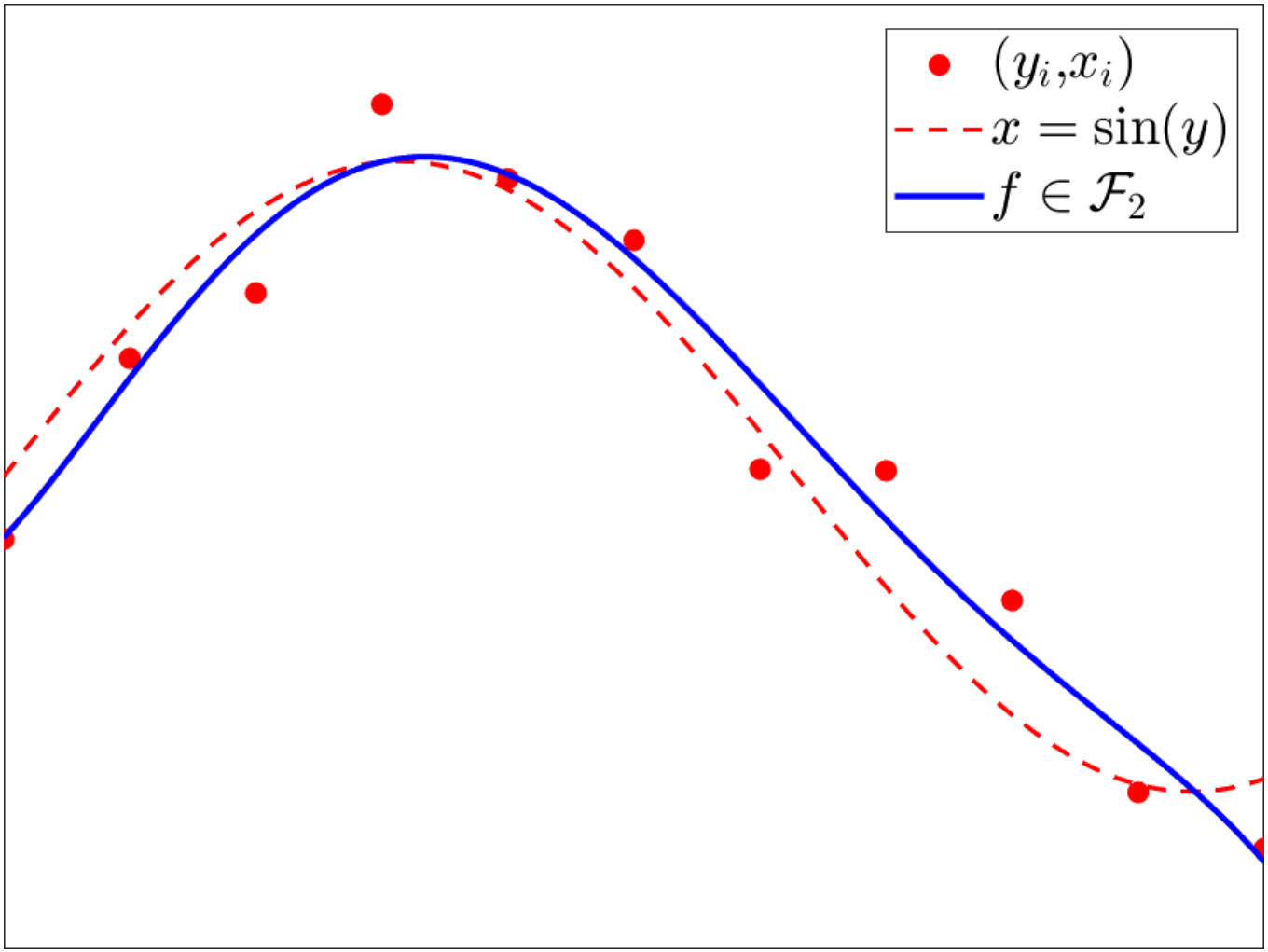
$X = \mathbb{R}, \quad Y = \mathbb{R},$

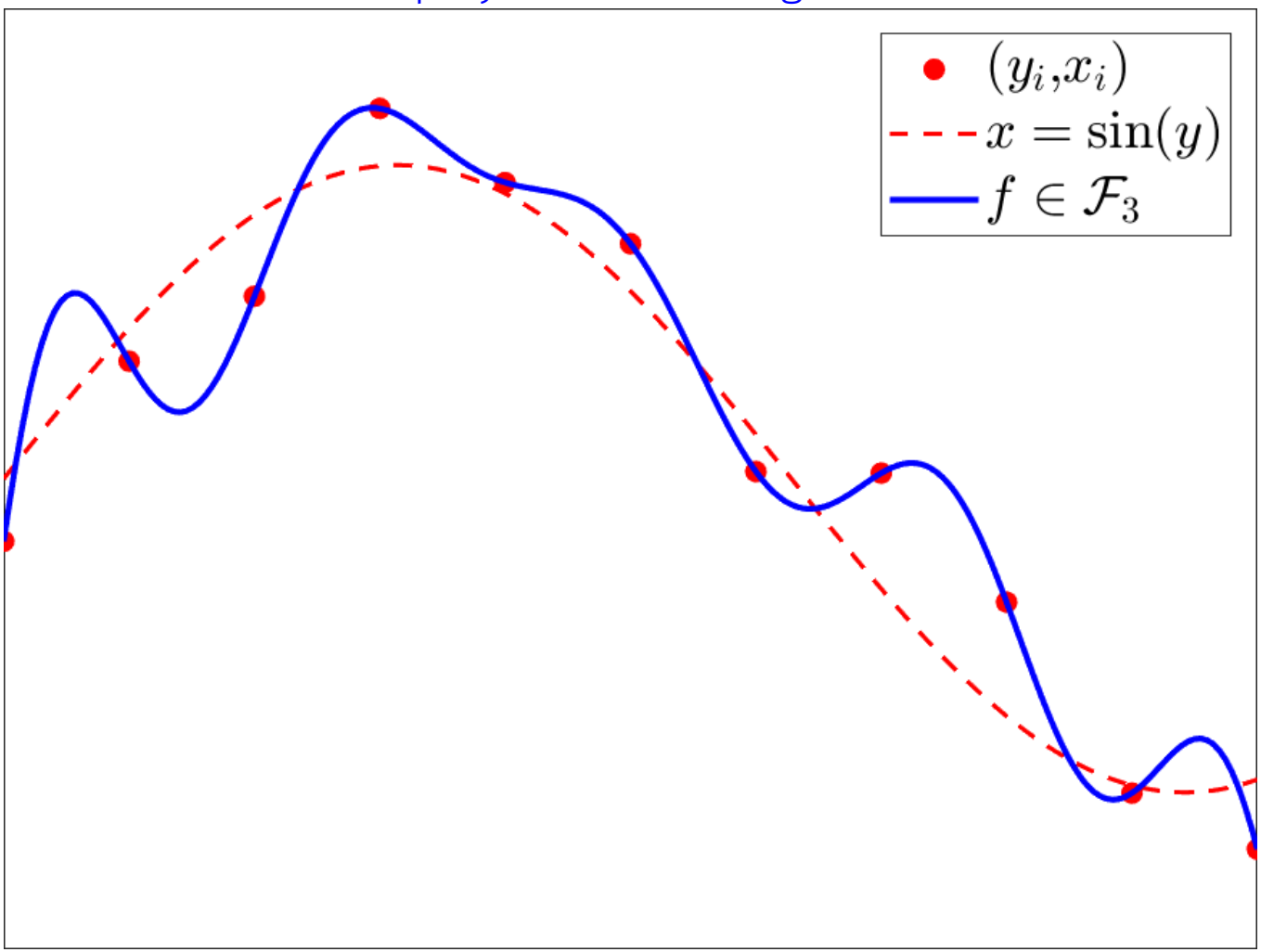
$\mathcal{F}$: polynomials of degree 5



How we usually see regression...

$$X = \mathbb{R}, \quad Y = \mathbb{R},$$

$\mathcal{F}$: polynomials of degree 10



... but denoising can be seen as a regression task!

Learned reconstruction for inverse problems:
suppose that $y = \mathsf{A}x + \boldsymbol{\varepsilon}$ (forward model): $f(y) \approx x \rightsquigarrow$ find $f \approx \mathsf{A}^{-1}$.

Denoising: let $X = Y$ and $\mathsf{A} = \mathsf{Id}$, i.e.: $y = x + \boldsymbol{\varepsilon}$.
Why not using simply $f = \mathsf{A}^{-1} = \mathsf{Id}$?

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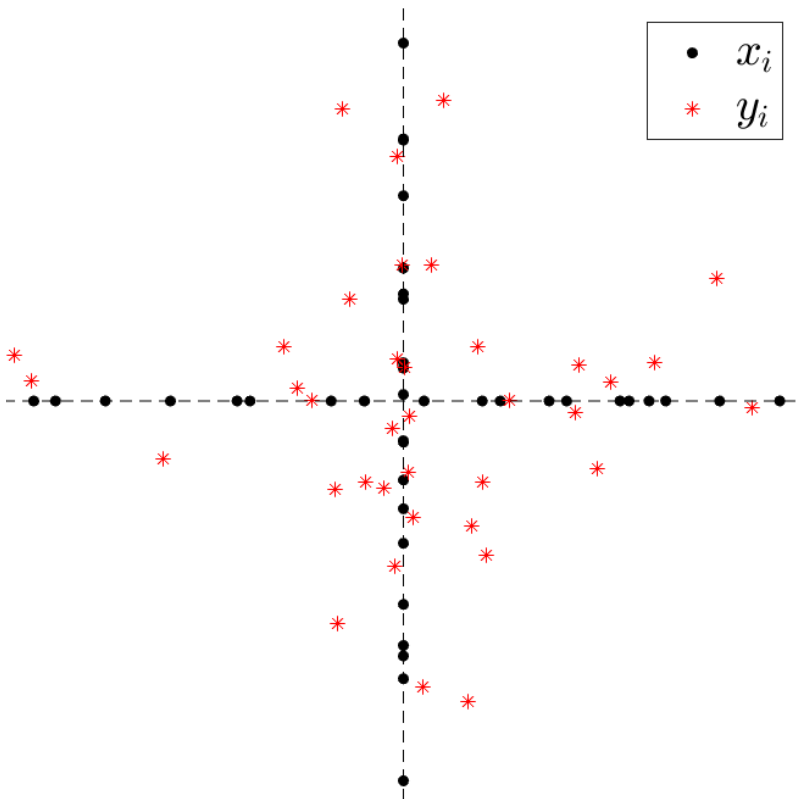

The simplest example:

- $X = \mathbb{R}^2$;
- $\varepsilon \sim \mathcal{N}(0, 0.1^2\,\mathsf{Id})$;
- x : uniformly distributed on $\{0\} \times [-1, 1] \cup [-1, 1] \times \{0\}$;
- $y = x + \varepsilon$.
- $f = \mathsf{Id}$
  relative error: 20.70%

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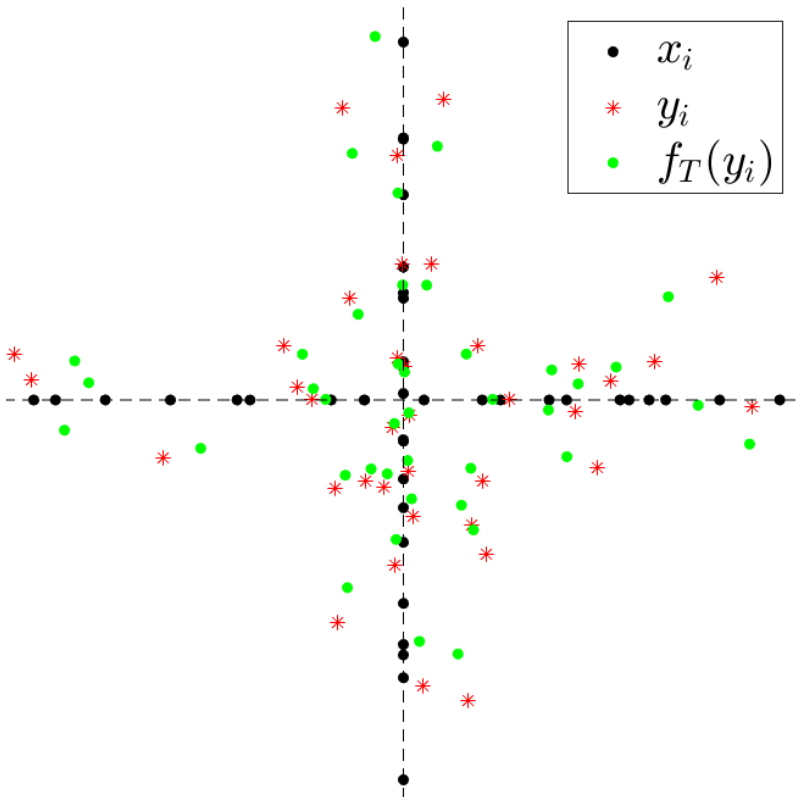

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- Variational denoiser: Tikhonov

$$f_T(\mathsf{y}) = \arg\min_{\mathsf{x}} \left\{ \frac{1}{2}\|\mathsf{x} - \mathsf{y}\|_2^2 + \frac{\lambda}{2}\|\mathsf{x}\|_2^2 \right\}$$
$$= \frac{1}{1+\lambda}\mathsf{y}$$

relative error: 19.21%

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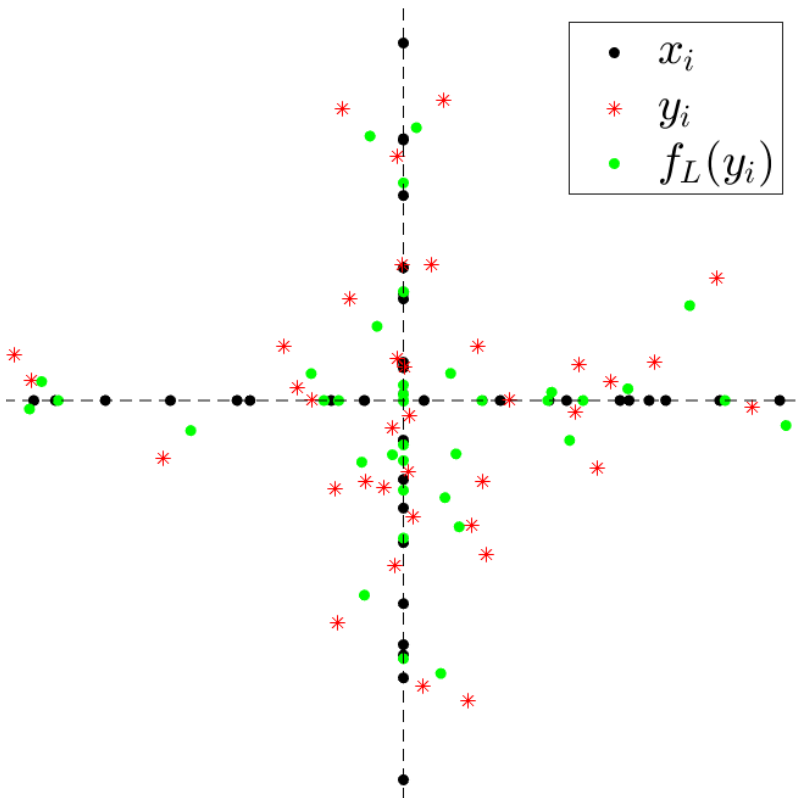

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- $y = x + \varepsilon$.
- Variational denoiser: Lasso

$$f_L(y) = \mathsf{prox}_{\lambda\|\cdot\|_1}(\mathsf{y}) = \mathsf{ST}(\mathsf{y}; \lambda)$$

$$[f_L(\mathsf{y})]_j = \begin{cases} \mathsf{sign}(y_j)(|y_j| - \lambda) \text{ if } |y_j| > \lambda \\ 0 \text{ if } |y_j| \leq \lambda \end{cases}$$

relative error: 18.41%

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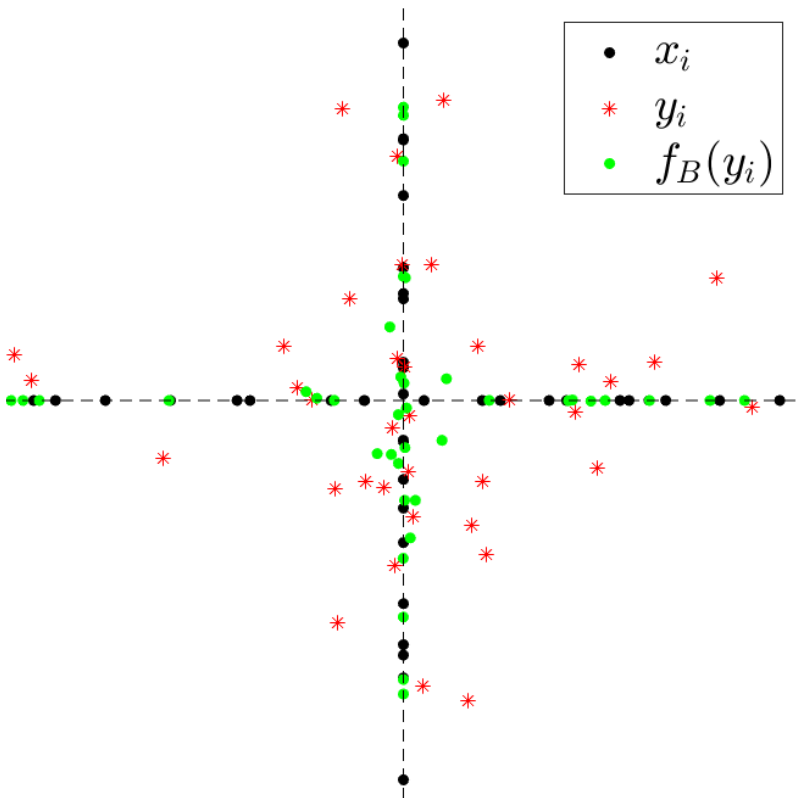

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- $y = x + \varepsilon$.
- Learned denoiser (ideal case)
  $f_B = \ldots$ - Bayes denoiser
  relative error: 16.56%

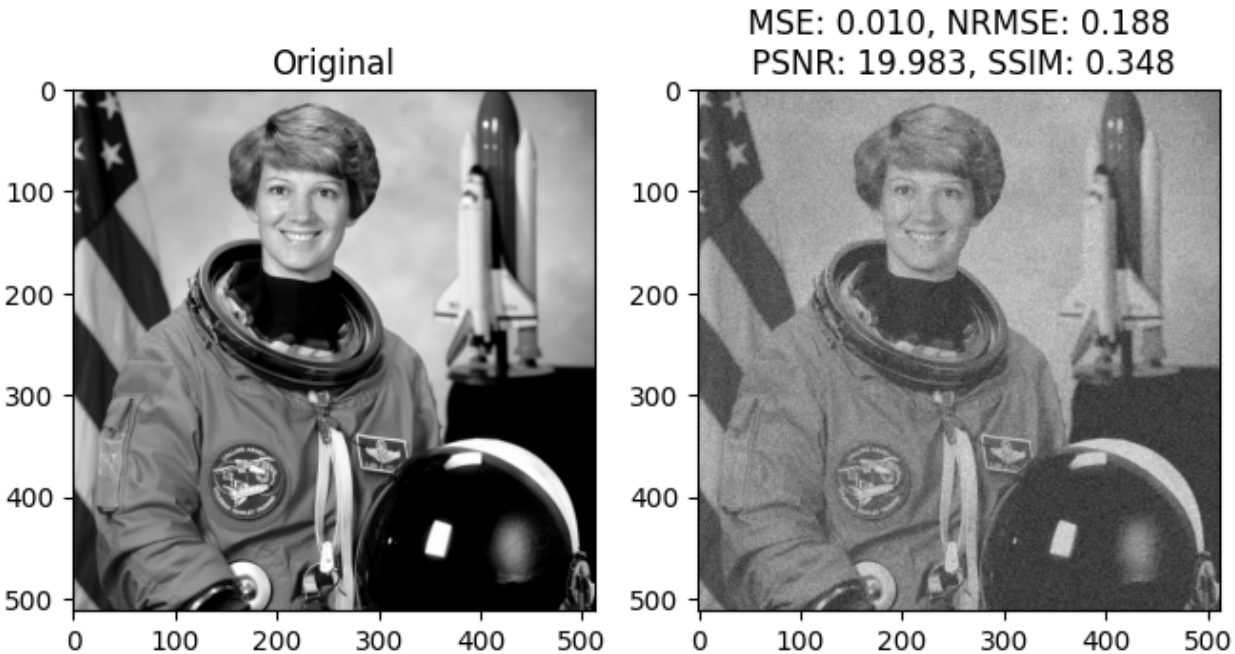
Image Denoising as a regression problem

Setup:

- Each image is a vector $\mathbf{x} \in \mathbb{R}^n$, where $n = \text{height} \times \text{width} \times \text{channels}$
- Noisy image: $\mathbf{y} = \mathbf{x} + \boldsymbol{\varepsilon}$, Clean image: $\mathbf{x}$, Noise: $\boldsymbol{\varepsilon}$
- **Goal:** learn $f : \mathbb{R}^n \to \mathbb{R}^n$ such that $f(\mathbf{y}) \approx \mathbf{x}$

Loss Function:

$$\ell(f(\mathbf{y}), \mathbf{x}) = \text{MSE}(f(\mathbf{y}), \mathbf{x}) = \frac{1}{n}\sum_{j=1}^{n}([f(\mathbf{y})]_j - [\mathbf{x}]_j)^2 = \frac{1}{n}\|f(\mathbf{y}) - \mathbf{x}\|_2^2$$



Audience-driven slide

In your experience,
what are key ingredients of statistical learning?

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My personal list

A. a training dataset;

B. a loss function;

C. a (parametric) hypothesis class;

D. an optimization algorithm.

A statistical learning perspective on image denoising

A. Training datasets

Supervised datasets and beyond

1) Supervised setting

A dataset of paired noisy and clean images $\{(\mathbf{y}_i, \mathbf{x}_i)\}_{i=1}^{N}$ is available.

- empirical loss minimization.

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A dataset of clean images $\{(\mathbf{x}_i)\}_{i=1}^N$ is available $+$ the noise model is known (e.g., $y = x + \varepsilon$, $\varepsilon \sim \mathcal{N}(0, \sigma^2 \mathbf{I})$)

- sample $\{\varepsilon_i\}_{i=1}^N \sim_{i.i.d.} \pi_\varepsilon$, define $\mathbf{y}_i = \mathbf{x}_i + \varepsilon_i$;
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A dataset of clean images $\{(\mathbf{x}_i)\}_{i=1}^N$ is available.

- learn the prior distribution π_x (e.g. via Tweedie's formula);
- use a model-adaptive algorithm to identify the noise model.

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A dataset of noisy images $\{(\mathbf{y}_i)\}_{i=1}^N$ is available.

- ad-hoc techniques (when available).

A statistical learning perspective on image denoising

B. Loss function

Loss minimization and regularization

- **Loss function, ℓ:** measures the quality of a single denoised image.
  Es.: $\ell(\mathbf{x}', \mathbf{x}) = \|\mathbf{x}' - \mathbf{x}\|_2^2, \quad \ell(\mathbf{x}', \mathbf{x}) = \mathrm{PSNR}(\mathbf{x}', \mathbf{x})$.
- **Expected loss, L:** measures the quality of a denoiser f, using the full knowledge of $\pi_{y,x}$ (ideal). Es. $L(f) = \mathbb{E}_{(y,x)\sim\pi_{(y,x)}}[\ell(f(y), x)]$.
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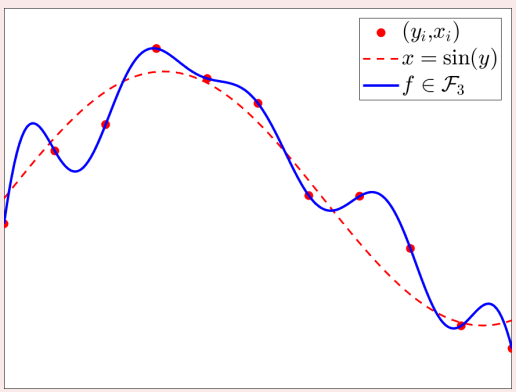
The risk of overfitting



Minimizing $\hat{L}$ among all possible $f\colon Y \to X$ is not a good idea:

- the result f might work poorly on new data $y_{m+1}, y_{m+2}, \ldots$
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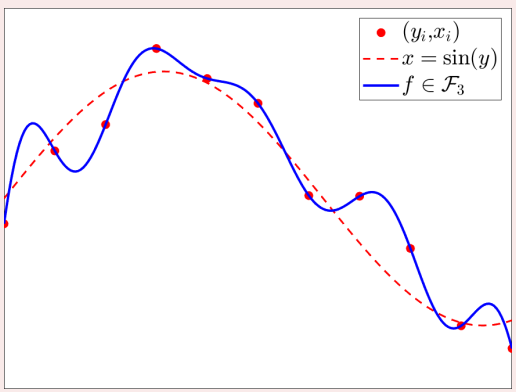

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Explicit regularization

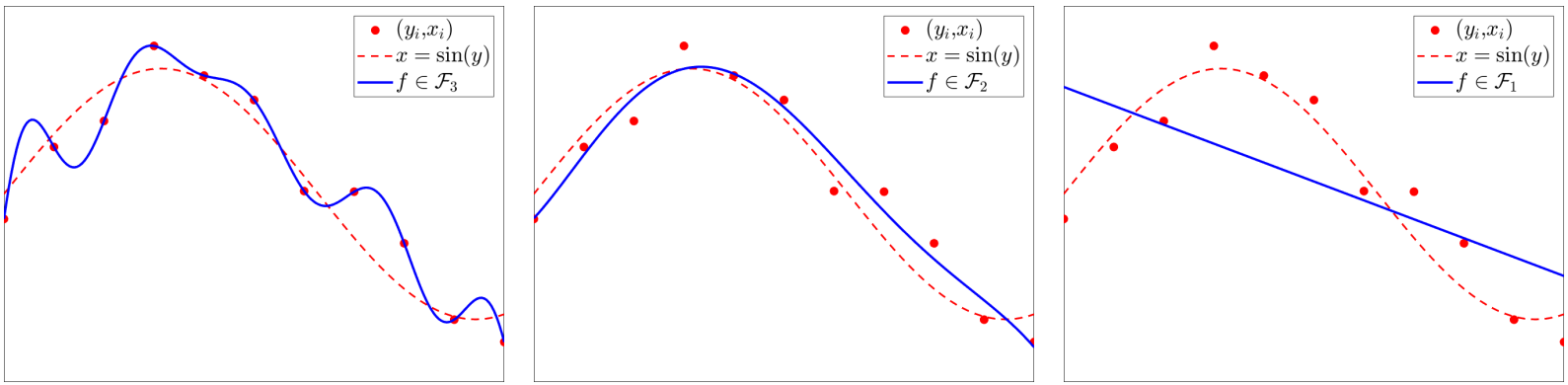
One possible solution: minimize $\hat{L}(f) + R(f)$.
Ex. $R(f) = \|f\|_{\mathcal{H}}$, $\mathcal{H}$ a suitable function space (e.g. a RKHS $\rightsquigarrow$ kernel methods).

A statistical learning perspective on image denoising

C. Parametric hypothesis classes

Implicit regularization via parametric hypothesis spaces

Idea: restricting the space $\mathcal{F}$ in which optimizing $\hat{L}$ induces an implicit regularization. Ex: $\mathcal{F} =$ polynomials of degree $10,\ 5,\ 1$.



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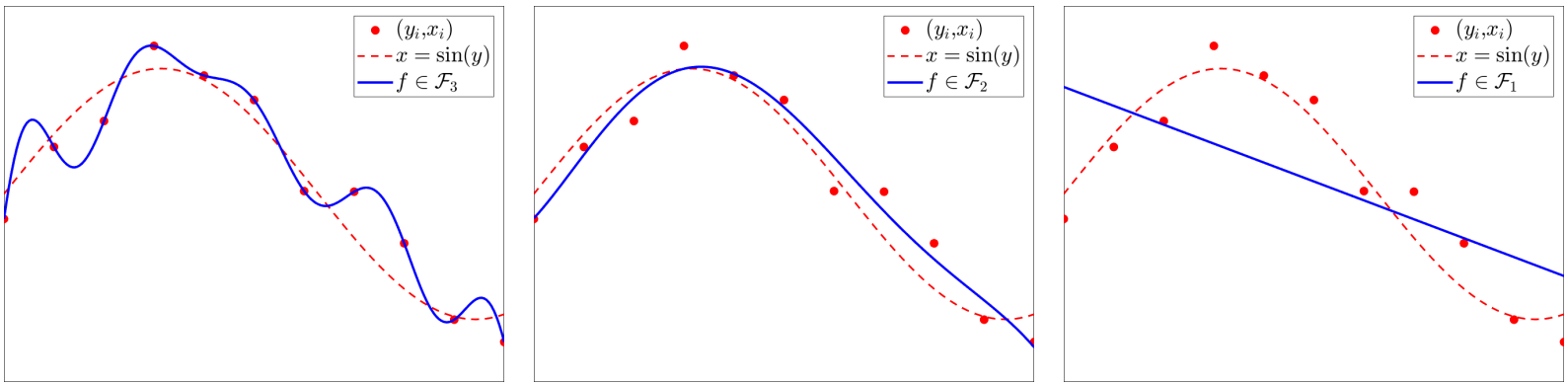


Parametric hypothesis spaces

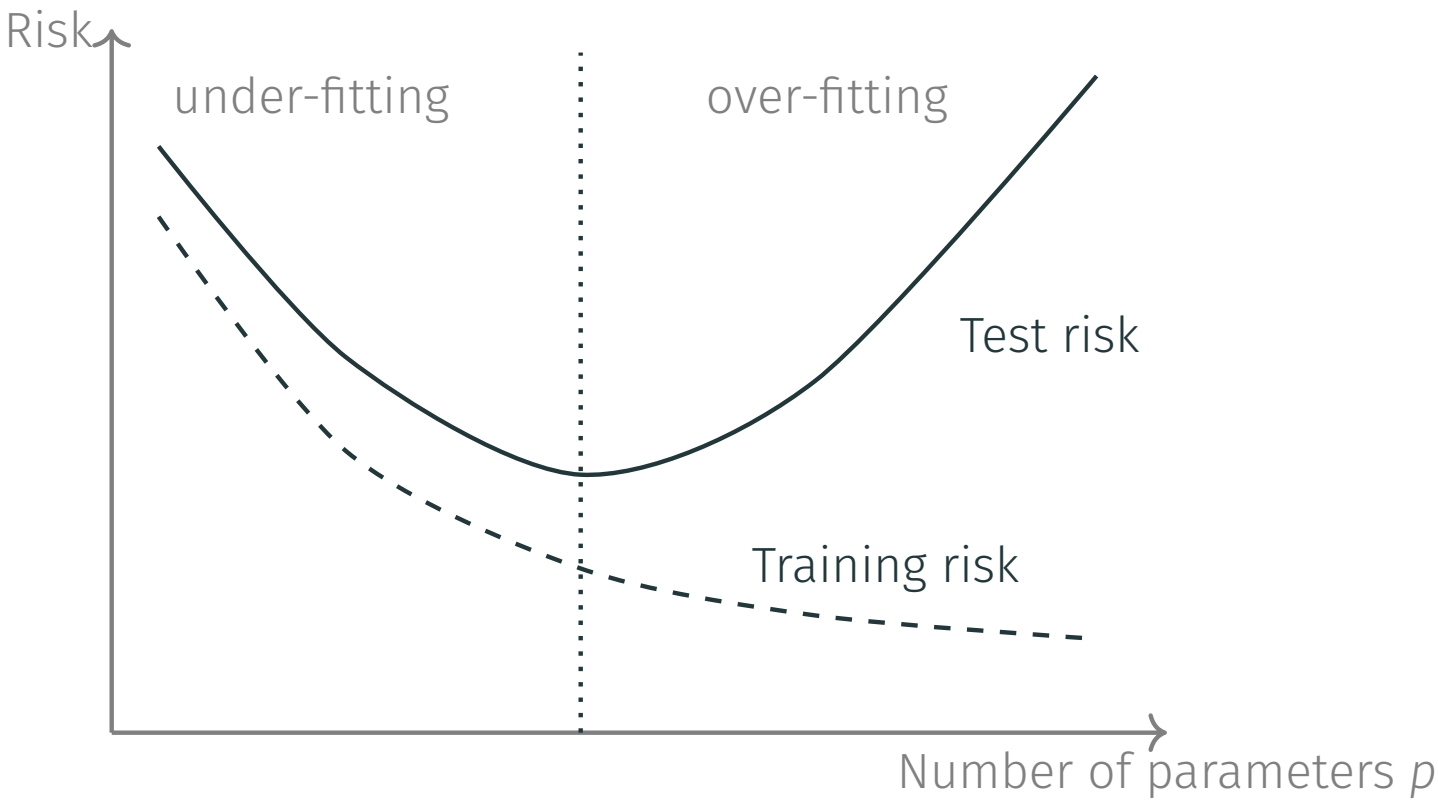
Spaces of functions depending on (p) parameters:

$\mathcal{F} = \{f_\theta : Y \to X, \quad \theta \in \Theta \cong \mathbb{R}^p\}$. Examples $(X = Y = \mathbb{R})$:

- polynomials: $f_\theta(y) = \theta_1 + \theta_2 y + \ldots + \theta_p y^{p-1}$;
- linear splines: $f_\theta(y) = \theta_1 + \theta_2 \chi_{[\theta_3, +\infty)}(y) + \ldots \theta_{p-1} \chi_{[\theta_p, +\infty)}(y)$;
- Neural Networks, e.g. $f_\theta(y) = \theta_1 \sigma(\theta_2 y) + \ldots + \theta_{p-1} \sigma(\theta_p y)$.

A bias-variance tradeoff

A variation of a picture from [M. Belkina, D. Hsuc, S. Maa, S. Mandal, *Reconciling modern machine-learning practice and the classical bias–variance trade-off*, 2019] - part 1



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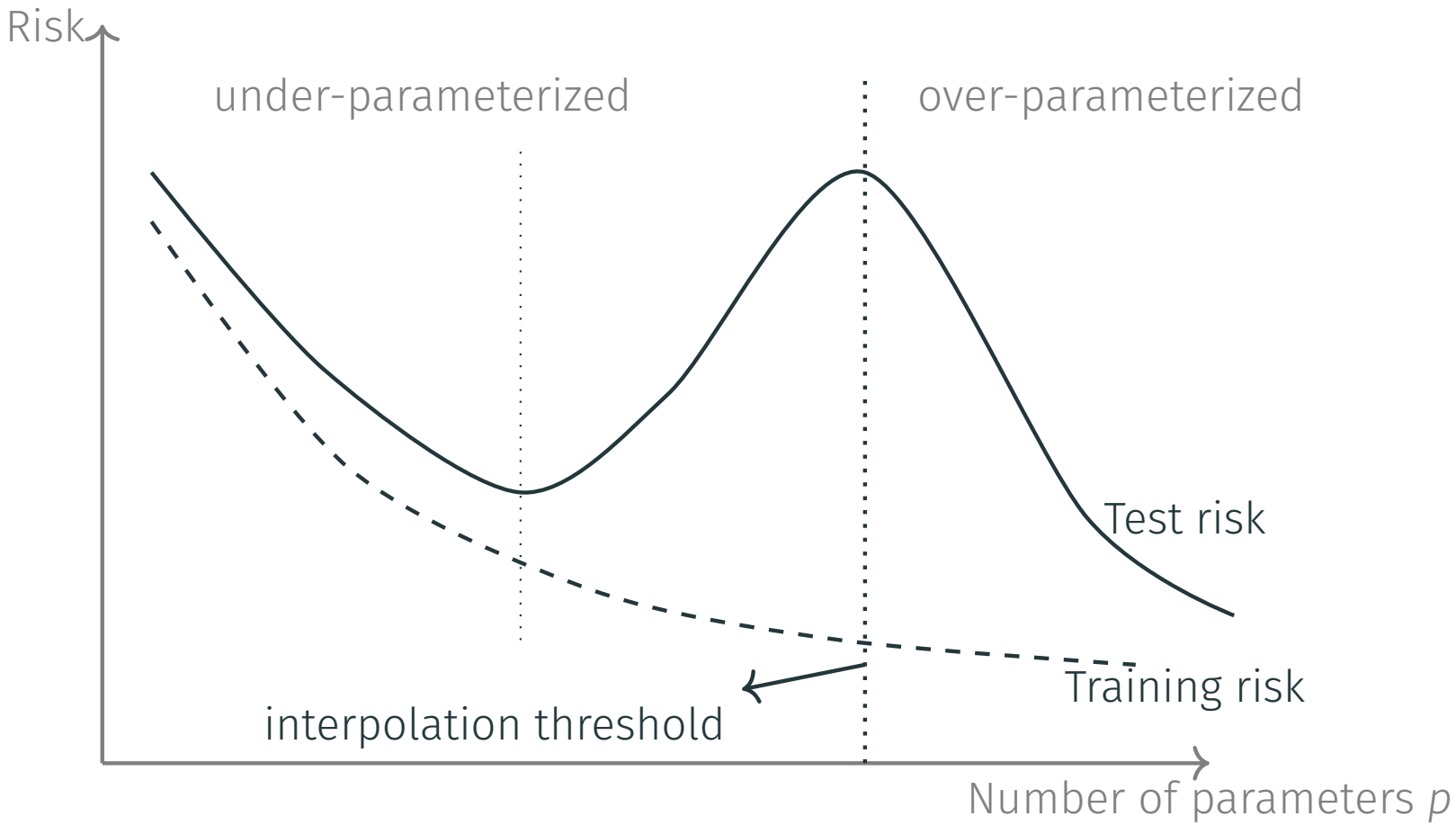

Neural Networks: a working expression

Given a nonlinear function $\sigma : \mathbb{R} \to \mathbb{R}$ and its element-wise version s.t. $[\boldsymbol{\sigma}(\mathbf{x})]_j = \sigma([\mathbf{x}]_j)$, given $W_l \in \mathbb{R}^{n_{l-1} \times n_l}$ and $\mathbf{b}_l \in \mathbb{R}^{n_l}$, $l = 1, \dots, L$, let

$$f_\theta(x) = W_{L+1}\boldsymbol{\sigma}(W_L \cdots \boldsymbol{\sigma}(W_1\mathbf{x} + \mathbf{b}_1) \cdots + \mathbf{b}_L) + \mathbf{b}_{L+1}.$$

The learnable parameters are $\theta = \{W_1, \dots, W_L, \mathbf{b}_1, \dots, \mathbf{b}_L\}$.

Neural Networks: very expressive parametric functions from $\mathbb{R}^n \to \mathbb{R}^n$

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Examples:

- **Multilayer Perceptron (MLP)**: fully connected layers (full matrices W_l), $\sigma =$ ReLU, tanh - target: vector data.
- **Convolutional Neural Network (CNN)**: convolutional layers ($W_l\mathbf{x} = \mathbf{K}_l * \mathbf{X}$) - target: image data.

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Theorem (Universal Approximation):

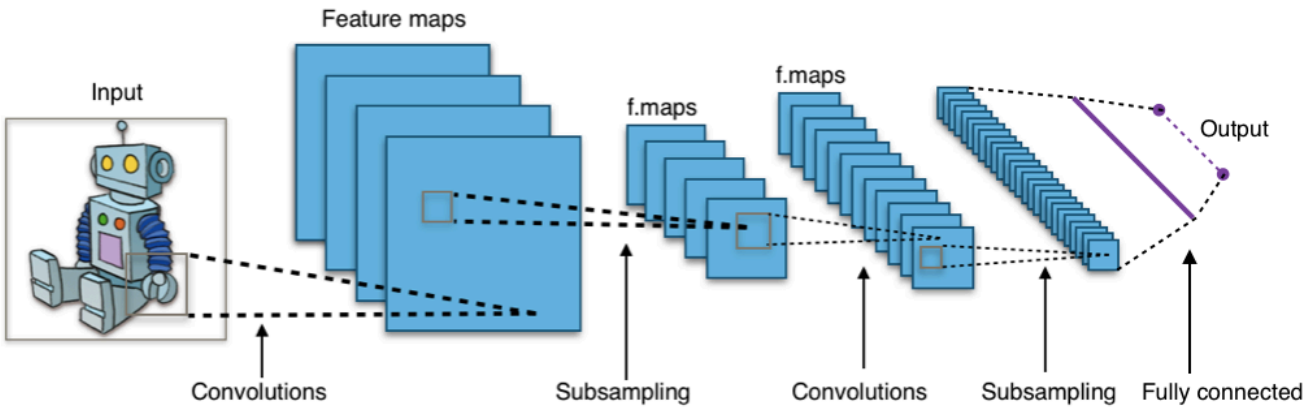
Under reasonable assumptions on σ, any continuous function $f : [0,1]^n \to \mathbb{R}^n$ can be approximated arbitrarily well (in the L^∞ norm) by a NN with $L = 1$, provided that $W_1 \in \mathbb{R}^{n_1 \times n}$ and $W_2 \in \mathbb{R}^{n \times n_1}$, and n_1 is sufficiently large. *(Cybenko, 1989; Hornik, 1991)*

Variants: arbitrary width, higher regularity, avoid the curse of dimensionality.

Network architectures for images: Convolutional Neural Networks (CNNs)

Key Idea: exploit spatial locality and translational invariance using local linear operations.

- Layers apply convolutions: $X \mapsto \sigma(K * X + b)$;
- Common operations: ReLU, pooling, normalization;
- Weight sharing between layers (parameter reduction).



Credits: Wikipedia, by Aphex34

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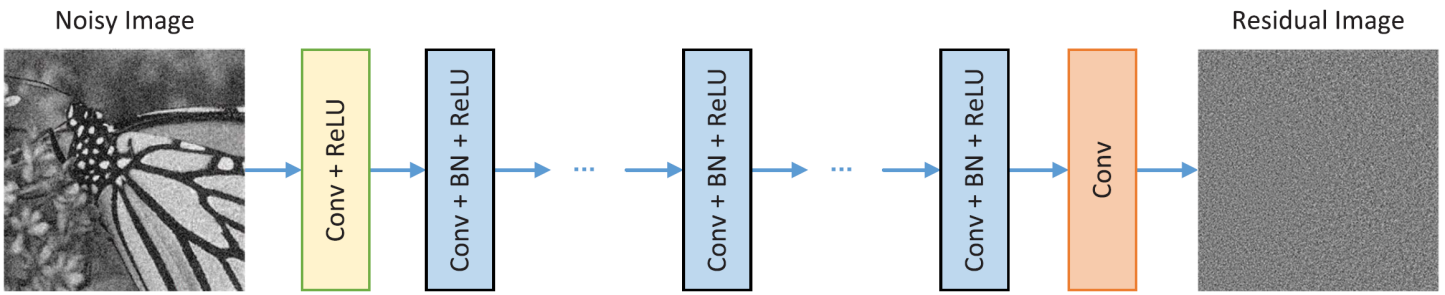


Credits: [Zhang et al., 2017]

Image denoising example:

- **DnCNN** (Zhang et al., 2017): deep CNN trained to remove additive Gaussian noise.

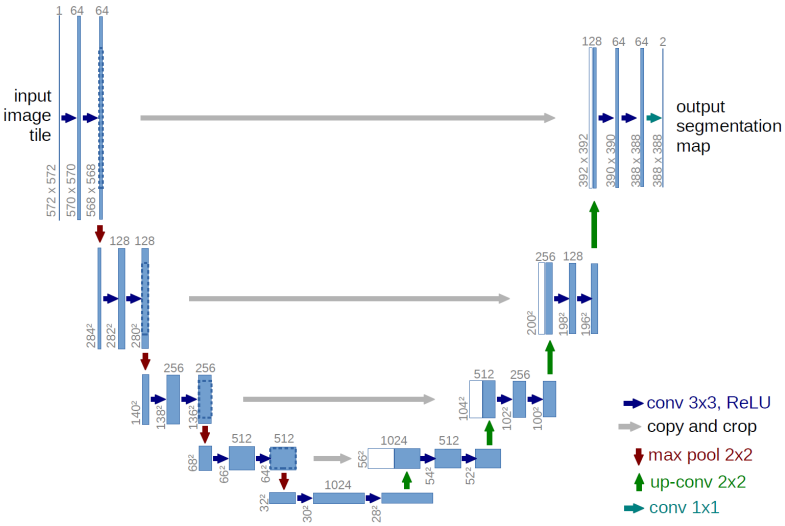
Warning: input dimensions!

In these examples, the input of the network is not a vectorized image $\mathbf{x} \in \mathbb{R}^n$ but a tensor $\mathbf{X}$ of size: height $\times$ width $\times$ channels.

Network architectures for images: U-Nets

Key Idea: extract and process local features with symmetric skip connections.

- An encoder branch extracts coarse features (convolution + downsampling)
- A decoder branch reconstructs full-resolution output (upsampling)
- Skip connections copy feature maps to enhance spatial details



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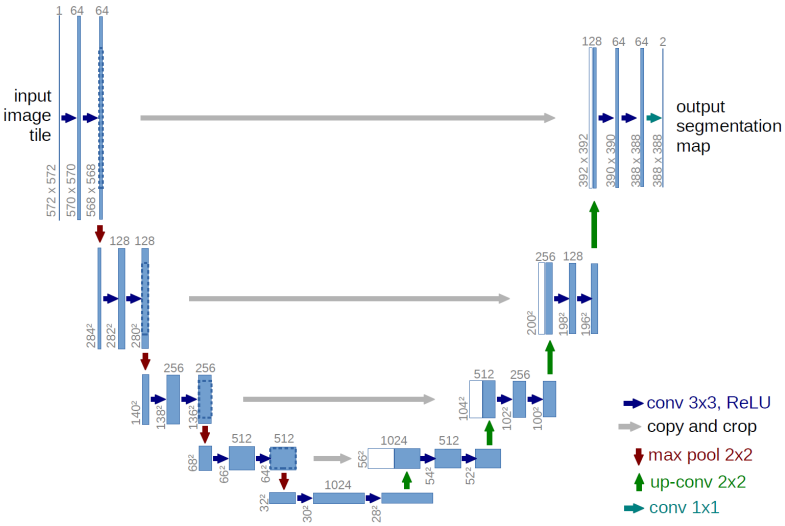

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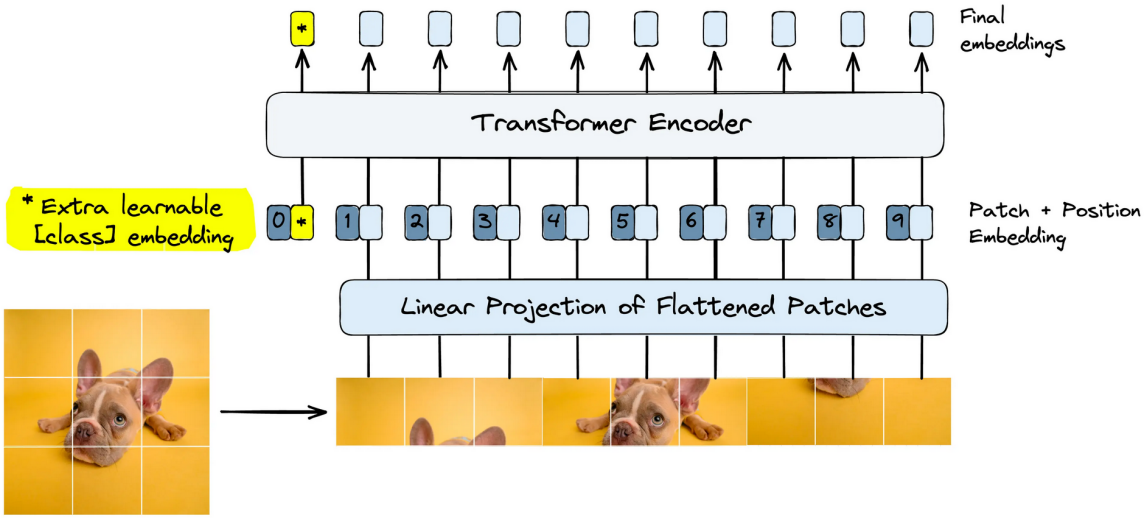
Image denoising example:

- U-Net variants are widely used in medical image denoising, see e.g. **Noise2Noise** (Lehtinen et al., 2018), **DRUNET** (Devalla et al., 2018), ...

Network architectures for images: Vision Transformers

Key Idea: Process images as sequences of patches with self-attention, removing convolutional inductive bias.

- Image is split into fixed-size patches (e.g., 16×16)
- Each patch is embedded into a vector (via linear projection)
- Transformer encoder applies global attention across all patches.



https://www.pinecone.io/learn/series/image-search/vision-transformers/

Image denoising example:

- **Restormer** (Zamir et al., 2022): uses self-attention over multi-resolution image representations; can be applied to unsupervised settings.

A statistical learning perspective on image denoising

D. Optimization algorithms

How to train your network - part I

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Stochastic Gradient Descent: just an intuition

- Exploit that $\hat{\mathcal{L}}(\theta) = \sum_{i=1}^{N} \ell(f_\theta(\mathbf{y}_i), \mathbf{x}_i)$: replace the full sum with the one on a randomly subsampled batch.
- Combined with momentum, acceleration, and adaptive step sizes, it provides efficient versions (Adam, AdaGrad, RMSProp)

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Validation set:

[What?] A separate dataset used to evaluate generalization.

[Why?] Hyperparameter tuning (parameters of the network - number of layers, channels - or of the optimizer - learning rate, batch size).

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Irreducible error
(if data are noisy,
I can't reduce this!)

Implementation aspects

Image processing in Python

The most important libraries for image processing:

- **numpy**: process tensors representing images - can handle arithmetic operations, cropping...
- **skimage**: basic image processing operations (thresholding, blurring,...) and metrics (MSE, PSNR, SSIM,...)
- **matplotlib**: image visualization, color adjustment,...
- **pytorch**: neural network - definition, training, testing.

A small tutorial: tomorrow!