

PhD Winter School 2023:
ADVANCED METHODS for
MATHEMATICAL IMAGE ANALYSIS

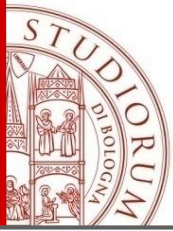
Computational Imaging Lab (A-B)

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Monica Pragliola

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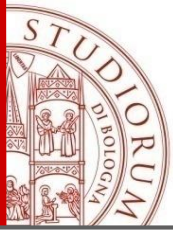
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Experimentation in Matlab: outline

- Presentation of the provided Matlab package:
 - provided scripts and functions, input and output folders,...
 - MAIN_2D script: short explanation of this main script
 - MAIN_2D script: activation/deactivation of all testing functionalities
- Experimental testing of unconstrained TIK- L_2 , TV- L_2 , TIK- L_1 , TV- L_1 models, with TIK- L_2 solved directly (FFT or PCG), TV- L_2 , TIK- L_1 , TV- L_1 solved by ADMM:
 - For TV- L_2 , TIK- L_1 , TV- L_1 , inspection of ADMM iterations
 - Evaluation/comparison of the obtained restoration results
- Experimental testing of discrepancy-constrained TV- L_2 model:
 - Comparison with the unconstrained TV- L_2 model



Experimentation in Matlab: outline

- The performance of all considered variational models solved by some specific minimization algorithm will be evaluated in terms of **accuracy** and of **efficiency**. In particular, we adopt two scalar measures for both accuracy and efficiency:

- **ACCURACY**:
 - 1) Improved Signal-to-Noise-Ratio (**ISNR**)
 - 2) Improved Structural Similarity Index (**ISSIM**)

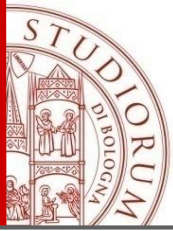
$$\text{ISNR}(u^*, u_{\text{true}}, b) = \text{SNR}(u^*, u_{\text{true}}) - \text{SNR}(b, u_{\text{true}}), \quad \text{SNR}(x, u_{\text{true}}) = 10 \log_{10} \left(\frac{\|Ku_{\text{true}} - E[Ku_{\text{true}}]\|_2^2}{\|Ku_{\text{true}} - x\|_2^2} \right) [\text{dB}]$$

$$\text{ISSIM}(u^*, u_{\text{true}}, b) = \text{SSIM}(u^*, u_{\text{true}}) - \text{SSIM}(b, u_{\text{true}}), \quad \text{SSIM}(x, u_{\text{true}}) \text{ defined according to [1],}$$

type "help ssim" in Matlab

- **EFFICIENCY**:
 - 1) **number of performed iterations** (for iterative algorithms)
 - 2) **CPU time** spent by the algorithm for obtaining the restoration

Clearly, accuracy will also be evaluated by visually inspecting the obtained results!!



Experimentation in Matlab: outline

- By means of the provided Matlab software, we can test experimentally 5 models:

TIK - L_2 - U $u^*(\mu) = \arg \min_{u \in \mathbb{R}^d} \left\{ J(u; \mu) = \frac{1}{2} \|Du\|_2^2 + \frac{\mu}{2} \|Au - b\|_2^2 \right\}$

TIK - L_1 - U $u^*(\mu) = \arg \min_{u \in \mathbb{R}^d} \left\{ J(u; \mu) = \frac{1}{2} \|Du\|_2^2 + \mu \|Au - b\|_1 \right\}$

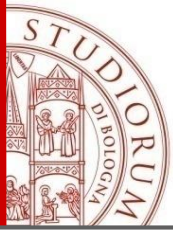
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TV - L_2 - DC $u^*(\delta) \in \arg \min_{u \in \mathbb{R}^d} \left\{ J(u; \delta) = \sum_{i=1}^d \sqrt{(D_h u)_i^2 + (D_v u)_i^2} + \iota_{B(\delta)}(Au - b) \right\},$
 $B(\delta) = \left\{ x \in \mathbb{R}^d : \|x\|_2^2 \leq \delta^2 \right\}, \quad \delta = \delta(\tau) = \tau \sqrt{d} \hat{\sigma}_n$

U \rightarrow Unconstrained

DC \rightarrow Discrepancy-Constrained



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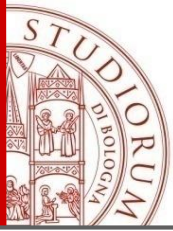
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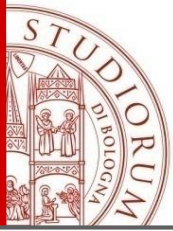
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When we speak about «**the accuracy**» of a variational model, we mean the **maximum** accuracy achievable by that model by letting its free scalar parameter (μ or τ) span its entire domain \mathbb{R}_{++}

$$B(\tau) = \left\{ x \in \mathbb{R}^d : \|x\|_2^2 \leq \tau^2 (d \hat{\sigma}_n^2) \right\}$$