

***PhD Winter School 2023:***  
**ADVANCED METHODS for**  
**MATHEMATICAL IMAGE ANALYSIS**

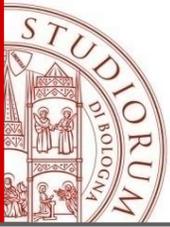
***Computational Imaging Lab (A-B)***

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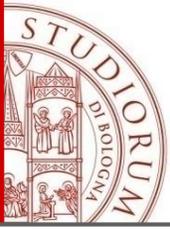
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# Experimentation in Matlab: outline

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- Presentation of the provided Matlab package:
  - provided scripts and functions, input and output folders,...
  - MAIN\_2D script: short explanation of this main script
  - MAIN\_2D script: activation/deactivation of all testing functionalities
- Experimental testing of unconstrained TIK- $L_2$ , TV- $L_2$ , TIK- $L_1$ , TV- $L_1$  models, with TIK- $L_2$  solved directly (FFT or PCG), TV- $L_2$ , TIK- $L_1$ , TV- $L_1$  solved by ADMM:
  - For TV- $L_2$ , TIK- $L_1$ , TV- $L_1$ , inspection of ADMM iterations
  - Evaluation/comparison of the obtained restoration results
- Experimental testing of discrepancy-constrained TV- $L_2$  model:
  - Comparison with the unconstrained TV- $L_2$  model



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- The performance of all considered variational models solved by some specific minimization algorithm will be evaluated in terms of **accuracy** and of **efficiency**. In particular, we adopt two scalar measures for both accuracy and efficiency:

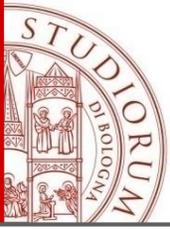
- **ACCURACY:** 1) Improved Signal-to-Noise-Ratio (**ISNR**)  
2) Improved Structural Similarity Index (**ISSIM**)

$$\text{ISNR}(u^*, u_{true}, b) = \text{SNR}(u^*, u_{true}) - \text{SNR}(b, u_{true}), \quad \text{SNR}(x, u_{true}) = 10 \log_{10} \left( \frac{\|Ku_{true} - E[Ku_{true}]\|_2^2}{\|Ku_{true} - x\|_2^2} \right) \text{ [dB]}$$

$$\text{ISSIM}(u^*, u_{true}, b) = \text{SSIM}(u^*, u_{true}) - \text{SSIM}(b, u_{true}), \quad \text{SSIM}(x, u_{true}) \text{ defined according to [1], type "help ssim" in Matlab}$$

- **EFFICIENCY:** 1) **number of performed iterations** (for iterative algorithms)  
2) **CPU time** spent by the algorithm for obtaining the restoration

Clearly, accuracy will also be evaluated by visually inspecting the obtained results!!



# Experimentation in Matlab: outline

- By means of the provided Matlab software, we can test experimentally 5 models:

$$\text{TIK} - L_2 - U \quad u^*(\mu) = \arg \min_{u \in \mathbb{R}^d} \left\{ J(u; \mu) = \frac{1}{2} \|Du\|_2^2 + \frac{\mu}{2} \|Au - b\|_2^2 \right\}$$

$$\text{TIK} - L_1 - U \quad u^*(\mu) = \arg \min_{u \in \mathbb{R}^d} \left\{ J(u; \mu) = \frac{1}{2} \|Du\|_2^2 + \mu \|Au - b\|_1 \right\}$$

$$\text{TV} - L_2 - U \quad u^*(\mu) = \arg \min_{u \in \mathbb{R}^d} \left\{ J(u; \mu) = \sum_{i=1}^d \sqrt{(D_h u)_i^2 + (D_v u)_i^2} + \frac{\mu}{2} \|Au - b\|_2^2 \right\}$$

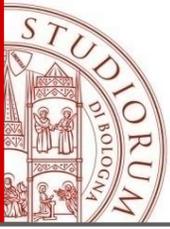
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$$\text{TV} - L_2 - DC \quad u^*(\delta) \in \arg \min_{u \in \mathbb{R}^d} \left\{ J(u; \delta) = \sum_{i=1}^d \sqrt{(D_h u)_i^2 + (D_v u)_i^2} + \iota_{B(\delta)}(Au - b) \right\},$$

$$B(\delta) = \left\{ x \in \mathbb{R}^d : \|x\|_2^2 \leq \delta^2 \right\}, \quad \delta = \delta(\tau) = \tau \sqrt{d} \hat{\sigma}_n$$

U → Unconstrained

DC → Discrepancy-Constrained



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**TIK - L<sub>1</sub> - U**  $u^*(\mu) = \arg \min_{u \in \mathbb{R}^d} \left\{ J(u; \mu) = \frac{1}{2} \|Du\|_2^2 + \mu \|Au - b\|_1 \right\}$

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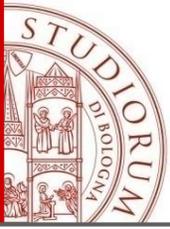
$B(\delta) = \{x \in \mathbb{R}^d : \|x\|_2^2 \leq \delta^2\}, \delta = \delta(\tau) = \tau \sqrt{d \hat{\sigma}_n}$

known  
↓

$B(\tau) = \{x \in \mathbb{R}^d : \|x\|_2^2 \leq \tau^2 (d \hat{\sigma}_n^2)\}$

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**TV - L<sub>2</sub> - DC**      $u^*(\tau) \in \arg \min_{u \in \mathbb{R}^d} \left\{ J(u; \tau) = \sum_{i=1}^d \sqrt{(D_h u)_i^2 + (D_v u)_i^2} + \iota_{B(\tau)}(Au - b) \right\},$

$B(\delta) = \{x \in \mathbb{R}^d : \|x\|_2^2 \leq \delta^2\}, \delta = \delta(\tau) = \tau \sqrt{d} \hat{\sigma}_n$  known  
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When we speak about «**the accuracy**» of a variational model, we mean the **maximum** accuracy achievable by that model by letting its free scalar parameter ( $\mu$  or  $\tau$ ) span its entire domain  $\mathbb{R}_{++}$

$$B(\tau) = \left\{ x \in \mathbb{R}^d : \|x\|_2^2 \leq \tau^2 (d \hat{\sigma}_n^2) \right\}$$