1. Self-scheduling tools for producers – Mixed-integer linear programming

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Optimal Response of a Thermal Unit to an Electricity Spot Market

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***Abstract*—This paper addresses the optimal response of a thermal unit to an electricity spot market. The objective is to maximize the unit profit from selling both energy and spinning reserve in the spot market. The paper proposes a 0/1 mixed-integer linear programming approach that allows a rigorous modeling of i) nonconvex and nondifferentiable operating costs, ii) exponential start-up costs, iii) available spinning reserve taking into account ramp rate restrictions, and iv) minimum up and down time constraints. This approach overcomes the modeling limitations of dynamic programming approaches and is computationally efficient. Results from realistic case studies are reported.**

***Index Terms*—Electricity spot market, mixed-integer optimization, optimal response.**

I. Introduction

ELECTRIC power utilities are experiencing a major restructuring process. This process is intended to open the power sector to market forces with the ultimate target of decreasing consumer prices. In this new competitive electric power generation environment, a fundamental task is to determine the optimal response of a thermal unit to the spot market. The objective is to find out the start-up and shut-down schedule as well as the hourly production of the unit so that the expected benefit from selling both energy and spinning reserve to the spot market is maximized. This problem has been recently referred to as decentralized unit commitment. For the analysis of this paper the market consists of generation companies (GENCO's), distribution companies (DISTCO's) and an independent market operator (IMO) to match offers and demands [1], [2].

Perfect market competition is assumed, i.e. no participant has the capability of altering clearing prices. Although hourly prices (energy and reserve) depend on competitor actions, they are considered known. This assumption is reasonable as the effect of the response of one thermal unit to the spot market does not change prices considerably. Forecasting procedures can be used to estimate hourly market prices for the next day. Under different price profiles, the solution to the considered problem provides consistent information to generate bids in response to the spot market. For instance, if the unit bids with cost equal to 0 for the hours with scheduled power output over 0 MW, it ensures it will be selected by the IMO. This is the minimum risk bidding strategy. If the GENCO decides to increase the scheduled benefit it may try to increase the forecast system marginal price by bidding a price just below the forecast spot price. Thus, the unit ensures to be accepted by the IMO and the profits will be higher than the ones computed by the algorithm presented in this paper.

The dimension of this optimal response problem is small but its mathematical features are complex. A precise model of the unit operating costs requires the use of nondifferentiable and nonconvex functions [3]. A detailed modeling of start-up costs requires the use of exponential functions [3]. Furthermore, taking into account ramp limitations, a precise modeling of the contribution of the unit to the spinning reserve of the system requires the use of complex restrictions usually formulated as nonlinear constraints [4]. Minimum up and down time constraints also require the enforcement of conditions usually expressed as nonlinear constraints [4]. Few references address bidding related issues for the power sector. Brokerage systems are proposed in [5]–[8] and references [9]–[11] provide scheduling algorithms in a competitive market.

The optimal response problem addressed in this paper is similar to any of the unit subproblems of a unit commitment solved by Lagrangian relaxation. In this unit commitment Lagrangian relaxation framework, the unit maximum profit problem has been traditionally solved using dynamic programming [12]–[14]. However, a dynamic programming approach has several important drawbacks:

1) Non-differentiable and nonconvex operating costs are complex to model in order to efficiently compute the profit of every state in every hour.
2) The start-up cost is a function of the time the unit has been shut down. This cost depending on time has to be modeled approximately by increasing the number of states considered in every hour.
3) To take into account ramp constraints, the modeling of the unit available spinning reserve has to be done approximately by further increasing the number of states considered in every hour.

Drawbacks 2 and 3 result in inaccuracies and additional computational burden.

This paper proposes a 0/1 mixed-integer linear programming approach that allows a rigorous modeling of i) nonconvex and nondifferentiable operating costs, ii) exponential start-up costs, iii) available spinning reserve taking into account ramp rate restrictions, and iv) minimum up and down time constraints. This approach is based on the formulation stated by Dillon *et al.* [15]. It overcomes the modeling limitations of dynamic programming approaches and is computationally efficient.

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The proposed approach is useful i) to determine the optimal bidding strategy of a unit in the spot market and ii) to solve efficiently and accurately the subproblems that result from addressing a (centralized) unit commitment problem using Lagrangian relaxation.

This paper is organized as follows. Section II provides the notation used throughout the paper. In Section III the optimal response problem is formulated. In Section IV this problem is formulated as a mixed-integer linear programming problem. Section V provides results from different realistic case studies. Finally, conclusions are stated in Section VI.

II. Notation

The notation used throughout the paper is stated below:

Functions:

$b_j(\cdot)$ start-up cost of unit j at hour k [\$], which is a nonlinear function of the time unit j has been shutdown at hour k, $s_j(k)$,

$\tilde{b}_j(k)$ stairwise approximation of the start-up cost of unit j at hour k [\$],

$d_j(\cdot)$ variable cost of unit j at hour k [\$/h], which is a nonlinear function of the power output of unit j at hour k, $p_j(k)$,

$\tilde{d}_j(k)$ piecewise linear approximation of the variable cost of unit j at hour k [\$/h].

Constants:

$\lambda(k)$ spot price of energy at hour k [\$/MWh],

$\mu(k)$ spot price of spinning reserve at hour k [\$/MW],

A_j fixed cost of unit j [\$/h],

C_j shut-down cost of unit j [\$/h],

DT_j minimum down time of unit j [h],

$F_\ell(j)$ slope of block ℓ of the variable cost of unit j [\$/MWh],

K_j^i cost of the ith discrete interval of the start-up cost of unit j [\$/h]

ND number of discrete intervals of the start-up cost function,

NL number of blocks of the piecewise linearization of the variable cost function,

$\overline{P}_j$ capacity of unit j [MW],

$\underline{P}_j$ minimum power output of unit j [MW],

RD_j ramp-down limit of unit j [MW/h],

RU_j ramp-up limit of unit j [MW/h],

$\overline{S}_j$ large enough constant (e.g., the maximum number of hours unit j can be off),

SD_j shut-down ramp limit of unit j [MW/h],

SU_j start-up ramp limit of unit j [MW/h],

T number of hours of the planning period,

$T_{\ell j}$ upper limit of block ℓ of the variable cost of unit j [MW],

U_j^0 time periods unit j has been on at the beginning of the planning horizon (end of hour 0) [h],

UT_j minimum up time of unit j [h].

Variables:

$\delta_\ell(j, k)$ variable used for the linearization of the variable cost function of unit j at hour k, it represents the ℓth power block [MW],

$m_j(k)$ dummy variable used for the discretization of the start-up cost function of unit j at hour k [h],

$p_j(k)$ power output of unit j at hour k [MW],

$\overline{p}_j(k)$ maximum power output of unit j at hour k [MW],

$\underline{p}_j(k)$ minimum power output of unit j at hour k [MW],

$s_j(k)$ time periods that unit j has been shut-down at hour k[h]

$t_\ell(j, k)$ 0/1 variable which is equal to 1 if the power output of unit j at hour k has exceeded block ℓ,

$v_j(k)$ 0/1 variable which is equal to 1 if unit j is committed at hour k,

$w_j^i(jk)$ 0/1 variable which is equal to 1 if unit j is started-up at the beginning of hour k and it has been off for i hours,

$x_j(k)$ number of hours unit j has been on (+) or off (−) at the end of hour k,

$y_j(k)$ 0/1 variable which is equal to 1 if unit j is started-up at the beginning of hour k,

$z_j(k)$ 0/1 variable which is equal to 1 if unit j is shut-down at the beginning of hour k.

Sets:

I set of discrete intervals of the start-up cost function,

K set of indices of the hours of the planning period,

L set of blocks of the piecewise linearization of the variable cost function.

III. Original Formulation

The optimal response of a thermal unit j to a perfect electricity spot market can be formulated as follows:

Maximize

$$\sum_{k\in K}\{\lambda(k)p_j(k) - A_j v_j(k) - d_j(p_j(k)) - C_j z_j(k) - b_j(s_j(k))y_j(k) + \mu(k)[\overline{p}_j(k) - p_j(k)]\} \quad (1)$$

subject to:

$$\underline{p}_j(k)v_j(k) \le p_j(k) \le \overline{p}_j(k)v_j(k) \qquad \forall\, k \in K \quad (2)$$

$$\overline{p}_j(k) = \text{Min}\{\overline{P}_j[v_j(k) - z_j(k+1)]z_j(k+1)SD_j,\ p_j(k-1) + RU_j v_j(k-1) + y_j(k)SU_j\} \quad \forall\, k \in K \quad (3)$$

$$\underline{p}_j(k) = \text{Max}[\underline{P}_j,\ p_j(k-1) - RD_j v_j(k)] \qquad \forall\, k \in K \quad (4)$$

$$[x_j(k-1) - UT_j][v_j(k-1) - v_j(k)] \ge 0 \qquad \forall\, k \in K \quad (5)$$

$$[x_j(k-1) + DT_j][v_j(k) - v_j(k-1)] \le 0 \qquad \forall\, k \in K \quad (6)$$

$$y_j(k) - z_j(k) = v_j(k) - v_j(k-1) \qquad \forall\, k \in K \quad (7)$$

$$y_j(k) + z_j(k) \le 1 \qquad \forall\, k \in K \quad (8)$$

and

$$v_j(k),\, y_j(k),\, z_j(k) \in \{0,\, 1\} \qquad \forall\, k \in K \tag{9}$$

For unit consistency, it should be noted that time periods of 1 hour are considered.

The objective function (1) represents the benefit from selling power and spinning reserve. It includes revenues from selling both energy and spinning reserve as well as production costs, start-up costs and shut-down costs. A more sophisticated revenue function taking into account competitor actions can be incorporated in the above framework. However, the objective of this paper is to provide a precise description and mathematical treatment of the technical constraints a unit faces when bidding in a spot market.

Constraints (2) force the thermal unit to work below its maximum power output, and above its minimum power output. Constraints (3) state that the available maximum power output at every hour depends on ramp rate limits. These constraints are explained below in Section IV-C. Constraints (4) update the minimum power output taking into account the ramp rate limits. These constraints are explained below in Section IV-D. Constraints (5) and (6) enforce feasibility in terms of minimum up and minimum down time constraints respectively. The remaining constraints preserve the logic of running, start-up, and shut-down status changes.

From a mathematical point of view, the above formulation of the problem is mixed-integer and nonlinear.

IV. Sorting Out Nonlinearities

This section provides an alternative linear formulation of problem (1)–(9). Nonlinear constraints such as minimum up and down time constraints, and maximum and minimum power output constraints are converted into linear constraints. The nonlinear startup cost is discretized so that a linear function of the time the unit has been de-committed is attained. As a byproduct a linear off-time counter is derived. Finally, nonconvex and nondifferentiable variable costs are expressed as a piecewise linear function.

A. Linear Expression of Minimum Up Time Constraints

Nonlinear constraints (5) are replaced by the equivalent linear constraints (10)–(12) below:

$$\sum_{k=1}^{L_j}[1 - v_j(k)] = 0 \tag{10}$$

$$\sum_{i=k}^{k+UT_j-1} v_j(i) \geq UT_j y_j(k) \quad \forall\, k = L_j + 1 \cdots T - UT_j + 1 \tag{11}$$

$$\sum_{i=k}^{T}[v_j(i) - y_j(k)] \geq 0 \quad \forall\, k = T - UT_j + 2 \cdots T \tag{12}$$

where $L_j = \text{Min}[T,\, (UT_j - U_j^0)v_j(0)]$.

Equation (10) ensures that the unit will satisfy the minimum up time constraint if it has been on at hour 0 for fewer hours than the minimum up time. Equation (11) enforces the minimum up time constraint for all the possible sets of consecutive hours of size UT_j. Equation (12) enforces the minimum up time constraint for the last $UT_j - 1$ hours, i.e. if unit j is started-up in one of these hours it will remain committed until the last hour of the time horizon.

Several remarks on indices i and k should be made. If $L_j < 1$ (i.e., either the unit is initially de-committed or $UT_j \leq U_j^0$), constraints (10) are not included in the formulation. Constraints (11) are only considered for positive values of index k. Moreover, in case $L_j + 1 > T - UT_j + 1$, constraints (11) are not to be included. Finally, constraints (12) are only introduced in the formulation for positive values of index k and if $T - UT_j + 2 \leq T$, i.e. $UT_j \geq 2$, otherwise they should be removed from the formulation.

B. Linear Expression of Minimum Down Time Constraints

Nonlinear constraints (6) are replaced by the equivalent linear constraints (13)–(15) below:

$$\sum_{k=1}^{F_j} v_j(k) = 0 \tag{13}$$

$$\sum_{i=k}^{k+DT_j-1} [1 - v_j(i)] \geq DT_j z_j(k) \quad \forall\, k = F_j + 1 \cdots T - DT_j + 1 \tag{14}$$

$$\sum_{i=k}^{T}[1 - v_j(i) - z_j(k)] \geq 0 \quad \forall\, k = T - DT_j + 2 \cdots T \tag{15}$$

where $F_j = \text{Min}\{T,\, [DT_j - s_j(0)][1 - v_j(0)]\}$.

Equation (13) ensures that the unit will satisfy the minimum down time constraint if it has been off at hour 0 for fewer hours than the minimum down time. Equation (14) enforces the minimum down time constraint for all the possible sets of consecutive hours of size DT_j. Equation (15) enforces the minimum down time constraint for the last $DT_j - 1$ hours, i.e. if unit j is shut-down in one of these hours it will remain de-committed until the last hour of the time horizon.

It should be noted that constraints (13)–(15) are identical to constraints (10)–(12) just by changing $v_j(k)$, $z_j(k)$, DT_j and $s_j(0)$ by $1 - v_j(k)$, $y_j(k)$, UT_j and U_j^0 respectively.

The same considerations regarding subindices i and k stated in Section IV-A should be taken into account for constraints (13)–(15).

C. Linear Expression of the Maximum Power Output Constraint

Variable $\overline{p}_j(k)$ represents the maximum power output of unit j at hour k. This variable considers the ramp-up rate limit, and the startup and shut-down ramp rates limits, and it is used in the spinning reserve term of the objective function. Mathematically, this variable can be expressed as follows:

$$\begin{aligned}\overline{p}_j(k) = \text{Min}\{&\overline{P}_j[v_j(k) - z_j(k+1)] + z_j(jk+1)SD_j,\\ &p_j(k-1) + RU_j v_j(k-1) + y_j(k)SU_j\}\\ &\forall\, k \in K\end{aligned} \tag{3}$$

As it was stated in Section III, constraints (3) update the available maximum power output taking into account the ramp rate limits. The available maximum power output is the minimum of i) the unit capacity if the unit is committed and it has not been started-up in the current hour and it will not be shut-down the next hour, ii) the shut-down ramp rate limit if it will be de-committed the next hour, iii) 0 if the unit is de-committed, iv) the start-up ramp rate limit if the unit is started-up, and v) the power output of the previous hour plus the ramp-up rate limit in the case the unit remains committed.

This variable can be defined in an alternative linear way as follows:

$$\overline{p}_j(k) \leq \overline{P}_j[v_j(k) - z_j(k+1)] + z_j(k+1)SD_j \quad \forall\, k \in K \tag{16}$$

$$\overline{p}_j(k) \leq p_j(k-1) + RU_j v_j(k-1) + SUk_j y_j(k) \quad \forall\, k \in K \tag{17}$$

$$\overline{p}_j(k) \geq 0 \qquad \forall\, k \in K \tag{18}$$

If unit j is off at hour k [i.e. $v_j(k) = 0$] the maximum available power output will be 0, which is enforced by constraints (16) and (18). Otherwise, $v_j(k) = 1$ and $\overline{p}_j(k)$ should be i) smaller than the unit capacity $\overline{P}_j$, ii) smaller than the power output in the previous period plus the maximum ramp-up rate limit, iii) smaller than the start-up ramp rate limit in case the unit is started at the beginning of period k, and iv) smaller than the shut-down ramp rate limit in case the unit is de-committed at the beginning of period $k+1$. All these limits are enforced by constraints (16) and (17). It should be noted that constraints (16)–(18) are linear. It should also be noted that constraints (16), (17) provide a value of $\overline{p}_j(k)$ identical to the value provided by equation (3) if and only if variable $\overline{p}_j(k)$ is multiplied by a positive constant in the objective function (in case of maximization) such as $\mu(k)$. Otherwise $\overline{p}_j(k)$ will take a value so that the benefit is maximum but it does not have to be equal to the value provided by equation (3).

Finally, the power output should be smaller than $\overline{p}_j(k)$, so that the nonlinear right hand side of equation (2) becomes:

$$p_j(k) \leq \overline{p}_j(k) \qquad \forall\, k \in K \tag{19}$$

which is linear.

D. Linear Expression of the Minimum Power Output Constraint

As variable $\underline{p}_j(k)$ only appears in constraints (2) there is no need to define it as it was done for $\overline{p}_j(k)$. The power output should be greater than the nominal minimum power output $\underline{P}_j$ [equation (20) below] and should take into account the maximum ramp-down rate limit and the shut-down ramp rate limit [equation (21) below]. Additionally, if the unit is de-committed, the power output must be 0 [equation (20) below]. This can be formulated as follows:

$$\underline{P} v_j(k) \leq p_j(k) \qquad \forall\, k \in K \tag{20}$$

$$p_j(k-1) - p_j(k) \leq RD_j v_j(k) + SD_j z_j(k) \qquad \forall\, k \in K \tag{21}$$

It should be noted that the both previous constraints are linear.

E. Time Counter

In order to express the start-up costs as a function of the time a unit has been off, it is necessary to define an off-time counter $s_j(k)$ and to include it in the formulation.

The counter of the hours a unit has been off can be expressed through the following condition:

$$\text{If } v_j(k) = 0 \text{ then } s_j(k) = s_j(k-1) + 1, \text{ else } s_j(k) = 0$$

Mathematically, as stated in [16], this conditional expression can be formulated with linear constraints as:

$$s_j(k) \leq s_j(k-1) + 1 \qquad \forall\, k \in K \tag{22}$$

$$s_j(k) + (\overline{S}_j + 1)v_j(k) \geq s_j(k-1) + 1 \qquad \forall\, k \in K \tag{23}$$

$$s_j(k) - \overline{S}_j[1 - v_j(k)] \leq 0 \qquad \forall\, k \in K \tag{24}$$

$$s_j(k) \geq 0 \qquad \forall\, k \in K \tag{25}$$

where $\overline{S}_j$ is a large enough constant (e.g., the maximum number of hours unit j can be off).

F. Stairwise Formulation of the Start-Up Cost Function

The start-up cost is modeled as a nonlinear (exponential) function of the number of hours a unit has been off. As the number of hours is discrete the start-up cost is also a discrete function that can be formulated as a 0/1 linear expression (Fig. 1). It should be noted that if the number of intervals, ND, is large enough this model of the start-up cost asymptotically approximates the original exponential model. The larger the number of intervals the more precise the discretization will be. It should also be noted that the number of intervals can be large because the problem addressed is small. This linear formulation is presented below:

$$\tilde{b}_j(k) = \sum_{i=1}^{ND} K_j^i w_j^i(k) \qquad \forall\, k \in K \tag{26}$$

$$\sum_{i=1}^{ND} w_j^i(k) = y_j(k) \qquad \forall\, k \in K \tag{27}$$

$$s_j(k-1) = \sum_{i=1}^{ND-1} i w_j^i(k) + m_j(k) \qquad \forall\, k \in K \tag{28}$$

$$m_j(k) \leq \overline{S}_j[w_j^{ND}(k) - y_j(k) + 1] \qquad \forall\, k \in K \tag{29}$$

$$m_j(k) \geq ND w_j^{ND}(k) \qquad \forall\, k \in K \tag{30}$$

$$w_j^i(k) \in \{0, 1\} \qquad \forall\, i \in I, \forall\, k \in K \tag{31}$$

Equation (26) represents the stairwise linear start-up cost function $\tilde{b}_j(k)$, which is a constant of value K_j^i if unit j has been off for i hours (corresponding to the evaluation of the exponential start-up cost for t equal to i). If unit j is started-up when it has been off for ND hours or longer $\tilde{b}_j(k)$ is equal to K_j^{ND}. This is modeled through a new binary variable $w_j^i(k)$ which is equal to 1 if unit j is started-up at hour k and has been off for i hours. If unit j is started-up, constraints (27) force only one of these binary variables to be equal to 1. Every constraint (28) relates variables $w_j^i(k)$ with the time counter $s_j(k)$ through a dummy variable, $m_j(k)$, whose limits are imposed by constraints (29) and (30). Variable $m_j(k)$ is used either when unit j is off at hour k or when unit j is started-up at hour k and has been off for ND hours or longer. Finally,

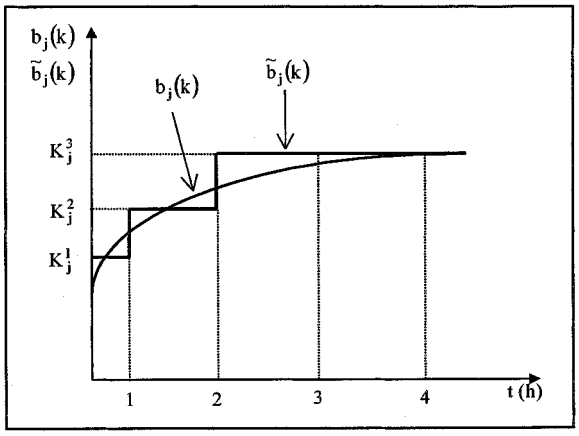

Fig. 1. Nonlinear start-up cost function ($b_j(k)$) and its stairwise approximation ($\tilde{b}_j(k)$).

constraints (31) state that variables $w_j^i(k)$ are binary. It should be noted that all the above constraints are linear. This stairwise approximation is shown in Fig. 1 for the case of 3 intervals (i.e. ND $= 3$).

G. Linear Piecewise Approximation of Non-Convex and Non-Differentiable Variable Costs

Variable costs are usually modeled as nonlinear (quadratic) functions of the power output. However, generators have a number of steam admission valves that are opened in sequence as the power output is increased. This is particularly so for combined cycle gas turbines. Thus, the variable cost is a nonconvex function [3]. This nonconvex function can be represented by a piecewise linear curve by using binary variables (Fig. 2). The size of the problem under consideration is small and therefore the number of linear blocks can be as large as needed to precisely model the actual nonconvex variable cost function.

The piecewise linear variable cost $\tilde{d}_j(k)$ can be expressed as follows:

$$\tilde{d}_j(k) = \sum_{\ell=1}^{NL} F_\ell(j)\delta_\ell(j, k) \qquad \forall\, k \in K \tag{32}$$

where $F_\ell(j)$ is the slope of power block ℓ for unit j and $\delta_\ell(j, k)$ is the power generated by unit j at hour k in block ℓ.

Equation (32) substitutes the nonlinear variable cost function $d_j(\cdot)$ in objective function (1).

The following constraints express the piecewise linearization:

$$p_j(k) = \sum_{\ell=1}^{NL} \delta_\ell(j, k) + \underline{P}_j v_j(k) \qquad \forall\, k \in K \tag{33}$$

$$(T_{1j} - \underline{P}_j)t_1(j, k) \le \delta_1(j, k) \qquad \forall\, k \in K \tag{34}$$

$$\delta_1(j, k) \le (T_{1j} - \underline{P}_j)v_j(k) \qquad \forall\, k \in K \tag{35}$$

$$(T_{\ell j} - T_{\ell-1j})t_\ell(j, k) \le \delta_\ell(j, k) \qquad \ell = 2 \cdots NL - 1, \quad \forall\, k \in K \tag{36}$$

$$\delta_\ell(j, k) \le (T_{\ell j} - T_{\ell-1j})t_{\ell-1}(j, k) \qquad \ell = 2 \cdots NL - 1, \quad \forall\, k \in K \tag{37}$$

$$\delta_{NL}(j, k) \le (\overline{P}_j - T_{NL-1j})t_{NL-1}(j, k) \qquad \forall\, k \in K \tag{38}$$

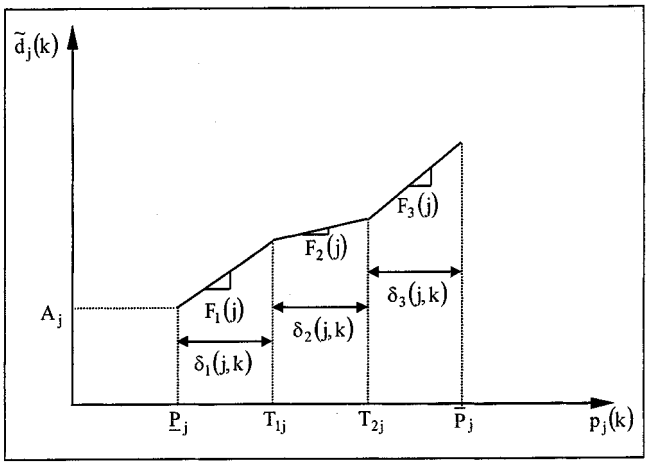

Fig. 2. Nonconvex variable cost function and its linear approximation.

$$\delta_{NL}(j, k) \ge 0 \qquad \forall\, k \in K \tag{39}$$

$$t_\ell(j, k) \in \{0, 1\} \qquad \forall\, \ell \in L, \forall\, k \in K \tag{40}$$

Constraints (33) state that power output of unit j at hour k is the sum of the power generated in each block plus the minimum power output. Constraints (34)–(39) set the limits of the power generated in each block. This power must be greater than 0 and smaller than the "power length" of each block. This is ensured through a binary variable $t_\ell(j, k)$ which is equal to 1 if the power output of unit j at hour k has exceeded block ℓ. Finally, constraints (40) state that variables $t_\ell(j, k)$ are binary.

H. Final Linear Formulation

The new linear formulation is presented below:

Maximize

$$\sum_{k \in K} \{\lambda(k)p_j(k) - A_j v_j(k) - \tilde{d}_j(k) - \tilde{b}_j(k) - C_j z_j(k) + \mu(k)[\overline{p}_j(k) - p_j(k)]\} \tag{41}$$

subject to:

Equations (26) and (32) which express linearized start-up cost and production cost respectively, (19), (20) which set limits on power output, (16)–(18) and (21) which enforce ramp rate limits, (10)–(12) and (13)–(15) which model minimum up and down time constraints respectively, (22)–(25) representing time counter constraints, (7)–(9) which impose the logic of start-ups and shutdowns, (27)–(31) expressing the stairwise approximation of the start-up cost, and (33)–(40) which model the linearization of the variable cost.

V. Case Study

The data for the unit considered in this study, given in Table I, is based on the data of reference [4]. A coal-fired unit has been chosen. Its original characteristics have been altered so that the following examples show the differences between a typical formulation and a precise formulation as the one proposed in this paper. Thus, ramp up and down rate limits have been decreased and start-up and shut-down ramp rates have been added. The start-up cost is modeled through 11 intervals (Table II) and the variable cost is linearized in 3 blocks as shown in Fig. 2. The

TABLE I
CHARACTERISTICS OF THE THERMAL UNIT

UT_j (h)	DT_j (h)	C_j ($)	$\overline{P}_j$ (MW)	$\underline{P}_j$ (MW)	SU_j (MW)	SD_j (MW)	RU_j (MW/h)	RD_j (MW/h)	A_j ($)
10	9	120	294	112	170	160	60	50	4480

TABLE II
STAIRWISE START-UP COST ($)

K_j^1	K_j^2	K_j^3	K_j^4	K_j^5	K_j^6	K_j^7	K_j^8	K_j^9	K_j^{10}	K_j^{11}
667	1333	1867	2267	2533	2667	2733	2767	2813	2853	2867

TABLE III
PIECEWISE LINEAR VARIABLE COST

	LOWER LIMIT (MW)	UPPER LIMIT (MW)	SLOPE ($/MWh)
BLOCK #1	112	190	40.00
BLOCK #2	190	230	36.67
BLOCK #3	230	294	43.33

slopes and limits of the power blocks are presented in Table III. It is to be noted that this variable cost function is nonconvex and nondifferentiable. Finally, an initial status is provided. At the beginning of the planning horizon, the unit has been committed for 11 hours and the power output is equal to 170 MW.

The profile of the spot prices of energy is based on the curve of marginal prices obtained in the Spanish de-regulated electricity market on January 28th, 1999 (www.mercaelectrico.comel.es). This profile of prices is plotted in Figs. 3 and 4. The prices for spinning reserve are all zero but for hours 12 and 20 (hours with the highest spot price), which are equal to $67/MW.

The aim of this case study is to show how the linear formulation presented in this paper models precisely i) constraints typically addressed as nonlinear (minimum up and down time constraints, and start-up and shut-down ramp rate limits), ii) nonconvex and nondifferentiable variable costs, and iii) exponential start-up costs. It is also shown how the scheduling of the unit is affected by the inclusion of a spinning reserve term in the revenue equation.

The optimal response of the unit is shown in Fig. 3. In hour 1, although the benefit is negative (the energy spot price is smaller than the variable cost), the unit cannot be shut-down due to the shut-down ramp rate limit. It should be noted that the unit is decommitted in hours of negative benefit. However, the unit remains off in hours 9 and 10 when the spot price is larger than $43.33/MWh (the maximum slope of the variable cost piecewise linear approximation). This is due to the minimum down time of 9 hours. It should also be noted that there is a reduction in power output in hour 15 (230 MW). This power output reduction corresponds to the upper limit of the second block of the variable cost (230 MW). Generation does not exceed this limit because the slope of the next block is greater than the spot price ($43.33/MWh versus $40.60/MWh). The total benefit over the 24 hour period is $27 898.8.

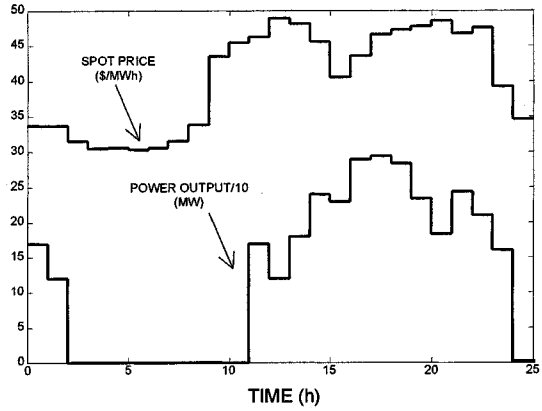

Fig. 3. Optimal response of the thermal unit for the base case.

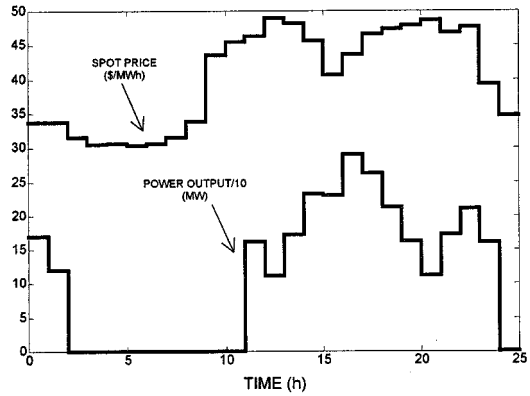

Fig. 4. Optimal response with approximate available spinning reserve.

In order to show the differences between usual approximate models and the one proposed in this paper, several simplifications have been introduced on this base case. The models usually found in the literature [17] i) do not model nonconvex and nondifferentiable variable costs, ii) consider constant start-up costs instead of nonlinear (exponential) start-up costs, and iii) do not model the actual available spinning reserve but a spinning reserve based on the unit capacity [i.e., to use $\overline{P}_j$ instead of $\overline{p}_j(k)$]. These approximations may deteriorate the quality of the solution obtained, leading to a solution far from optimal.

The first simplification consists of considering a linear variable cost of constant slope. In this case, nonconvex and nondifferentiable variable cost functions are not modeled, which is a usual approximation considered when solving this kind of problems [17]. The slope is equal to $40/MWh which corresponds to the mean of the slopes of the three blocks of the base case. The production of the unit is identical to the one obtained in the base case except for hours 15 and 16. In hour 15 the power output in the base case was 230 MW because the spot price was smaller than the variable cost slope of the next power block. The production in hour 16 is limited by the ramp up rate (60 MW/h). For the simplified case, the spot price is greater than the variable cost slope and the unit is at full power (294 MW) in both hours. The total benefit is $27 638.33.

The second simplification shows the effect of using a constant start-up cost regardless the number of hours the units has been off before being started-up. This is a usual approximation that leads to sub-optimal solutions. The constant start-up cost is equal to \$2667 which is the start-up cost if the unit has been off for 6 hours at the time it is started-up (K_j^6). As a result, the generation does not change with respect to the base case and the unit is again started-up at hour 11. The total benefit is \$28 048.8 which is \$150 higher than the benefit of the base case. This is the difference between the constant start-up cost and the start-up cost if the unit has been off for 9 hours before being started-up (K_j^9).

The last simplification on the base case shows the need of a precise modeling of the available maximum power output. This variable, $\overline{p}_j(k)$, is substituted by the unit capacity, $\overline{P}_j$, in the objective function, i.e. the revenue from selling spinning reserve is not modeled precisely. The results are shown in Fig. 4. The benefit is \$34 898.88 which is considerably higher than the one obtained in the base case. This revenue is unreal as the unit benefits from fictitious spinning reserve that cannot be served by the unit. Moreover, it should be noted a reduction of generation in hours 11–14 and 18–21 so that the difference between production and the unit capacity is maximum, i.e. to obtain the maximum benefit. The actual benefit of this solution is \$25 299.08, which represents a 10.28% benefit reduction with respect to the base case.

The model has been implemented on a SGI R10000 based processor using CPLEX under GAMS [18]. The computing time is negligible for all the cases analyzed.

VI. Conclusions

This paper addresses the optimal response of a thermal unit to a perfect electricity spot market. The objective is to maximize the unit profit from selling both energy and spinning reserve in the spot market. Perfect market competition is assumed. The paper proposes a 0/1 mixed-integer linear programming approach that overcomes the modeling limitations of dynamic programming and is computationally efficient. The proposed approach is useful to generate appropriate information to produce successful bids to the spot market. Results from realistic case studies are reported. With respect to benefits, it is shown how relevant is the precise modeling of the available spinning reserve and cost nonlinearities.

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Optimal Response of a Power Generator to Energy, AGC, and Reserve Pool-Based Markets

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***Abstract*—The electric power industry all over the world is undertaking major regulatory and operational changes. The understanding of power supply as a public service is being replaced by the notion that competitive markets constitute a more appropriate framework to supply reliable and cheap electric energy to consumers. As mandated by FERC orders 888 and 889, this paper considers simultaneously energy, AGC, and reserve markets in a pool framework. It provides a tool to enable a power generator to determine optimally its degree of involvement in each one of these markets. A real-world case study is analyzed and results are presented.**

***Index Terms*—AGC market, energy market, mixed-integer optimization, pool-based electricity markets, reserve market.**

NOMENCLATURE

The notation used throughout the paper is stated below in the following.

Functions

$b(\cdot)$	Start-up cost in period k [in dollars per hour], which is a nonlinear function of the time the unit has been shut-down before period k, $s(k)$.
$c(k)$	Operating cost in period k [in dollars per hour].
$d(\cdot)$	Variable cost in period k [in dollars per hour], which is a nonlinear function of the average power output in period k, $p^{\text{avg}}(k)$.

Constants

α_i	Weighting factor associated to commodity i, where $i = \{G$ (energy), AGC, TMSR, TMNSR, TMOR$\}$.
$\lambda_{\text{AGC}}(k)$	Forecasted price of AGC in period k [\$/MW & h].
$\lambda_G(k)$	Forecasted price of energy in period k [\$/MWh].
$\lambda_{\text{TMNSR}}(k)$	Forecasted price of 10 min nonspinning reserve (nonsynchronized reserve) in period k [\$/MW & h].
$\lambda_{\text{TMOR}}(k)$	Forecasted price of 30 min operating reserve (nonsynchronized reserve) in period k [\$/MW & h].
$\lambda_{\text{TMSR}}(k)$	Forecasted price of 10 min spinning reserve (synchronized reserve) in period k [\$/MW & h].
A	Fixed cost [in dollars per hour].
$\overline{\text{AGC}}$	Maximum contribution to AGC [in megawatts].
C	Shut-down cost [in dollars per hour].
$\overline{P}$	Maximum capacity [in megawatts].
$\underline{P}$	Minimum power output [in megawatts].
RD	Ramp-down limit [in megawatts per hour].
RH	Regulating high limit (as defined by FERC) [in megawatts].
RL	Regulating low limit (as defined by FERC) [in megawatts].
RU	Ramp-up limit [in megawatts per hour].
SD	Shut-down ramp limit [in megawatts per hour].
SU	Start-up ramp limit [in megawatts per hour].
$\overline{\text{TMNSR}}$	Amount of capacity that can be delivered in ten minutes [in megawatts].
$\overline{\text{TMOR}}$	Amount of capacity that can be delivered in thirty minutes [in megawatts].
$\overline{\text{TMSR}}$	Maximum contribution to TMSR [in megawatts].

Variables

$a(k)$	0/1 variable which is equal to 1 if AGC is provided in period k.
$\text{AGC}(k)$	Capacity assigned to AGC in period k [in megawatts].
$\text{AGC}^{\text{avg}}(k)$	Average value of the AGC contribution in period k [in megawatts].
$p(k)$	Power output in period k [in megawatts].
$p^{\text{avg}}(k)$	Average power output in period k [in megawatts].
$\overline{p}(k)$	Maximum available synchronized power in period k (taking into account ramp rates) [in megawatts].
$s(k)$	Time periods during which the unit has been shut-down before period k [in hours].
$\text{TMNSR}(k)$	Capacity assigned to the 10 min nonspinning reserve in period k [in megawatts].
$\text{TMNSR}^{\text{avg}}(k)$	Average value of the 10 min nonspinning reserve contribution in period k [in megawatts].
$\text{TMOR}(k)$	Capacity assigned to the 30 min operating reserve in period k [in megawatts].
$\text{TMOR}^{\text{avg}}(k)$	Average value of the 30 min operating reserve contribution in period k [in megawatts].
$\text{TMSR}(k)$	Capacity assigned to the 10 min spinning reserve in period k [in megawatts].
$\text{TMSR}^{\text{avg}}(k)$	Average value of the 10 min spinning reserve contribution in period k [in megawatts].

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$v(k)$	0/1 variable which is equal to 1 if the unit is on-line in period k.
$y(k)$	0/1 variable which is equal to 1 if the unit is started-up at the beginning of period k.
$z(k)$	0/1 variable which is equal to 1 if the unit is shut-down at the beginning of period k.

Sets

Π	Set of production constraints.
K	Set of indices of the intervals (hours) of the market horizon.

I. Introduction

THE ELECTRIC power industry in the U.S. and all over the world is undertaking major regulatory and operational changes. The underlying rationale behind all these changes is to move from a centralized operation approach to a competitive one. That is, the understanding of power supply as a public service is being replaced by the notion that a competitive market is a more appropriate framework to supply reliable and cheap electric energy to consumers. This new competitive paradigm materializes into two market structures: 1) a power pool and 2) a floor to facilitate bilateral contracts among producers and consumers.

A pool is an e-commerce marketplace where producers and consumers submit production and consumption bids, respectively. The market operator clears the market (energy, AGC, or reserve) using market rules and produces a market-clearing price and the sets of accepted production and consumption bids. Typically, the market operator clears the market once a day. Markets for energy, AGC and reserve can be considered simultaneously. Producers, consumers, and the regulatory authority agree upon the market rules before their implementation.

In a monthly or yearly framework, the structure to allow competitive trade among producers and consumers is the floor for bilateral contracts. A bilateral contract is an agreement between a producer and a consumer so that the producer supplies electric energy to the consumer at a given price. Physical bilateral contracts are medium-term decisions lasting from one month to one year. Financial contracts to hedge price volatility are possible and desirable.

The power pool is commonly denominated Power Exchange (PX). Usually, producers are referred to as GENerating COmpanies (GENCOs) and consumers can be referred to as CONsumption COmpanies (CONCOs). Energy Service COmpanies (ESCOs) buy energy from bilateral contracts and from the power pool to sell it to different types of customers with the purpose of maximizing their own benefits. The market-clearing entity is often known as the Market Operator (MO). Power transactions are carried out through the transmission and distribution networks. The TRANSmission COmpanies (TRANSCOs) provide the wires to materialize the power transactions. Transmission companies are usually highly regulated entities that provide a nondiscriminatory access to their wires at a regulated fee. Analogously, DIStribution COmpanies (DISCOs) offer a nondiscriminatory use of their distribution wires at a regulated fee. The entity in charge of ensuring the technical feasibility of the power transactions agreed upon at the Power Exchange is denominated the Independent System Operator (ISO). The ISO has usually the authority to modify power transactions already scheduled if system security is at stake. These modifications should be minimal. In the technical literature, confusingly, the MO is called sometimes ISO, and CONsumption and Energy Service COmpanies are referred to as DISCOs. See [1] and [2] for additional details.

This paper considers simultaneously energy, regulation and reserve markets in a pool framework as stated in FERC orders 888 [3] and 889 [4]. It does not consider bilateral contracts. This paper proposes a tool to enable a power generator to determine optimally its degree of involvement in each one of these markets. The related problem of determining how to bid to achieve the energy, AGC, and reserve optimal schedules is beyond the scope of this paper. Moreover, it is considered that all those markets have enough liquidity to allow the generator to sell the optimal amounts it desires of energy, AGC, and reserves. The following unbundled pool-based markets are considered simultaneously:

1) energy market;
2) AGC market;
3) 10 min spinning reserve (TMSR) market;
4) 10 min nonspinning reserve (TMNSR) market;
5) 30 min operating reserve (TMOR) market.

All these markets are cleared once a day on an hourly basis, either simultaneous or successively.

The energy pool allows energy transactions and it is the most important trade structure in a pool-based electricity market. Producers and consumers submit energy production and consumption bids, respectively. The market operator clears the market using market rules and produces a market-clearing energy price and the sets of accepted production and consumption energy bids.

In the AGC market, generators offer a regulation band within an upper and a lower limit at a given price for every hour. The market operator selects AGC offers following an increasing price order until enough AGC power has been allocated to meet system requirements. Reserve markets work in a similar fashion. In the TMSR market, offers refer to synchronized power whereas in the TMNSR and the TMOR markets, offers refer to nonsynchronized power.

Facing those five markets, a generator must decide how much energy/power to allocate to every market so that its profit is maximized and all its technical constraints are satisfied. This paper provides a tool that a generator can use to optimally allocate its resources and therefore to determine the optimal amount of energy/power to offer in each market. It is assumed that the generator should be able to deliver all of its contracted services and that bids can only be called in the market in which they were placed. The tool is based on a mixed-integer mathematical programming model that can be solved using a branch and cut solver such as [5].

The model assumes that sufficiently accurate price forecasts are readily available for energy, AGC, and the three types of reserves. These forecasts can be obtained from historical price time series using an appropriate forecasting tool, either based on

time series, transfer functions or neural networks [6]–[9]. Electricity price forecast errors are typically below 5% [10]. To consider uncertainty in prices requires a stochastic programming approach that is not used in this paper. However, price uncertainty is better considered in the process of elaborating the actual bidding strategies.

All generator constraints including intertemporal ones are enforced. Intertemporal constraints are ramp-up and ramp-down limits, start-up and shut-down ramp limits, minimum up time and minimum down time.

This paper assumes that the considered GENCO does not have the capability of altering market-clearing prices. Therefore, its maximum profit problem decomposes by generator, and this is why the problem addressed in this paper is the one corresponding to a single generator. The problem of determining the coordinated actions of all the generators of a price-maker GENCO to alter market-clearing prices to its own benefits is outside the scope of this paper.

In a typical situation, most of the revenue of the GENCO corresponds to energy, unless AGC prices are particularly high. Nevertheless, volatility is usually higher for AGC prices than for energy prices and therefore a not-too-risky GENCO will mostly rely on the energy market. Usually, potential revenues are comparatively much smaller in reserve markets than in AGC and energy markets, and therefore reserve markets normally play a secondary role on the GENCO total revenue.

This paper is built upon previous work by Rau [11] and Cheung *et al.* [12] where hybrid markets are considered. In these markets competitive pricing coexists with centralized load balance procedures. The model reported in this paper considers pure competitive markets.

The remainder of this paper is organized as follows. In Section II the problem of optimal multimarket response is formulated. Section III presents and analyzes results for a realistic case study. Finally, Section IV provides conclusions. The Appendix further details the problem formulation.

II. Formulation

The optimal response of a thermal unit to an electricity multimarket, including energy, AGC, and different types of reserves, can be formulated as follows:

$$\begin{aligned}\text{Maximize} \sum_{k\in K} [&\alpha_G\lambda_G(k)p^{\text{avg}}(k)\\ &+\alpha_{\text{AGC}}\lambda_{\text{AGC}}(k)\text{AGC}^{\text{avg}}(k)\\ &+\alpha_{\text{TMSR}}\lambda_{\text{TMSR}}(k)\text{TMSR}^{\text{avg}}(k)\\ &+\alpha_{\text{TMNSR}}\lambda_{\text{TMNSR}}(k)\text{TMNSR}^{\text{avg}}(k)\\ &+\alpha_{\text{TMOR}}\lambda_{\text{TMOR}}(k)\text{TMOR}^{\text{avg}}(k)-c(k)]\end{aligned} \quad (1)$$

subject to

$$c(k)=Cz(k)+Av(k)+d(p^{\text{avg}}(k))+b(s(k))y(k) \quad \forall k\in K \quad (2)$$

$$p^{\text{avg}}(k)=\frac{p(k)+p(k-1)}{2} \qquad \forall k\in K \quad (3)$$

$$\text{AGC}^{\text{avg}}(k)=\frac{\text{AGC}(k)+\text{AGC}(k-1)}{2} \qquad \forall k\in K \quad (4)$$

$$\text{TMSR}^{\text{avg}}(k)=\frac{\text{TMSR}(k)+\text{TMSR}(k-1)}{2} \quad \forall k\in K \quad (5)$$

$$\text{TMNSR}^{\text{avg}}(k)=\frac{\text{TMNSR}(k)+\text{TMNSR}(k-1)}{2} \quad \forall k\in K \quad (6)$$

$$\text{TMOR}^{\text{avg}}(k)=\frac{\text{TMOR}(k)+\text{TMOR}(k-1)}{2} \quad \forall k\in K \quad (7)$$

$$\text{AGC}(k)\le \min\left\{(RH-RL),\,\overline{\text{AGC}}\right\}a(k) \qquad \forall k\in K \quad (8)$$

$$\text{TMSR}(k)\le \overline{\text{TMSR}}v(k) \qquad \forall k\in K \quad (9)$$

$$\text{TMNSR}(k)\le \overline{\text{TMNSR}} \qquad \forall k\in K \quad (10)$$

$$\text{TMOR}(k)\le \overline{\text{TMOR}} \qquad \forall k\in K \quad (11)$$

$$a(k)\le v(k) \qquad \forall k\in K \quad (12)$$

$$p(k)\ge \underline{P}v(k)+(RL-\underline{P})\,a(k) \qquad \forall k\in K \quad (13)$$

$$p(k)+\text{AGC}(k)\le \left(RH-\overline{P}\right)a(k)+\overline{P}v(k) \quad \forall k\in K \quad (14)$$

$$p(k)+\text{AGC}(k)+\text{TMSR}(k)\le \overline{p}(k) \qquad \forall k\in K \quad (15)$$

$$\begin{aligned}&p(k)+\text{AGC}(k)+\text{TMSR}(k)+\text{TMNSR}(k)+\text{TMOR}(k)\\ &\quad\le SUy(k)+\overline{P}[1-y(k)] \qquad \forall k\in K\end{aligned} \quad (16)$$

$$\begin{aligned}&p(k)+\text{AGC}(k)+\text{TMSR}(k)+\text{TMNSR}(k)+\text{TMOR}(k)\\ &\quad -p(k-1)-\text{AGC}(k-1)-\text{TMSR}(k-1)\\ &\quad -\text{TMNSR}(k-1)-\text{TMOR}(k-1)\\ &\quad\le RU[1-y(k)]+SUy(k) \qquad \forall k\in K\end{aligned} \quad (17)$$

$$\begin{aligned}&p(k)+\text{AGC}(k)+\text{TMSR}(k)+\text{TMNSR}(k)+\text{TMOR}(k)\\ &\quad\le SDz(k+1)+\overline{P}[1-z(k+1)] \qquad \forall k\in K\end{aligned} \quad (18)$$

$$\begin{aligned}&p(k-1)+\text{AGC}(k-1)+\text{TMSR}(k-1)\\ &\quad +\text{TMNSR}(k-1)+\text{TMOR}(k-1)-p(k)\\ &\quad -\text{AGC}(k)-\text{TMSR}(k)-\text{TMNSR}(k)-\text{TMOR}(k)\\ &\quad\le RD[1-z(k)]+SDz(k) \qquad \forall k\in K\end{aligned} \quad (19)$$

$$\overline{p}(k),\,p(k)\in\Pi \qquad \forall k\in K \quad (20)$$

$$a(k),\,v(k),\,y(k),\,z(k)\in\{0,\,1\} \qquad \forall k\in K. \quad (21)$$

For unit consistency, it should be noted that time periods of 1 h are considered.

The objective function (1) represents the benefit from selling energy, AGC, synchronized spinning reserve, and nonsynchronized reserves (ten minutes nonspinning reserve and thirty minutes operating reserve). It should be noted that average values of each commodity are considered to take into account its variation over each pricing interval. That is, constant pricing intervals span one hour while the considered saleable commodities (energy, AGC, and reserves) change during each hourly interval. Therefore, the hourly revenue associated to each commodity is calculated as the price in the interval times the average value of the commodity over the interval. It should be noted that, in what respect to energy, the above criterion implies a linear variation of the power output throughout every hour.

In order to take into account the different risks involved in the above markets, the revenue associated to each one is weighted by GENCO-selected weighting factors, so as to materialize the preference (or risk aversion) of the GENCO participating in those markets. The operating cost includes production costs, start-up costs, and shut-down costs, which are formulated in (2). The production cost used in this paper is a piecewise linear function which allows modeling nonconvex and nondifferentiable costs. The exponential start-up cost can be modeled precisely by linear expressions using binary variables. The expressions for these two cost functions are stated in the Appendix.

Constraints (3)–(7) express the average value of each commodity in every period which is used in (1). Constraints (8) set the limits for the capacity assigned to AGC in each period. The AGC capacity contribution is limited by the regulating range and by its maximum bound [11]. Constraints (9)–(11) set the upper limits of the spinning reserve and the nonsynchronized reserves respectively. Constraints (12) state that a unit may provide AGC if and only if it is on-line. Constraints (13) force the thermal unit to work above its minimum power output when it is on-line. Additionally, if the unit is providing AGC the power output should be greater than or equal to the regulating low limit. On the other hand, constraints (14) enforce the unit to work below its capacity. Analogously, these constraints state that the AGC contribution plus the power output should be below the regulating high limit [11]. Constraints (15) introduce the effect of ramp rate limits on the power output, the AGC contribution and the spinning reserve contribution through a new variable, $\overline{p}(k)$, which represents the maximum available synchronized power. Constraints (16) and (17) impose, respectively, the ramp-up and the start-up ramp rate limits on the sum of all the commodities. Analogously, ramp-down and shut-down ramp rates are enforced on the sum of all the commodities by constraints (18) and (19).

The block of constraints (20) expresses in a compact way the operating constraints for every time period, i.e., generation output limits, and all intertemporal constraints including ramp-up and ramp-down limits, start-up and shut-down ramp limits, minimum up time, and minimum down time. These constraints are further described in the Appendix. Finally, constraints (21) express the binary nature of some variables used in the formulation.

From a mathematical point of view, the above formulation of the problem is mixed-integer and linear.

III. Case Study

Data for the considered power unit as well as price values are given in this section. The characteristics of the unit are based on the data of [13].

The start-up cost in $ is provided in Table I. Eleven one-hour steps are considered to model the exponential variation of this cost.

A ten-block nonconvex variable cost is considered and given in Table II. A nonconvex cost has been selected to illustrate the capability of the proposed formulation to handle this type of costs.

TABLE I
START-UP COST (IN DOLLARS)

1	2	3	4	5	6	7	8	9	10	11
250	500	700	850	950	1000	1025	1038	1056	1070	1075

TABLE II
VARIABLE COST

	Cost ($/MWh)	Power (MW)
Block 1	18.846	130
Block 2	17.523	148
Block 3	17.836	166
Block 4	18.000	184
Block 5	18.624	202
Block 6	18.978	220
Block 7	19.146	238
Block 8	18.379	256
Block 9	19.000	274
Block 10	19.272	294

The shut-down cost is considered constant and equal to $56 and the fixed cost is $500.

Power, AGC, and reserve limits are provided in Table III.

Minimum up time and minimum down time for the considered unit are both 4 h. In the period before the market horizon the unit has been running for 11 h and produced 170 MW.

Price data are provided in Table IV. Energy prices are the prices obtained in the electricity market of California on April 23rd, 2000 (www.calpx.com). This price profile is plotted in Fig. 1. The prices for AGC and spinning reserve are considered constant and equal to $15/MW & h and $10/MW & h, respectively. Prices of TMNSR are selected to follow the shape of energy prices. Finally, prices for TMOR are different than zero only for peak hours. It should be emphasized that, in general, price profiles should be obtained using appropriate forecasting engines such as the ones reported in [6]–[10].

For the sake of simplicity all weighting factors (α_i) have been set to 1.

Table V shows the optimal allocation of the unit resources in every period of the market horizon. The maximum available synchronized power $\overline{p}(k)$ is also shown in Table V. Note that this variable includes the effect of the intertemporal constraints such as ramp-up, start-up, and shut-down limits.

Fig. 1 shows the energy price profile (data) and the unit power output for the 24-h market horizon.

Some comments are in order. Despite the low energy price, during the first period the unit remains on-line due to the initial status and the shut-down ramp rate limit. The unit is off-line during the valley periods in which the price of energy is very low (hours 2–7), while meeting the minimum down time requirement. The unit is started-up in period 8 when the energy prices begin to rise. The algorithm determines the unit power output and its AGC contribution so that the maximum profit is obtained and ramp rate limits are met. It should be noted that the power output follows the shape of the energy price profile except for hours 18 and 19 (Fig. 1) in which it decays although the energy prices are higher than in previous hours. This reduction in power is a side effect of the large TMOR price in hours 20–22. Thus, it is more profitable for the unit to reduce the power output

TABLE III
POWER OUTPUT, AGC, AND RESERVE LIMITS

Energy						AGC			TMSR	TMNSR	TMOR
$\underline{P}$	$\overline{P}$	SU	SD	RU	RD	RL	RH	$\overline{AGC}$	$\overline{TMSR}$	$\overline{TMNSR}$	$\overline{TMOR}$
112	294	170	160	60	50	120	200	200	50	50	147

TABLE IV
PRICE DATA

Hour	Energy ($/MWh)	AGC ($/MW & h)	TMSR ($/MW & h)	TMNSR ($/MW & h)	TMOR ($/MW & h)
1	9.9953	15	10	4.00	0
2	8.5029	15	10	3.80	0
3	7.7549	15	10	3.75	0
4	7.7507	15	10	3.70	0
5	7.2978	15	10	3.50	0
6	7.5068	15	10	3.60	0
7	10.0546	15	10	4.00	0
8	13.6770	15	10	4.30	0
9	21.8620	15	10	6.00	0
10	22.3454	15	10	6.10	0
11	23.2917	15	10	6.20	0
12	23.5578	15	10	6.30	0
13	21.4120	15	10	5.90	0
14	20.7028	15	10	5.85	0
15	20.4556	15	10	5.80	0
16	23.0123	15	10	6.25	0
17	23.0104	15	10	6.25	0
18	24.1537	15	10	6.40	0
19	24.9977	15	10	6.50	0
20	32.2507	15	10	12.00	10
21	33.9976	15	10	12.50	10
22	29.7608	15	10	11.80	10
23	23.7982	15	10	6.40	0
24	11.1547	15	10	4.15	0

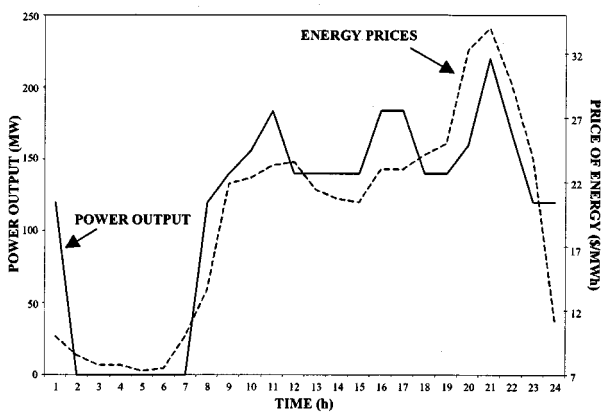

Fig. 1. Power output versus energy price.

in hours 18 and 19, so that there is a higher amount of nonsynchronized power in hours 20 and 21. In these peak hours the unit begins to increase its power output again.

Since prices of AGC are higher than prices of synchronized reserve, TMSR (see Fig. 2), the unit is mostly contributing to AGC, while the TMSR contribution is different than 0 only in periods 11, 12, 17, 18, 22 and 23 when the synchronized power is large. The evolution of AGC and TMSR contributions along the periods is shown in Fig. 2, where AGC and TMSR prices are also represented.

Regarding the nonsynchronized reserves it should be emphasized that TMNSR contribution is active almost all over the

TABLE V
UNIT OPTIMAL ALLOCATION OF PRODUCTION RESOURCES

Hour k	p(k) (MW)	AGC(k) (MW)	TMSR(k) (MW)	$\overline{p}$(k) (MW)	TMNSR(k) (MW)	TMOR(k) (MW)
1	120	40	0	160	0	0
2	0	0	0	0	50	0
3	0	0	0	0	50	0
4	0	0	0	0	50	0
5	0	0	0	0	50	0
6	0	0	0	0	50	0
7	0	0	0	0	50	0
8	120	50	0	170	0	0
9	140	40	0	180	50	0
10	156	44	0	200	50	0
11	184	16	16	216	50	0
12	140	60	44	244	50	0
13	140	60	0	200	50	0
14	140	60	0	200	50	0
15	140	60	0	200	50	0
16	184	16	0	200	50	0
17	184	16	44	244	50	0
18	140	60	44	244	50	0
19	140	60	0	200	50	44
20	160	40	0	200	50	44
21	220	0	0	220	50	24
22	170	30	50	280	44	0
23	120	80	30	230	50	0
24	120	60	0	180	50	0

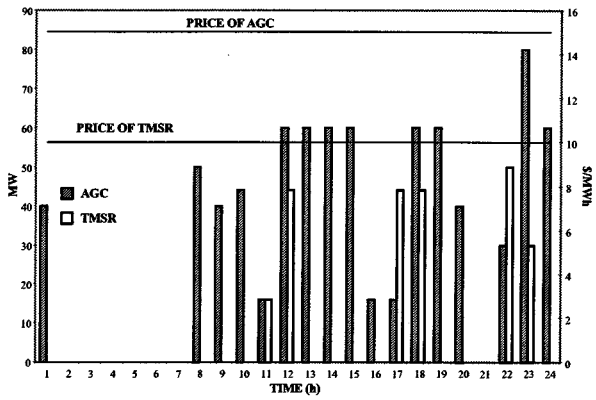

Fig. 2. AGC and TMSR contributions.

market horizon. The reason for these results comes from the fact that the regulating high limit, RH, for AGC is 200 MW and the total capacity is 294 MW. Therefore, as AGC prices are higher than TMSR prices, the synchronized power is limited by RH, and in most of the periods there is enough power to be sold either as TMNSR or TMOR. The TMNSR contribution is at its maximum limit, 50 MW, in all the periods except in periods 1 and 8 when the unit is shutting-down and starting-up, respectively; and in period 22 which precedes period 23 wherein there is a high decline in the TMNSR price with respect to the TMSR price and enough synchronized power is available.

Finally, TMOR contribution is greater than 0 only in periods 19, 20, and 21 so that $\mathrm{TMOR}^{\mathrm{avg}}(k)$ is different from 0 in periods 20, 21, and 22 in which, as mentioned previously, TMOR prices are high. During the remaining periods in which TMOR prices are 0, it is indifferent to the unit whether to contribute to TMOR or not.

Table VI shows revenues, costs, and profits for the unit.

TABLE VI
REVENUES, COST, AND PROFIT (IN DOLLARS)

Energy revenues	62729.4
AGC revenues	11430.0
TMSR revenues	2280.0
TMNSR revenues	6645.4
TMOR revenues	900.0
Total revenues	83984.8
Total cost	61273.6
Profit	22711.2

The model has been implemented on a SGI R12000, 400 MHz based processor with 500 MB of RAM using CPLEX 7.0 under GAMS [5]. The computing time required to achieve the optimal solution is 51.51 s.

IV. CONCLUSION

As stated in FERC orders 888 and 889, this paper considers a pool-based electricity market in which energy, AGC, and reserve are traded through independent auctions. This paper develops a tool that enables a power generator to optimally determine how to allocate its power among the energy, AGC and reserve markets, while enforcing all generator constraints, including intertemporal constraints. The result of this model provides the generator with the information it needs to successfully bid in a pool-based electricity market. The usefulness of this tool is demonstrated by means of a real-world case study.

APPENDIX

The linear expressions of nonconvex and nondifferentiable production costs, the linear exact expressions of exponential start-up costs, and the set of operating constraints, are presented in this Appendix.

The following set of constraints formulate the nonconvex and nondifferentiable variable production cost as a function of the average value of the power output.

$$\tilde{d}(k) = \sum_{\ell=1}^{NL} [F_\ell \delta_\ell(k) + t_\ell(k) \sum_{m=1}^{\ell-1} F_m (T_m - T_{m-1})] \quad \forall k \in K \tag{A1}$$

$$p^{\text{avg}}(k) = \sum_{\ell=1}^{NL} [\delta_\ell(k) + t_\ell(k) T_{\ell-1}] \qquad \forall k \in K \tag{A2}$$

$$\sum_{\ell=1}^{NL} t_\ell(k) = 1 \qquad \forall k \in K \tag{A3}$$

$$\delta_\ell(k) \le (T_\ell - T_{\ell-1}) t_\ell(k) \qquad \forall \ell = 1 \cdots NL,\ \forall k \in K \tag{A4}$$

$$\delta_\ell(k) \ge 0 \qquad \forall \ell = 1 \cdots NL,\ \forall k \in K \tag{A5}$$

$$t_\ell(k) \in \{0, 1\} \qquad \forall \ell = 1 \cdots NL,\ \forall k \in K \tag{A6}$$

where $\tilde{d}(k)$ is the piecewise linear variable cost in period k [\$/h] which replaces the nonlinear variable cost, $d(\cdot)$, in (2), F_ℓ is the slope of block ℓ of the variable cost [\$/MWh], NL is the number of blocks of the variable cost, $\delta_\ell(k)$ is the average power produced in the block ℓ in period k [MW], $t_\ell(k)$ is a 0/1 variable which is equal to 1 if block ℓ determines the average power in period k, and T_ℓ is the upper limit of block ℓ [MW].

Finally, the start-up cost and the set of operating constraints can be modeled as follows [14].

$$\tilde{b}(k) = \sum_{i=1}^{ND} K^i w^i(k) \qquad \forall k \in K \tag{A7}$$

$$\sum_{i=1}^{ND} w^i(k) = y(k) \qquad \forall k \in K \tag{A8}$$

$$s(k-1) = \sum_{i=1}^{ND-1} i w^i(k) + m(k) \qquad \forall k \in K \tag{A9}$$

$$m(k) \le \overline{S}[w^{ND}(k) - y(k) + 1] \qquad \forall k \in K \tag{A10}$$

$$m(k) \ge ND w^{ND}(k) \qquad \forall k \in K \tag{A11}$$

$$w^i(k) \in \{0, 1\} \qquad \forall i \in I,\ \forall k \in K \tag{A12}$$

$$s(k) \le s(k-1) + 1 \qquad \forall k \in K \tag{A13}$$

$$s(k) + (\overline{S} + 1) v(k) \ge s(k-1) + 1 \qquad \forall k \in K \tag{A14}$$

$$s(k) - \overline{S}[1 - v(k)] \le 0 \qquad \forall k \in K \tag{A15}$$

$$s(k) \ge 0 \qquad \forall k \in K \tag{A16}$$

$$p(k) \le \overline{p}(k) \qquad \forall k \in K \tag{A17}$$

$$\overline{p}(k) \le \overline{P}[v(k) - z(k+1)] + z(k+1) SD \quad \forall k \in K \tag{A18}$$

$$\overline{p}(k) \le p(k-1) + RU v(k-1) + SU y(k) \quad \forall k \in K \tag{A19}$$

$$p(k-1) - p(k) \le RD v(k) + SD z(k) \quad \forall k \in K \tag{A20}$$

$$\sum_{k=1}^{G} [1 - v(k)] = 0 \tag{A21}$$

$$\sum_{j=k}^{k+UT-1} v(j) \ge UT y(k) \quad \forall k = G+1 \cdots T - UT + 1 \tag{A22}$$

$$\sum_{j=k}^{T} [v(j) - y(k)] \ge 0 \qquad \forall k = T - UT + 2 \cdots T \tag{A23}$$

$$\sum_{k=1}^{F} v(k) = 0 \tag{A24}$$

$$\sum_{j=k}^{k+DT-1} [1 - v(j)] \ge DT z(k) \quad \forall k = F+1 \cdots T - DT + 1 \tag{A25}$$

$$\sum_{j=k}^{T} [1 - v(j) - z(k)] \ge 0 \qquad \forall k = T - DT + 2 \cdots T \tag{A26}$$

$$y(k) - z(k) = v(k) - v(k-1) \qquad \forall k \in K \tag{A27}$$

$$y(k) + z(k) \le 1 \qquad \forall k \in K \tag{A28}$$

where

$$F = \mathrm{Min}\{T, [DT - s(0)][1 - v(0)]\}$$
$$G = \mathrm{Min}[T, (UT - U^0)v(0)].$$

In the above formulation $\tilde{b}(k)$ is the stairwise linear start-up cost in period k [\$/h] that substitutes the exponential start-up cost, $b(\cdot)$, in (2), K^i is the cost of the ith discrete interval of the start-up cost [\$/h], $w^i(k)$ is a 0/1 variable which is equal to 1 if the unit is started-up at the beginning of period k and it has been off for i periods, ND is the number of discrete intervals of the start-up cost function, $m(k)$ is a dummy variable used for the discretization of the start-up cost function in period k [h], $\overline{S}$ is a large enough constant, G is the number of intervals the unit must be initially on-line due to the minimum up time constraint [h], UT is the minimum up time [h], F is the number of intervals the unit must be initially off-line due to the minimum down time constraint [h], DT is the minimum down time [h], T is the number of intervals of the market horizon, and U^0 is the number of periods the unit has been on-line at the beginning of the market horizon (end of hour 0) [h].

Constraints (A7)–(A16) express the stairwise linear start-up cost, including the formulation of a time counter. It should be noted that if ND is equal to the number of intervals of the market horizon T this linear expression is exact. Constraints (A17)–(A20) introduce the effect of ramp rate limits on the power output through variable $\overline{p}(k)$, which represents the maximum available synchronized power in period k. Constraints (A21)–(A23) and (A24)–(A26) enforce the minimum up and down time constraints, respectively. Finally, constraints (A27) and (A28) preserve the logic of running, start-up, and shut-down status changes [15].

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Self-Scheduling of a Hydro Producer in a Pool-Based Electricity Market

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***Abstract*—This paper addresses the self-scheduling of a hydro generating company in a pool-based electricity market. This company comprises several cascaded plants along a river basin. The objective is to maximize the profit of the company from selling energy in the day-ahead market. This paper proposes a 0/1 mixed-integer linear programming model to account, in every plant, for the nonlinear and nonconcave three-dimensional (3-D) relationship between the power produced, the water discharged, and the head of the associated reservoir. Additionally, start-up costs due mainly to the wear and tear are considered. Finally, different realistic case studies are analyzed in detail.**

***Index Terms*—Hydroelectric producer, mixed-integer LP, pool-based electricity market.**

NOMENCLATURE

The notation used throughout the paper is stated as follows:

Constants:

M	Conversion factor equal to 3.6×10^{-3} [Hm^{3}s/m^{3}h].
$\bar{P}_i$	Capacity of plant i [MW].
$P0_1(i)$	Minimum power output of plant i for performance curve 1 (lower interval of water content) [MW].
$P0_2(i)$	Minimum power output of plant i for performance curve 2 (intermediate interval of water content) [MW].
$P0_3(i)$	Minimum power output of plant i for performance curve 3 (higher interval of water content) [MW].
Q_i	Future value of the stored water in the reservoir associated with plant i at the end of the market time horizon [\$/Hm^3].
SU_i	Start-up cost of plant i [\$].
T	Number of periods of the market time horizon.
$\underline{U}_i$	Minimum water discharge of plant i [m^3/s].
$\bar{U}_i$	Maximum water discharge of plant i [m^3/s].
$\bar{U}_\ell(i)$	Maximum water discharge of block ℓ of plant i [m^3/s].
$W_i(k)$	Forecasted natural water inflow of the reservoir associated to plant i in period k [Hm^3/h].
$X_i(0)$	Initial water content of the reservoir associated to plant i [Hm^3].
$\underline{X}_i$	Minimum content of the reservoir associated to plant i [Hm^3].
$\bar{X}_i$	Maximum content of the reservoir associated to plant i [Hm^3].
XL_i	Lower level of the content of the reservoir associated with plant i used in the discretization of the performance curves [Hm^3].
XU_i	upper level of the content of the reservoir associated to plant i, used in the discretization of the performance curves [Hm^3],
$\lambda(k)$	Forecasted price of energy in period k [\$/MWh].
$\rho1_\ell(i)$	Slope of the block ℓ of the performance curve 1 of plant i [MW/m^3/s].
$\rho2_\ell(i)$	Slope of the block ℓ of the performance curve 2 of plant i [MW/m^3/s].
$\rho3_\ell(i)$	Slope of the block ℓ of the performance curve 3 of plant i [MW/m^3/s].
τ_{ji}	Time delay between reservoir of plant j and reservoir of plant i [h].

Variables:

$d_1(i,k)$	0/1 variable used for the discretization of the performance curves.
$d_2(i,k)$	0/1 variable used for the discretization of the performance curves.
$p_i(k)$	Power output of plant i in period k [MW].
$s_i(k)$	Spillage of the reservoir associated to plant i in period k [m^3/s].
$u_i(k)$	Water discharge of plant i in period k [m^3/s].
$u_\ell(i,k)$	Water discharge of block ℓ of plant i in period k [m^3/s].
$v_i(k)$	0/1 variable which is equal to 1 if plant i is on-line in period k.
$w_\ell(i,k)$	0/1 variable which is equal to 1 if water discharged by plant i has exceeded block ℓ in period k.
$x_i(k)$	Water content of the reservoir associated to plant i in period k [Hm^3].
$y_i(k)$	0/1 variable which is equal to 1 if plant i is started-up at the beginning of period k.
$z_i(k)$	0/1 variable which is equal to 1 if unit i is shut-down at the beginning of period k.

Sets:

I	Set of indices of the plants belonging to the hydro company.
K	Set of indices of the periods of the market time horizon.
L	Set of indices of the blocks of the piecewise linearization of the unit performance curve.
Ω_i	Set of upstream reservoirs of plant i.

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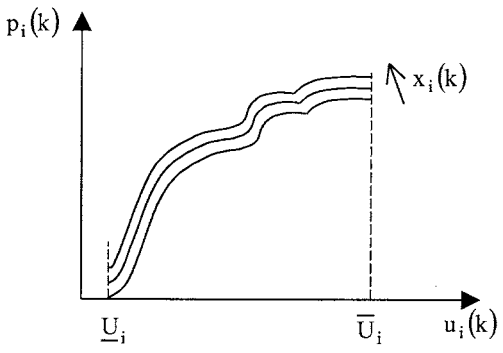

Fig. 1. Hill chart.

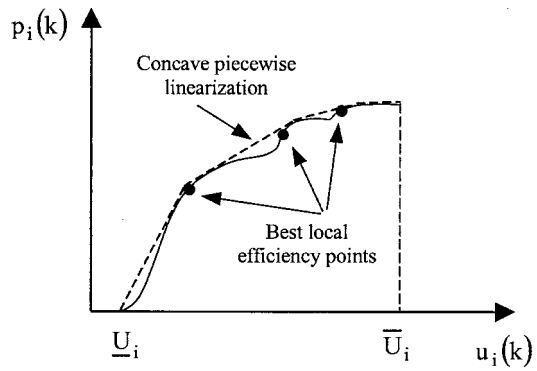

Fig. 2. Traditional approximation of the unit performance curve.

I. Introduction

THIS paper considers a day-ahead electricity market for energy based on a pool. In this framework, generating companies (GENCOs) and consumer or energy service companies submit bids for selling and buying electric energy for the next 24 hr. An independent agent, namely, the market operator (PX), is in charge of clearing the market, determining the accepted and rejected buying and selling bids. The market clearing procedure also provides the market clearing price for every period. These electricity markets based on a pool are spread worldwide (Spain [1], New England [2], Scandinavia [3]).

In this new environment, GENCOs face new challenging problems with the ultimate goal of maximizing their profits. One of those problems consists of determining, in the short term, the optimal self-schedule of the units belonging to the GENCO so that the profit from selling energy is maximized without regard to the power balance in the system. This problem is referred to as self-scheduling, and it is the problem addressed in this paper.

Another relevant problem is how to translate this information into a bidding strategy to ensure that in the day-ahead market, the GENCO achieves the maximum benefit. Bidding strategy development is outside the scope of this paper.

Recently, several contributions dealing with the problem of self-scheduling have been available in the technical literature [4], [5]. In [4], this problem is solved for a price-taker GENCO comprising one thermal unit. The same problem is solved in [5] for an oligopolistic GENCO owning several thermal units.

This paper addresses the self-scheduling problem for a hydro GENCO (H-GENCO) owning several plants cascaded along a river basin [6]. Specific features of hydro plants include i) spatial-temporal coupling among reservoirs and ii) for every plant, the nonlinear dependence between the power output, the water discharged, and the head of the associated reservoir are precisely accounted for through a 0/1 mixed-integer linear formulation. Additionally, start-up costs are also considered in this work.

In order to solve the self-scheduling problem, it is essential to use accurate models. These models should include the hydro generation characteristic describing the relationship between the head of the associated reservoir, the water discharged, and the power generated. This is a nonlinear and nonconcave 3-D relationship: the so-called Hill chart [7]. This relationship can be represented as a family of nonlinear and nonconcave curves, which are also known as unit performance curves, each for a specified value of the head (see Fig. 1).

In most optimization methods, the effect of the variation of the head has been neglected to avoid nonlinearities, which allows using a single unit performance curve. Furthermore, this single curve has been approximated either through a concave piecewise linear approximation (see Fig. 2) [8], [9] or by modeling just the so-called best local efficiency points (see Fig. 2) [10], [11]. All these simplifications may lead to inaccuracies. On the contrary, dealing with nonconcavities in a nonlinear optimization framework is difficult and may lead to multiple local optima and to unreliable solution procedures.

On the other hand, it is important to consider start-up costs for hydro units [12]. The start-up costs are due to

i) the loss of water during maintenance;
ii) wear and tear of the windings;
iii) wear and tear of the mechanical equipment;
iv) malfunctions in the control equipment;
v) loss of water during the start-up.

From the above causes, the main ones are ii) and iv). As a result, a start-up cost per megawatt unit nominal output that depends on the type of unit [12] can be derived.

The H-GENCO analyzed in this paper is considered to be a price-taker. In other words, it does not have market power, i.e., its schedule does not alter the hourly market clearing prices. Therefore, prices are assumed known. Several forecasting procedures are available to predict the day-ahead hourly market clearing prices, such as auto regressive models [3], [13], linear programming [14], or neural networks [15]. If the H-GENCO is a price-maker, a new source of complexity arises: It is necessary to model how the market clearing price changes with the H-GENCO total production. For every hour, the function modeling this variation is the so-called price-quota curve of the H-GENCO [5]. These curves have to be estimated and incorporated in the self-scheduling problem formulation of the H-GENCO.

The self-scheduling problem for an H-GENCO can be formulated as a 0/1 mixed-integer nonlinear nonconcave and moderate scale problem.

The main contributions of this paper are the following:

1) a discretization of the Hill chart into a set of nonconcave curves depending on the reservoir content so that the traditionally neglected effect of the head is modeled;

2) a precise piecewise linear approximation of each nonconcave unit performance curve that overcomes the inaccuracies of previous approximations (see Fig. 3);
3) the modeling of start-up costs to avoid unnecessary start-ups.

This paper provides a 0/1 mixed-integer linear model that allows an accurate representation of the variation of the performance curves with the reservoir head. The number of performance curves needed to represent accurately the head variation is usually small (below 5), and therefore, the associated computational burden is moderate (minutes on a PC). Furthermore, this paper also provides a 0/1 mixed-integer linear model to represent the nonconcavities of each performance curve, which can be made as accurate as needed while keeping the computational burden low. Finally, start-up costs are also modeled using binary variables.

The above contributions are included in the model, resulting in a 0/1 mixed-integer linear programming problem that can be solved efficiently by available branch and cut solvers [16].

The self-scheduling problem addressed in this paper is similar to any of the hydro subproblems of a short-term hydro-thermal coordination problem solved by Lagrangian relaxation. In this framework, the maximum profit problem faced by the hydro producer has been traditionally solved using network programming [17] or a combination of network programming and dynamic programming [6], without taking into account the three improvements explained above.

The proposed approach allows i) the determination of the self-scheduling of the H-GENCO in the day-ahead market and ii) the efficient and accurate solution of the subproblems that result from addressing a (centralized) short-term hydro-thermal coordination problem using Lagrangian relaxation.

The ultimate motivation of this paper is to provide the H-GENCO with a short-term self-scheduling tool to achieve maximum profit from selling energy in the day-ahead market while considering all its operating constraints.

The remaining of this paper is organized as follows. In Section II, the problem is formulated in detail. This section presents a discretization of the Hill chart. In addition, this section includes a precise modeling of the piecewise linearization of the nonconcave and nonlinear relationship for every plant between the water discharged and the power output. In Section III, results from realistic case studies are provided and discussed. Finally, some relevant conclusions are drawn in Section IV.

II. Formulation

The formulation of the self-scheduling problem of an H-GENCO is presented in Sections II-A–C. This problem is formulated as a 0/1 mixed-integer linear programming problem.

A. Objective Function

The goal of any participant in an electricity market is to maximize its own profit, which is computed as the difference between the revenues and the total operating costs. The operating costs comprise production costs and start-up costs. In the case of a hydro company, the production costs are negligible. As it is reported in [10] and [12], the start-up costs have real impact on the short-term scheduling of hydro generation. Start-up costs are mainly caused by the increased maintenance of windings and mechanical equipment and by malfunctions of the control equipment. Thus, the objective function to be maximized can be expressed as

$$\sum_{k\in K}\sum_{i\in I}[\lambda(k)\,p_i(k) - SU_i y_i(k)] + \sum_{i\in I} Q_i x_i(T). \qquad (1)$$

In (1), the first term is related to the revenues of each plant belonging to the H-GENCO, whereas the second term represents the start-up costs, which are defined as a constant dependent on the type of the plant [10], [12]. Finally, an extra term has been added to model in a simple way the future value of stored water in reservoirs.

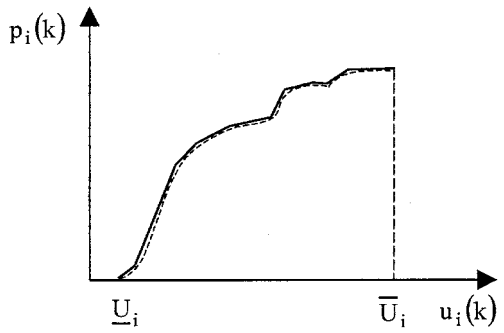

Fig. 3. Nonconcave piecewise linearization.

B. Hydro Constraints

The following expressions represent the set of constraints related to every hydro plant over the market time horizon. In this paper, the family of curves representing, for every plant, the relationship between the reservoir head, the water discharged, and the power output has been simplified to a prespecified number of curves, each one for an interval of the head. The generalization to any number of curves is straightforward. For the sake of simplicity, three curves are considered in this paper, corresponding to a low, medium, and high level of the reservoir content. Through the use of binary variables, the model accounts for these three curves.

Moreover, in order to consider the nonconcavities and nonlinearities of these unit performance curves [7], [10], a precise mixed-integer linear formulation has been developed.

Finally, water balance and logical status of commitment is enforced through additional constraints.

1) Simplification of the Three-Dimensional Unit Performance Curves: For each plant, the set of curves representing the relationship between the head, the power output, and the water discharge is reduced to three curves, according to two levels of the stored water in the reservoir. Fig. 4 depicts an illustrative example of this simplification. It should be noted that this figure represents a simplification of the Hill chart (see Fig. 1), where each unit performance curve has been approximated by a nonconcave piecewise linearization (see Fig. 3). If the content of the reservoir i in period k, $x_i(k)$, is below XL_i (low level), then curve 1 is used. If $x_i(k)$ is between XL_i and XU_i, i.e., intermediate level, then curve 2 is

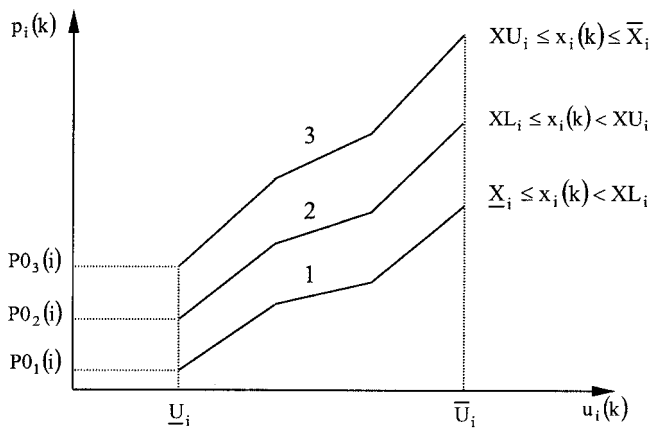

Fig. 4. Three-dimensional unit performance curves for plant i.

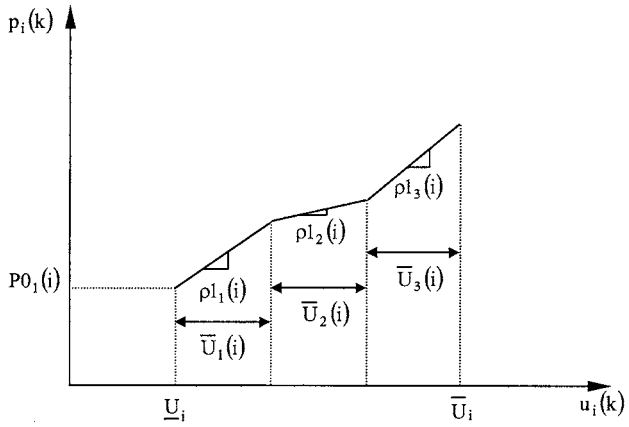

Fig. 5. Piecewise linear nonconcave unit performance curve.

used. Finally, if the stored water in period k is above XU_i (high level), then curve 3 is used. The generalization to any number of curves is straightforward.

This discretization is modeled through a 0–1 mixed-integer linear formulation, which is presented as

$$x_i(k) \geq XL_i[d_1(i,k) - d_2(i,k)] + XU_i d_2(i,k) \quad \forall i \in I,\ \forall k \in K \tag{2}$$

$$x_i(k) \leq \bar{X}_i d_2(i,k) + XL_i[1 - d_1(i,k)] + XU_i[d_1(i,k) - d_2(i,k)] \quad \forall i \in I,\ \forall k \in K \tag{3}$$

$$d_1(i,k) \geq d_2(i,k) \quad \forall i \in I,\ \forall k \in K \tag{4}$$

$$x_i(k) \geq \underline{X}_i \quad \forall i \in I,\ \forall k \in K. \tag{5}$$

In the formulation above, $d_1(i,k)$ and $d_2(i,k)$ are the binary variables used to choose the right curve according to the content level. If $d_1(i,k)$ and $d_2(i,k)$ are both equal to 0, constraints (2) and (3) force $x_i(k)$ to be in the lower interval so that curve 1 is used. If $d_1(i,k)$ is equal to 1 and $d_2(i,k)$ is equal to 0, constraints (2) and (3) force $x_i(k)$ to be in the intermediate interval, implying that the relationship between head, flow, and power output is represented by curve 2. Finally, if $d_1(i,k)$ and $d_2(i,k)$ are both equal to 1, constraints (2) and (3) force $x_i(k)$ to be in the higher level so that curve 3 is used. The formulation of each unit performance curve is presented in Section II-B2.

It should be noted that constraints (4) avoid the combination 0-1 for variables $d_1(i,k)$ and $d_2(i,k)$. It should also be noted that a higher number of levels can be modeled through the use of additional binary variables in a straightforward manner.

Finally, the set of constraints (5) sets the lower limit of the reservoir content in every period.

2) Piecewise Linear Formulation of the Unit Performance Curves: Unit performance curves are nonlinear and nonconcave [7], [10]. In this paper, these curves have been modeled through a piecewise linear formulation. Nonconcavities have also been modeled through the use of binary variables. Fig. 5 shows a three-piece linear nonconcave unit performance curve for a low content level. Fig. 5 is similar to Fig. 3 but simplified to illustrate the nonconcave linearization. Note that unlike in [10] and [11], the operation of the plant is not restricted to the local best efficiency points. In other words, the whole nonconcave unit performance curve is accurately modeled through the use of binary variables.

The formulation of the nonconcave unit performance curves is as follows:

$$p_i(k) - P0_1(i)v_i(k) - \sum_{\ell \in L} u_\ell(i,k)\rho 1_\ell(i) - \bar{P}_i[d_1(i,k) + d_2(i,k)] \leq 0 \quad \forall i \in I \quad \forall k \in K \tag{6}$$

$$p_i(k) - P0_1(i)v_i(k) - \sum_{\ell \in L} u_\ell(i,k)\rho 1_\ell(i) + \bar{P}_i[d_1(i,k) + d_2(i,k)] \geq 0 \quad \forall i \in I \quad \forall k \in K \tag{7}$$

$$p_i(k) - P0_2(i)v_i(k) - \sum_{\ell \in L} u_\ell(i,k)\rho 2_\ell(i) - \bar{P}_i[1 - d_1(i,k) + d_2(i,k)] \leq 0 \quad \forall i \in I \quad \forall k \in K \tag{8}$$

$$p_i(k) - P0_2(i)v_i(k) - \sum_{\ell \in L} u_\ell(i,k)\rho 2_\ell(i) + \bar{P}_i[1 - d_1(i,k) + d_2(i,k)] \geq 0 \quad \forall i \in I \quad \forall k \in K \tag{9}$$

$$p_i(k) - P0_3(i)v_i(k) - \sum_{\ell \in L} u_\ell(i,k)\rho 3_\ell(i) - \bar{P}_i[2 - d_1(i,k) - d_2(i,k)] \leq 0 \quad \forall i \in I \quad \forall k \in K \tag{10}$$

$$p_i(k) - P0_3(i)v_i(k) - \sum_{\ell \in L} u_\ell(i,k)\rho 3_\ell(i) + \bar{P}_i[2 - d_1(i,k) - d_2(i,k)] \geq 0 \quad \forall i \in I \quad \forall k \in K \tag{11}$$

$$u_i(k) = \sum_{\ell \in L} u_\ell(i,k) + \underline{U}_i v_i(k) \quad \forall i \in I \quad \forall k \in K \tag{12}$$

$$u_1(i,k) \leq \bar{U}_1(i)v_i(k) \quad \forall i \in I,\ \forall k \in K \tag{13}$$

$$u_1(i,k) \geq \bar{U}_1(i)w_1(i,k) \quad \forall i \in I,\ \forall k \in K \tag{14}$$

$$u_\ell(i,k) \leq \bar{U}_\ell(i)w_{\ell-1}(i,k) \quad \forall \ell \in L,\ \forall i \in I \quad \forall k \in K \tag{15}$$

$$u_\ell(i,k) \geq \bar{U}_\ell(i)w_\ell(i,k) \quad \forall \ell \in L,\ \forall i \in I \quad \forall k \in K. \tag{16}$$

Constraints (6) and (7) represent curve 1, i.e., the unit performance curve for the lower level (see Fig. 4). For this

curve, $d_1(i,k)$ and $d_2(i,k)$ are both equal to 0, as imposed by constraints (2)–(4). Therefore, both constraints force the power output to be equal to the minimum power output plus the blocks of the lower-level piecewise linear curve. It should be noted that if $d_1(i,k)$ and $d_2(i,k)$ are not both equal to 0, constraints (6) and (7) are deactivated through the inclusion of the capacity $\bar{P}_i$.

In an analogous fashion, couples of constraints (8) and (9), as well as (10) and (11), model curves 2 and 3, respectively (see Fig. 4), through the appropriate values of $d_1(i,k)$ and $d_2(i,k)$ stated in Section II-B1.

Constraints (12) state that the water discharge of plant i in period k is the sum of the water discharged in each block plus the minimum water discharge. Constraints (13)–(16) set the limits of the water discharged in each block. This discharge must be greater than 0 and smaller than the size of each block. This is ensured through a binary variable $w_\ell(i,k)$, which is equal to 1 if the water discharge of plant i in period k has exceeded block ℓ.

3) Water Balance: The continuity equation of the hydro reservoirs is formulated as

$$\begin{aligned} x_i(k) =& x_i(k-1) + W_i(k) \\ &+ M \sum_{j \in \Omega_i} [u_j(k-\tau_{ji}) + s_j(k-\tau_{ji})] \\ &- M u_i(k) - M\, s_i(k) \quad \forall\, i \in I \\ \forall\, k \in& K \end{aligned} \tag{17}$$

where M is a factor to convert water discharge units (m^3/s) into stored water units (Hm^3). For unit consistency, it should be noted that time periods of 1 hr are considered.

4) Logical Status of Commitment: The following constraints

$$y_i(k) - z_i(k) = v_i(k) - v_i(k-1) \quad \forall\, i \in I,\ \forall\, k \in K \tag{18}$$

are necessary to model the start-up and shut-down status of the plants [18]

Although variables $z_i(k)$ may seem superfluous since they only appear in (18), extensive numerical simulations have proven their ability in considerably reducing the computing time.

C. Types of Variables

Variables used in the formulation are

$$v_i(k), y_i(k), d_1(i,k), d_2(i,k) \in \{0,1\} \quad \forall\, i \in I \quad \forall\, k \in K \tag{19}$$

$$w_\ell(i,k) \in \{0,1\} \quad \forall\, \ell \in L \quad \forall\, i \in I,\ \forall\, k \in K \tag{20}$$

$$z_i(k) \in [0,\ 1] \quad \forall\, i \in I \quad \forall\, k \in K \tag{21}$$

$$p_i(k), s_i(k), u_i(k), x_i(k) \geq 0 \quad \forall\, i \in I \quad \forall\, k \in K \tag{22}$$

$$u_\ell(i,k) \geq 0 \quad \forall\, \ell \in L,\ \forall\, i \in I \quad \forall\, k \in K. \tag{23}$$

TABLE I
HYDRO DATA

#	$\underline{U}_i$	$\overline{U}_i$	$X_i(0)$	XL_i	XU_i	$\underline{X}_i$	$\overline{X}_i$	$W_i(k)$	SU_i
1	2	62	100	152	200	6	225	0.051	110
2	5	163	80	100	150	6	162	0.058	150
3	14	464	790	500	1000	6	1200	0.603	200
4	19	662	33	50	60	6	66	0.051	250
5	18	628	13	8	20	6	26	0.051	350
6	14	479	1200	1000	2000	6	2586	0.199	1500
7	29	985	50	40	100	6	115	0.500	2000
8	30	1028	90	100	150	6	181	0.048	1000

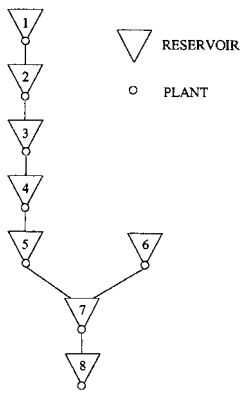

Fig. 6. Hydraulic topology of the river basin.

Variables $v_i(k)$, $y_i(k)$, $d_1(i,k)$, $d_2(i,k)$, and $w_\ell(i,k)$ are defined as binary in (19) and (20). Variables $z_i(k)$ can be defined as real variables belonging to the interval $[0,1]$, as stated in (21). Finally, constraints (22) and (23) state that the power output, spillage, discharge, reservoir content, and water discharged in each block in every period are all positive variables.

III. Case Study

Results from a realistic case study are reported in this section. The H-GENCO analyzed owns eight cascaded plants along a river basin. Table I shows the data of these plants. The spatial coupling among reservoirs is depicted in Fig. 6. For the sake of simplicity, 1-hr delays between any connected reservoirs have been considered. Additionally, forecasted water inflows are considered constant over the whole market time horizon, which is one day divided into 24 hourly periods. This case study considers that the final water content of each reservoir is identical to its initial value. Initial and final reservoir contents can be obtained by a medium-term planning procedure. Consequently, constants Q_i representing the future value of stored water are all equal to 0.

Table II shows the value of the slopes of the performance curves of each plant for the low interval of their corresponding reservoir contents $\rho 1_\ell(i)$. A piecewise linear approximation with four blocks has been implemented. The performance curves for the intermediate and high water contents are obtained by adding 0.05 and 0.1, respectively, to each slope in

TABLE II
PIECEWISE LINEAR APPROXIMATION OF THE PERFORMANCE CURVES

#	$\rho l_1(i)$	$\rho l_2(i)$	$\rho l_3(i)$	$\rho l_4(i)$	$\overline{U}_\ell(i)$
1	0.80	0.30	0.20	0.10	15.00
2	0.40	0.30	0.50	0.10	39.50
3	0.20	0.10	0.30	0.20	112.50
4	0.10	0.10	0.05	0.05	160.75
5	0.10	0.40	0.20	0.10	152.50
6	1.30	3.00	1.50	0.80	116.25
7	0.75	1.50	1.20	0.90	239.00
8	0.80	0.30	0.50	0.10	249.50

TABLE III
POWER OUTPUT LIMITS

#	$P0_1(i)$	$P0_2(i)$	$P0_3(i)$	$\overline{P}_i$
1	1.440	1.530	1.620	28.62
2	1.896	2.133	2.370	69.52
3	2.700	3.375	4.050	139.05
4	1.929	2.894	3.858	116.38
5	1.830	2.745	3.660	186.66
6	18.135	18.833	19.530	833.28
7	21.510	22.944	24.378	1159.63
8	23.952	25.449	26.946	550.90

Table II. For each plant, the blocks of the approximation are equally sized, as shown in the last column of the table.

Finally, Table III shows the minimum power output corresponding to each performance curve as well as the capacity of each plant.

The optimal solution is achieved in 22 min of CPU time, and the optimal value of the objective function is \$558 587.1. The model developed has been implemented on a SGI R12000, 400-MHz-based processor with 500 MB of RAM using CPLEX 7.0 under GAMS [16].

Fig. 7 shows the computed 24-hr generation schedule. It also shows the considered energy price profile corresponding to the electricity market in mainland Spain on March 3, 2001 [19]. It should be noted that the hydro production follows the shape of the price profile. For the sake of clarity, the production schedule of units 1 to 5 is also plotted in Fig. 8.

To illustrate the results obtained, plant 5 is selected. Most of the other plants behave in a similar fashion (see Figs. 7 and 8). Figs. 9–12 show the evolution of the water content, the profit, the water discharge, and the power output of plant 5 over the time span, respectively.

From Figs. 9–12, it should be noted that the plant is mainly operated on the hours with the highest prices.

As it can be seen in Fig. 9, the associated reservoir stores water in the periods 1–9 preceding the hours with the highest prices. Note that the water content is over 20 Hm^3 in periods 8–15 in order to use the high-level performance curve (see Table I), thereby yielding large profits. On the other hand, note that in hours 19–22, which also correspond to high prices, the water content decreases below 20 Hm^3 (medium level) in order to achieve the medium-term target of 13 Hm^3 in the last period of the time span. It should also be noted that only in hours 20 and 21, the production corresponds to the last block of the linearization of the performance curve, which is the most inefficient one, with a slope of 0.2 $MW/m^3/s$. This is due to the need to discharge water to reach the desired final water content.

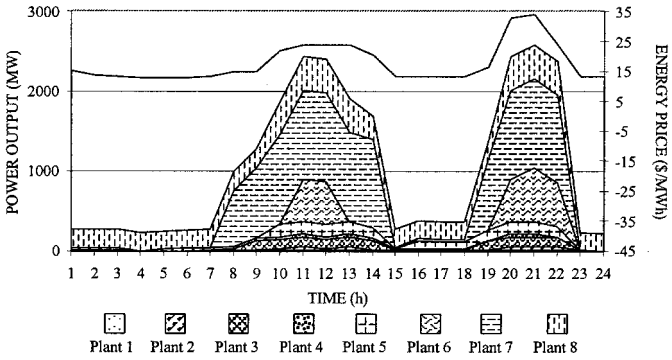

Fig. 7. Energy price profile and production schedule.

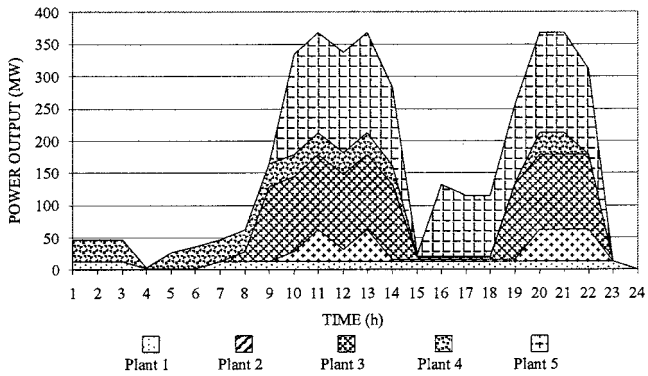

Fig. 8. Production schedule of plants 1 to 5.

The same test case has been run without considering the start-up costs. The optimal solution is achieved in 7 min of CPU time, and the optimal value of the objective function is \$567 156.3, i.e., a 1.53% increase on the total profit with respect to the original case. Aside from this difference in profit, the schedule of the plants is also affected by the omission of start-up costs. For instance, plant 1 is decommitted in hour 5, and plant 5 is decommitted in hours 15, 16, and 23.

Finally, this test case has also been run, including two simplifications that highlight how the two contributions modeled in this paper (discretization of the Hill chart and modeling of nonconcave performance curves) influence the computational burden and the objective function value. The first simplification consists of considering only one nonconcave efficiency curve for every plant. This simplification has been carried out by considering a lower level curve, a medium level curve, and a higher level curve for every plant in each case. The optimal solution was achieved for the three cases in 2 s of CPU time. The optimal values of the objective function were \$531 254.4, \$574 943.5, and \$619 109.1. In other words, the effect of not considering the nonlinear dependence between the power output, the water discharged, and the head of the associated reservoir materializes in deviations from the optimal solution of -5.1%, 2.9%,

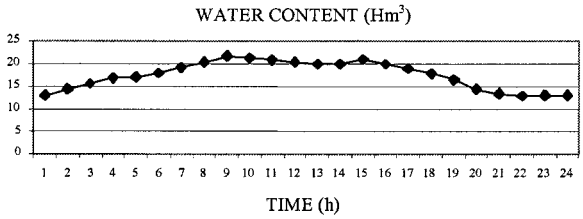

Fig. 9. Water content of reservoir associated with plant 5.

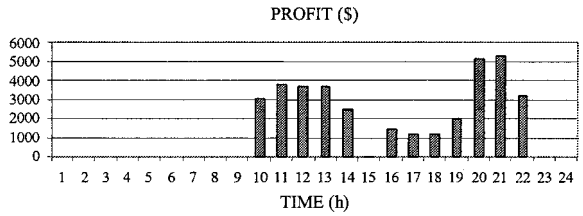

Fig. 10. Hourly profit of plant 5.

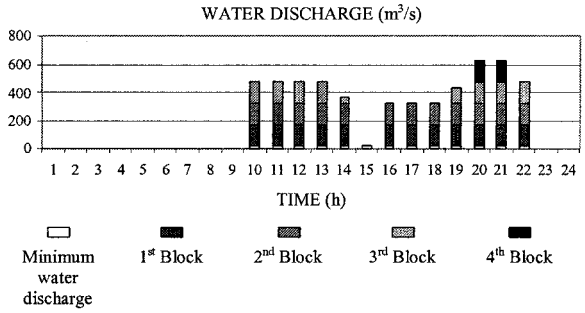

Fig. 11. Hourly water discharge of plant 5.

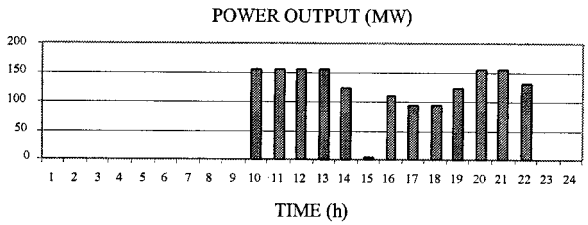

Fig. 12. Hourly power output of plant 5.

and 10.8%, respectively. Additionally, the scheduling of plants 3, 4, and 5 experiments changes with respect to the original case, mainly in hours 1–7 and 15–18, when the energy prices are low.

The second simplification considered a discretization of the Hill chart into three concave performance curves to quantify the effect of modeling several concave performance curves. The optimal solution was $642 452.7 and was attained in 4 min of CPU time. From these results, it can be concluded that the discretization of the Hill chart to model head variations takes up most of the computing time.

IV. Conclusions

This paper considers a pool-based electricity market for energy. In this framework, it provides a tool that allows a hydro generating company to optimally determine the short-term self-scheduling of its hydro plants cascaded along a river basin. The objective is to maximize the profit of the company from selling energy in the day-ahead market. This paper proposes a 0/1 mixed-integer linear programming model to account in each plant for the nonlinear nonconcave 3-D relationship between the reservoir head, the power output, and the water discharged. Additionally, start-up costs are considered. The model has been successfully tested on realistic case studies.

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Risk-Constrained Self-Scheduling of a Thermal Power Producer

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***Abstract*—This paper addresses the self-scheduling problem of a price-taker power producer. It focuses on risk modeling, emphasizing the tradeoff existing between maximum profit and minimum risk. The paper analyzes a self-scheduling model that considers simultaneously profit and risk. This model is formulated as a mixed-integer quadratic programming problem, which is solved using commercially available software. Relevant results from a realistic case study are discussed.**

***Index Terms*—Pool-based electricity market, price-taker power producer, profit versus risk tradeoff, risk-constrained self-scheduling.**

NOMENCLATURE

Variables:

c_t	Production cost during hour t.
p_t	Power production during hour t.
R	Total revenue.
V	$T \times T$ covariance matrix of random variables $\lambda_1, \ldots, \lambda_T$.
λ_t	Market-clearing price of hour t (random variable).
Λ_d	Vector of the T (24) prices for day d.

Constants:

D	Number of days for which true and estimate prices are available.
T	Considered time periods in one day (typically 24).
α	Factor used to estimate the covariance matrix.
β	Weighting factor to incorporate risk into the expected profit objective function.
Π	Feasible operating region of the generating machine.

Miscellaneous:

$\mathrm{E}_{\lambda_1,\ldots,\lambda_T}$	Expected value operator with respect to random variables $\lambda_1, \ldots, \lambda_T$.
$\mathrm{Var}_{\lambda_1,\ldots,\lambda_T}$	Variance operator with respect to random variables $\lambda_1, \ldots, \lambda_T$.
est	Superscript that indicates estimate value.
true	Superscript that indicates true value.
exp	Superscript that indicates expected value.

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I. INTRODUCTION

THIS PAPER considers a day-ahead electric energy market based on a pool. Within this pool, producers and consumers submit production and consumption bids to the market operator, which clears the market using an appropriate market-clearing procedure. This procedure results in 24 hourly energy prices to be paid by consumers and to be charged by producers. Appropriate forecasting techniques can be used to predict these 24 day-ahead hourly prices. Furthermore, the covariance matrix that expresses the statistical dependence of these prices among themselves can also be estimated [1]. It should be noted that modeling bilateral contracts requires longer market horizons than those considered in this paper.

In the above framework, this paper addresses the self-scheduling problem faced by a power producer. The effect of risk is explicitly recognized in formulating this self-scheduling problem taking into account the variance of the market-clearing prices. Therefore, the tradeoff of maximum profit versus minimum risk is properly addressed. Within the framework stated in [2], the analysis performed can be extended to simultaneously consider AGC, spinning reserve, and nonspinning reserve markets.

Note that any producer participating in an electric energy pool should self-schedule its units to maximize its expected profit assuming a given level of risk. This optimal self-schedule is then used by the producer to derive an appropriate bidding strategy to the pool [3]. This strategy is designed so that the optimal self-schedule is accepted by the pool operator through its market-clearing procedure (based on unit commitment or on auctions).

The producer considered in this paper is a price-taker, i.e., a producer with no capability of altering the market-clearing prices. Therefore, its power plants do not have to seek coordination among themselves in determining their respective production strategies because coordination does not change market-clearing prices. Therefore, the self-scheduling of each generator is independent of the self-scheduling of the others. For the sake of simplicity and without loss of generality, a producer owning a single generator is considered in this paper.

The objective of the producer is to maximize the expected value of profit from selling energy in the day-ahead market, assuming a certain risk level. Therefore, a precise modeling of risk is embedded in the considered maximum profit problem.

The profit maximization problem faced by the producer is therefore a risk-constrained self-scheduling problem that is formulated as a mixed-integer quadratic programming problem. A commercial software [4] is used to solve this problem.

This paper is built upon previous results on self-scheduling of power producers as stated in [3] and [5]. Its relevant contribution

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consists in modeling risk in a rigorous manner and providing a tradeoff decision framework involving profit versus risk.

Relevant references on self-scheduling include [5] and [6]. Although several references are available on risk management in pool markets for a long-term time span, only few references address risk-related problems in the short-term. They include the following references. Paper [7] presents a unit commitment problem with uncertainty in the market prices. It uses a real option model to manage the price risk and the price behavior is modeled through scenarios (trinomial trees). Paper [8] presents "Value-at-Risk" and hedging instruments to manage market price risk for suppliers, distributors and traders. In [9], the authors analyze how to deal with the risk associated with operating conditions for models that include the transmission network. In [10], the authors explain how the participants in the former California power market had a risk exposure related to prices and propose the "Value-at-Risk" method to quantify this exposure. [11] presents a methodology to analyze the risk associated to short-term operational planning in the presence of load uncertainty.

The risk analysis carried out in this paper frames itself in a pool-based electric energy market that is cleared using a simple auction procedure, and where bilateral contracts are not significant. This is actually the case of the electricity market of mainland Spain [12].

If the producer has the possibility of signing long/medium-term contracts to hedge risk, the decision framework becomes more complex and versatile. In the long/medium-term, the producer should decide which contracts to sign, and as a result of such decision, the power that the producer allocates to trade in the pool (short-term). The long/medium-term decision of which contracts to sign is better taken within a stochastic programming framework, similar to the one stated, for instance, in [13]–[16] or [17]. The power that has not been allocated to bilateral contracts should be traded in the spot market using a framework similar to the one developed in this paper. An overview of the cascaded decisions faced by a power producer in the long-, medium-, and short-term frameworks can be found in [18].

This paper is organized as follows. Section II provides a description of the revenue that the producer expects to achieve in the day-ahead energy market. This revenue is characterized as a random variable whose average value and variance (a measure of risk) are described mathematically. In Section III, the risk-constrained self-scheduling problem faced by the producer is formulated as a mixed-integer quadratic programming problem. Results from a realistic case study are described and analyzed in Section IV. Finally, Section V provides some relevant conclusions.

II. Revenue and Risk Characterization

The expected value of the revenue obtained by the generator in the day-ahead electric energy market is calculated as

$$R^{\exp} = \mathrm{E}_{\lambda_1,\ldots,\lambda_T}\left\{\sum_{t=1}^{T}\lambda_t p_t\right\}. \tag{1}$$

Note that expectation and summation operators can be swapped, i.e.,

$$R^{\exp} = \sum_{t=1}^{T}\mathrm{E}_{\lambda_t}\left\{\lambda_t\right\}p_t = \sum_{t=1}^{T}\lambda_t^{\mathrm{est}}p_t. \tag{2}$$

Therefore, the average value of the revenue of the generator in the day-ahead electric energy market is computed as the summation over time of the price estimate times the actual power production in each hourly time-period.

The mutual dependence among the 24 revenues can be measured through their variance. This variance is an appropriate measure of risk: the higher the variance, the higher the risk. The variance of the 24 revenues can be computed as

$$\mathrm{Var}_{\lambda_1,\ldots,\lambda_T}\left\{\sum_{t=1}^{T}\lambda_t\, p_t\right\} = \sum_{i=1}^{T}\sum_{j=1}^{T}V_{ij}\, p_i p_j \tag{3}$$

where $V = (V_{ij})$ is the $T \times T$ covariance matrix of prices $\lambda_1,\ldots,\lambda_T$ [1]. It should be noted that the variance of the total revenue can be formulated solely as a function of the covariance matrix of the prices because the only random variables involved are these prices.

The actual covariance matrix V for day d is

$$V = \mathrm{E}_{\lambda_1,\ldots,\lambda_T}\left\{\left(\Lambda_d^{\mathrm{true}} - \Lambda_d^{\mathrm{est}}\right)\left(\Lambda_d^{\mathrm{true}} - \Lambda_d^{\mathrm{est}}\right)^{\mathrm{T}}\right\} \tag{4}$$

where $\Lambda_d = [\lambda_1,\ldots,\lambda_T]^{\mathrm{T}}$ for day d.

If the true values of prices as well as their estimates are available up to day $d-1$, the covariance matrix of day d can be estimated as

$$V^{\mathrm{est}} = \frac{1}{D}\sum_{i=1}^{D}\left(\Lambda_i^{\mathrm{true}} - \Lambda_i^{\mathrm{est}}\right)\left(\Lambda_i^{\mathrm{true}} - \Lambda_i^{\mathrm{est}}\right)^{\mathrm{T}} \tag{5}$$

where D is a convenient number of days (up to and including day $d-1$) for which true and estimate prices are available.

Hourly price series of most electric energy markets present some characteristics that can cause problems if (5) is directly used. These characteristics are [19]: nonconstant mean and variance, multiple seasonality (corresponding to a daily and weekly periodicity, respectively), high volatility, and high percentage of unusual prices (mainly in periods of high demand). Therefore, a better estimate for the covariance matrix can be obtained using the following exponentially weighted moving-average equation [20]:

$$V^{\mathrm{est}} = (1-\alpha)\sum_{i=1}^{D}\alpha^{i-1}\left(\Lambda_{D-i+1}^{\mathrm{true}} - \Lambda_{D-i+1}^{\mathrm{est}}\right) \times \left(\Lambda_{D-i+1}^{\mathrm{true}} - \Lambda_{D-i+1}^{\mathrm{est}}\right)^{\mathrm{T}} \tag{6}$$

where D is greater than or equal to 24 to make the covariance matrix positive definite. To guarantee that covariance matrix V is positive definite, at least 24 error vectors should be linearly

independent, which is usually the case. Moreover, using more than 24 error vectors further ensures the positive definiteness of the covariance matrix V. Because past prices are weighted by the "smoothing constant" α $(0 < \alpha < 1)$, higher weights are assigned to the days closer to day d and these weights decay exponentially as the days considered are farther and farther away in the past from day d. Therefore, seasonality effects and outliers have less impact on variances and covariances as these effects occur in periods more and more distant from day d. It should be noted that both (5) and (6) render biased estimations of matrix V. However, as stated in [20], it can be shown that the bias approaches zero as D, the number of days considered in the estimation, increases. Nevertheless, as the value of D increases, precision may deteriorate due to possible outliers. Therefore, by trial and error, an appropriate value of D should be selected so that the bias approaches zero while estimation precision remains high.

III. Self-Scheduling

At the time of self-scheduling, any producer faces a tradeoff between maximum profit and minimum risk. If the producer decides to ignore risk, the resulting profit maximization self-scheduling problem can be formulated as [5]

$$\begin{aligned} &\underset{p_1,\ldots,p_T}{\text{maximize}} \sum_{t=1}^{T} \left(\lambda_t^{\text{est}} p_t - c_t\right) \\ &\text{subject to } p_1, \ldots, p_T \in \Pi \end{aligned} \tag{7}$$

where c_t is the production cost during hour t. This cost includes a quadratic operation cost that depends on the power output p_t, and fixed, start-up and shut-down costs. Further information on cost can be found in [5].

Operation constraints of the generator include:

- minimum and maximum power output limits;
- ramp-up and ramp-down limits;
- start-up and shut-down ramp limits;
- minimum up- and minimum down-time constraints.

The above constraints can be generally expressed as $p_1, \ldots, p_T \in \Pi$, where Π is the feasible operating region of the generator. A detailed description of the feasibility region Π can be found in [5].

On the other hand, if the producer seeks to minimize risk and it is not disturbed by low profit, the corresponding risk minimization self-scheduling problem can be formulated as

$$\begin{aligned} &\underset{p_1,\ldots,p_T}{\text{minimize}} \sum_{i=1}^{T} \sum_{j=1}^{T} V_{ij}^{\text{est}} p_i p_j \\ &\text{subject to } p_1, \ldots, p_T \in \Pi \end{aligned} \tag{8}$$

where both i and j are time indices.

It should be noted that problem (8) is of no practical use. However, it is formulated for derivation consistency and for the sake of clarification.

Generally, a producer is interested in finding a self-schedule that results in a large profit with low risk (variance). To combine these two conflicting objectives, the technique presented in [21] for portfolio selection can be used. This technique uses a single objective function with the help of a risk tolerance parameter β. The resulting self-scheduling problem has the form

$$\begin{aligned} &\underset{p_1,\ldots,p_T}{\text{maximize}} \sum_{t=1}^{T} \left(\lambda_t^{\text{est}} p_t - c_t\right) - \beta \sum_{i=1}^{T} \sum_{j=1}^{T} V_{ij}^{\text{est}} p_i p_j \\ &\text{subject to } p_1, \ldots, p_T \in \Pi. \end{aligned} \tag{9}$$

Note that the variance of the total revenue is equal to the variance of the total profit because all costs are considered deterministic.

Penalty parameter β allows adding the two conflicting terms that form the objective function of problem (9). Parameter β lies in the range $[0, \infty)$ and its actual value materializes the tradeoff between expected profit and risk; therefore, it depends on the preferences of the producer. A conservative producer places more emphasis on minimizing risk while deriving its self-scheduling, so it chooses a large value of β to increase the weight of the risk measure in (9). On the contrary, another producer may be prepared to assume higher risk in the hope of obtaining a higher profit, so its selected value for β is close to 0.

The solution of (9) for different values of β provides the efficient frontier [21]–[24], that is, the set of solutions for which expected profit cannot be increased without increasing profit variance, i.e., risk. As stated in [25], the selection of the weighting factor β depends on, among others, the financial situation of the

TABLE I
TECHNICAL DATA FOR THE GENERATING MACHINE

Minimum power [MW]	Maximum power [MW]	Start-up ramping limit [MW/h]	Shut-down ramping limit [MW/h]
112	294	170	160
Ramping-up limit [MW/h]	**Ramping-down limit [MW/h]**	**Minimum up-time [hours]**	**Minimum down-time [hours]**
60	50	4	4

TABLE II
COSTS DATA FOR THE GENERATING MACHINE

Fixed [$/h]	Linear [$/MWh]	Quadratic [$/MW^{2}h]	Start-up [$/start-up]	Shut-down [$/shut-down]
1150	18	0.035	1038	56

TABLE III
PRICE ESTIMATES

Hour #	Price estimate [$/MWh]	Hour #	Price estimate [$/MWh]
1	33.31	13	41.05
2	26.53	14	41.61
3	22.16	15	38.98
4	23.10	16	39.74
5	22.60	17	42.02
6	23.15	18	42.09
7	24.65	19	40.74
8	24.75	20	38.80
9	25.50	21	39.63
10	27.58	22	46.14
11	31.60	23	39.04
12	35.60	24	33.68

TABLE IV
ESTIMATE OF THE COVARIANCE MATRIX

Hour #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1.60	-0.40	0.40	0.14	0.10	0.09	0.10	0.58	0.27	0.40	0.09	0.10	0.27	0.19	-0.14	0.08	0.23	0.06	0.19	-0.16	0.02	-0.15	-0.19	-0.01
2	-0.40	0.37	0.08	0.00	0.05	0.05	0.06	-0.10	-0.05	-0.05	-0.12	0.01	-0.05	-0.08	0.04	0.05	0.02	0.17	-0.05	0.07	-0.20	0.02	-0.15	0.10
3	0.40	0.08	0.61	0.08	0.20	0.16	0.08	0.37	0.07	0.18	-0.29	0.08	0.10	0.12	0.15	0.16	0.29	0.18	0.06	0.03	-0.11	-0.32	-0.13	-0.10
4	0.14	0.00	0.08	0.32	0.09	0.15	0.11	0.10	0.06	-0.01	-0.07	-0.05	0.09	-0.06	0.09	0.08	0.08	0.02	0.01	0.00	-0.04	-0.02	0.06	-0.05
5	0.10	0.05	0.20	0.09	0.25	0.15	0.23	0.43	0.13	0.07	-0.11	-0.10	0.12	-0.07	0.00	0.17	0.19	0.15	0.04	0.01	-0.15	-0.32	-0.10	-0.15
6	0.09	0.05	0.16	0.15	0.15	0.19	0.31	0.32	0.18	0.10	-0.11	-0.05	0.08	-0.06	-0.01	0.14	0.13	0.12	0.04	-0.04	-0.17	-0.28	-0.12	-0.12
7	0.10	0.06	0.08	0.11	0.23	0.31	1.49	1.18	0.85	0.16	-0.18	-0.19	-0.20	-0.17	-0.40	0.32	0.09	0.24	0.00	-0.29	-0.65	-1.03	-0.71	-0.26
8	0.58	-0.10	0.37	0.10	0.43	0.32	1.18	1.76	0.83	0.55	0.01	-0.11	0.09	0.09	-0.38	0.43	0.39	0.27	0.07	-0.14	-0.56	-1.03	-0.59	-0.44
9	0.27	-0.05	0.07	0.06	0.13	0.18	0.85	0.83	1.21	0.35	0.18	0.07	0.11	-0.06	-0.29	0.27	0.10	0.11	0.02	-0.15	-0.39	-0.66	-0.33	-0.25
10	0.40	-0.05	0.18	-0.01	0.07	0.10	0.16	0.55	0.35	0.72	0.27	0.24	0.24	0.17	-0.20	0.12	0.15	0.11	0.08	-0.01	-0.20	-0.05	-0.23	-0.15
11	0.09	-0.12	-0.29	-0.07	-0.11	-0.11	-0.18	0.01	0.18	0.27	0.90	0.14	0.27	0.34	-0.11	-0.20	0.00	-0.08	0.00	0.12	0.03	0.44	0.18	0.01
12	0.10	0.01	0.08	-0.05	-0.10	-0.05	-0.19	-0.11	0.07	0.24	0.14	0.42	-0.03	0.25	-0.06	0.01	-0.05	-0.00	0.05	0.09	-0.07	0.33	-0.09	0.20
13	0.27	-0.05	0.10	0.09	0.12	0.08	-0.20	0.09	0.11	0.24	0.27	-0.03	0.48	-0.01	0.08	0.01	0.21	0.02	0.11	0.06	0.08	0.01	0.21	-0.15
14	0.19	-0.08	0.12	-0.06	-0.07	-0.06	-0.17	0.09	-0.06	0.17	0.34	0.25	-0.01	0.60	0.06	-0.04	0.03	-0.01	0.02	0.15	0.06	0.20	0.12	0.17
15	-0.14	0.04	0.15	0.09	0.00	-0.01	-0.40	-0.38	-0.29	-0.20	-0.11	-0.06	0.08	0.06	0.43	-0.07	0.11	-0.13	0.04	0.13	0.29	0.20	0.40	-0.05
16	0.08	0.05	0.16	0.08	0.17	0.14	0.32	0.43	0.27	0.12	-0.20	0.01	0.01	-0.04	-0.07	0.39	0.15	0.19	0.09	0.08	-0.24	-0.22	-0.08	-0.04
17	0.23	0.02	0.29	0.08	0.19	0.13	0.09	0.39	0.10	0.15	0.00	-0.05	0.21	0.03	0.11	0.15	0.31	0.07	0.11	0.08	-0.05	-0.12	0.05	-0.22
18	0.06	0.17	0.18	0.02	0.15	0.12	0.24	0.27	0.11	0.11	-0.08	-0.00	0.02	-0.01	-0.13	0.19	0.07	0.41	-0.07	0.05	-0.27	-0.19	-0.25	0.07
19	0.19	-0.05	0.06	0.01	0.04	0.04	0.00	0.07	0.02	0.08	0.00	0.05	0.11	0.02	0.04	0.09	0.11	-0.07	0.21	0.02	0.00	0.01	0.04	-0.01
20	-0.16	0.07	0.03	0.00	0.01	-0.04	-0.29	-0.14	-0.15	-0.01	0.12	0.09	0.06	0.15	0.13	0.08	0.08	0.05	0.02	0.24	0.03	0.32	0.23	0.09
21	0.02	-0.20	-0.11	-0.04	-0.15	-0.17	-0.65	-0.56	-0.39	-0.20	0.03	-0.07	0.08	0.06	0.29	-0.24	-0.05	-0.27	0.00	0.03	0.66	0.35	0.56	-0.02
22	-0.15	0.02	-0.32	-0.02	-0.32	-0.28	-1.03	-1.03	-0.66	-0.05	0.44	0.33	0.01	0.20	0.20	-0.22	-0.12	-0.19	0.01	0.32	0.35	1.71	0.45	0.26
23	-0.19	-0.15	-0.13	0.06	-0.10	-0.12	-0.71	-0.59	-0.33	-0.23	0.18	-0.09	0.21	0.12	0.40	-0.08	0.05	-0.25	0.04	0.23	0.56	0.45	0.84	-0.03
24	-0.01	0.10	-0.10	-0.05	-0.15	-0.12	-0.26	-0.44	-0.25	-0.15	0.01	0.20	-0.15	0.17	-0.05	-0.04	-0.22	0.07	-0.01	0.09	-0.02	0.26	-0.03	0.60

generating company, its willingness to take risks and the characteristics of the particular electricity market. A detailed discussion on how to obtain appropriate values for the weighting factor β is outside the scope of this paper.

It should be noted that modeling the feasible operating region of the generator (represented by Π) requires the use of binary variables to represent the commitment status as well as start-ups and shut-downs. Therefore, problem (9) is a mixed-integer quadratic programming problem.

If covariance matrix V is positive definite, problem (9) has a strict optimum that can be efficiently attained. If, on the other hand, V is positive semidefinite (only other possibility), nonstrict optima may exist and solution algorithms behave not as efficiently as in the positive definite case. Nevertheless, computational experience shows that appropriate solutions are obtained in both cases.

IV. CASE STUDY

The considered case study consists in the day-ahead self-scheduling of a power producer owning a single generating machine. Data for this machine is provided in Tables I and II.

Price forecasts are provided in Table III while an estimate of the covariance matrix is provided in Table IV. A time series of three months corresponding to the electric energy market of mainland Spain [12] has been used to estimate prices. Price estimates are obtained using a transfer function procedure [19], while the covariance matrix has been estimated through (6) using data for the last 24 days prior to the estimation day. Through numerical simulation it has been verified that good estimations for this covariance matrix are obtained with $\alpha = 0.98$ and $D = 24$. Using standard clustering techniques [26] to analyze this covariance matrix, the following can be concluded in terms of correlations affecting risk. Hours 20–24 (late-evening peak) are highly correlated among each other and show small correlations with the other hours of the day.

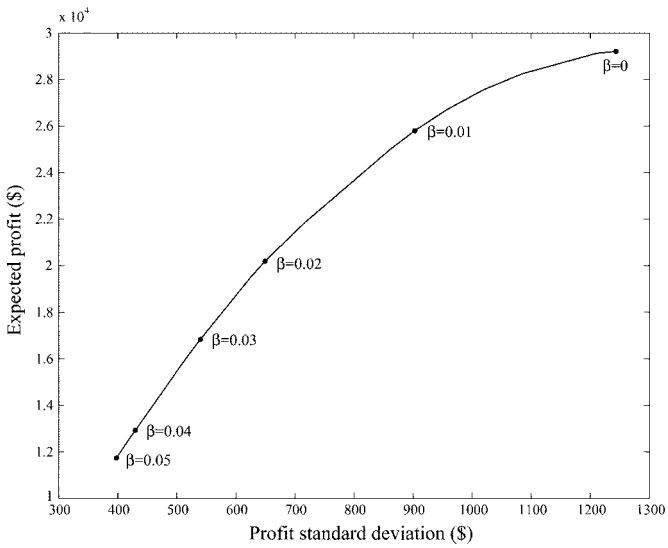

Fig. 1. Expected profit versus profit standard deviation.

The same pattern is presented by hours 5–9 (early-morning shoulder) and 11–14 (afternoon peak). The physical reasons for the above correlations lay on the structure and the composition of the load. However, the study of the load structure is beyond the scope of this paper.

The scheduling problem (9) is solved for different values of the parameter β, which allows assigning different weights to the risk term versus the profit term in the objective function. This allows constructing the efficient frontier. Fig. 1 depicts expected profit versus profit standard deviation and illustrates the efficient frontier. It can be observed that expected profit increases as variance also increases. Note that the decrement rate in expected profit is important, which makes the tradeoff profit versus risk relevant.

The expected profit achieved by a conservative producer ($\beta = 0.05$) is \$11 737.21 whereas the expected profit with maximum risk is equal to \$29 209.56.

TABLE V
SCHEDULING COMPARISON FOR TWO LEVELS OF RISKS

Hour #	Power [MW] β = 0	Power [MW] β = 0.05	Hour #	Power [MW] β = 0	Power [MW] β = 0.05
1	160.00	120.00	13	290.00	0.00
2	0.00	0.00	14	294.00	0.00
3	0.00	0.00	15	294.00	0.00
4	0.00	0.00	16	294.00	163.72
5	0.00	0.00	17	294.00	172.67
6	0.00	0.00	18	294.00	232.67
7	0.00	0.00	19	294.00	199.12
8	0.00	0.00	20	294.00	150.58
9	0.00	0.00	21	294.00	180.34
10	0.00	0.00	22	294.00	130.34
11	170.00	0.00	23	287.26	0.00
12	230.00	0.00	24	237.26	0.00

Table V illustrates scheduling differences for the cases of maximum risk ($\beta = 0$) and a low level of risk ($\beta = 0.05$) (see Fig. 1). This range on β is considered wide enough to reflect the variety of risk that electricity producers are willing to assume. As can be seen from this table, the consideration of different risk levels yields two results: 1) the risk averse producer ($\beta = 0.05$) is on-line during fewer time periods, and 2) in those periods in which it is on-line, its production is considerably lower compared to the case of maximum risk ($\beta = 0$). Note that intermediate values of β result in different intermediate schedules.

A clear conclusion can be drawn: the producer should make a decision on its desired level of risk before solving its day-ahead scheduling problem and using that information to bid in the electric energy market. Different levels of risk imply different self-scheduling results and different bidding strategies, and ultimately, different actual profits.

The CPU time required to solve problem (9) for a given value of β in a Dell PowerEdge 6600 with two processors at 1.60 GHz and 2 Gb of RAM memory is approximately 7 s.

V. CONCLUSION

Any power producer faces a profit versus risk tradeoff while determining its self-scheduling and its bidding strategy for the day-ahead electric energy market. This paper provides an appropriate tool to analyze the profit versus risk tradeoff faced by the producer. Moreover, it provides the producer with an instrument to efficiently self-schedule once a level of risk has been specified. Practical simulations using realistic electric energy markets show that risk levels affect profits in a significant manner. Therefore, any producer should be aware of the consequences of its selected level of risk on its self-scheduling, its bidding strategy, and ultimately, on its actual profits.

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Price Maker Self-Scheduling in a Pool-Based Electricity Market: A Mixed-Integer LP Approach

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***Abstract*— This paper addresses the self-scheduling problem faced by a price-maker to achieve maximum profit in a pool-based electricity market. An exact and computationally efficient mixed-integer linear programming (MILP) formulation of this problem is presented. This formulation models precisely the price-maker capability of altering market-clearing prices to its own benefits, through price quota curves. No assumptions are made on the characteristics of the pool and its agents. A realistic case study is presented and the results obtained are analyzed in detail.**

***Index Terms*—Electricity pool market, market power, mixed-integer linear programming (MILP), price maker.**

NOTATION

A. Functions

$\lambda_t(q_t)$ stepwise monotonically decreasing discontinuous function that, for every hour t, expresses the market-clearing price as a function of the price maker quota in that hour (see Fig. 1);

$c_{t,i}$ production cost for hour t of the ith unit belonging to the price maker;

R_t revenue of the price maker in hour t.

B. Constants

$b_{t,s}^{\max}$ megawatt size of step s of the price quota curve for hour t (see Figs. 2 and 3);

m number of units owned by the price maker;

n_t number of steps of the price quota curve in hour t;

$q_{t,s}^{\min}$ is the summation of power blocks from step 1 to step s-1 for hour t (note that $q_{t,1}^{\min} = 0, \forall\, t$; see Figs. 2 and 3).

T number of time periods considered;

$\lambda_{t,s}$ price corresponding to step number s of the price quota curve in hour t;

Π_i feasible operating region of unit i.

C. Variables

$b_{t,s}$ real variable representing the fractional value of the power block corresponding to step s to obtain quota q_t;

$p_{t,i}$ power produced by unit i in hour t;

q_t price maker quota in hour t;

$u_{t,s}$ binary variable that is equal to 1 if step s is the last step needed to obtain quota q_t in hour t and 0 otherwise.

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I. INTRODUCTION

The framework considered in this paper is a pool-based electricity market for energy. An auction mechanism to clear the market one day ahead on an hourly basis is assumed [1], [2]. No particular assumptions are made on generating companies; therefore, several price makers as well as several competitive fringe producers are market agents. The hourly load may be price elastic or not.

In the above context, this paper addresses the self-scheduling problem of a price maker, i.e., a generating company owning a portfolio of units that is capable of altering market-clearing prices. The objective function of this self-scheduling problem is to maximize the price maker profits. Once the optimal self-schedule is known, an appropriate bidding strategy to actually achieve this optimal schedule should be developed.

For every hour, it is assumed that the market-clearing price as well as the offer and demand curves are available once the market has been cleared. This is the case of several electricity markets like the market in mainland Spain [3], the former electricity market of California [4], and the electricity market of New England [5]. The above information is crucial because it allows small producers to forecast next-day market-clearing prices, and it also allows price makers to forecast their corresponding price quota curves. Note that several price makers can compete in the considered pool-based electricity market.

The price quota curve of a price maker for a given hour provides the market-clearing price as a function of the price maker quota (accepted production in that hour). This curve is described in detail in [1], [6]. Hourly price quota curves (also known as residual demand curves) allow formulating precisely the self-scheduling profit maximization problem that every price maker faces every day in a pool-based electricity market for energy.

This paper specifically addresses the day-ahead self-scheduling problem faced by a particular price maker. It provides an efficient yet simple mixed-integer linear programming (MILP) formulation that allows achieving the optimal solution for realistic case studies in moderate solution times. This LP formulation is exact in the sense that it is not an approximation of a nonlinear formulation, as is the case of the model presented in [7].

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Market power and price maker behavior have been analyzed mostly in a single period context (therefore, not using 0/1 variables), and continuity has been assumed in functions and variables. Particularly relevant works are [8]–[11]. This paper provides a complementary approach that does not require continuity and considers a multiperiod setting. On the other hand, it extends the work reported in [6], specifically in providing an exact and efficient mixed-integer formulation, which guarantees the achievement of the optimal solution.

The remainder of this paper is organized as follows. Section II describes price quota curves, presents a nonlinear formulation of the price maker self-scheduling problem, and provides its equivalent mixed-integer linear formulation. In Section III, the results from a realistic case study are analyzed in detail. Section IV provides some conclusions. Finally, an Appendix is presented to describe the running cost and the feasible operating region for generating units.

II. Problem Formulation

A. Price Quota Curve Description

For a given hour, the quota of a price maker is the amount of power it contributes to serve the demand in that hour. If the price maker exercises its market power by retaining production, the market-clearing price increases. The curve that expresses how the market-clearing price (for the whole market) changes as the quota of the price maker changes is called residual demand curve [1] or, more directly, price quota curve. Note that different price makers competing in the same electricity market present different price quota curves.

The price quota curve for a given hour, corresponding to a price maker, is a stepwise monotonically decreasing curve that expresses the actual market-clearing price in that hour as a function of the market quota (total accepted production) of the price maker. Price quota curves are stepwise because (producer/consumer) bids are assumed to be blocks of power at given prices.

The 24 hourly day-ahead price quota curves of a given price maker provide all the market information it needs to self-schedule optimally, i.e., to maximize its benefits. That is, these curves embody the effects of all interactions with competitors and the market functioning rules. Once these curves are available, the price maker self-scheduling problem can be precisely formulated without further regard to the effect of competitors, i.e., the self-scheduling problem of any price maker can be formulated independently of the problems of other producers.

The functioning of a day-ahead electricity market is as follows. First, each producer, either price maker or price taker, uses a self-scheduling algorithm to determine its optimal self-schedule. Then, each producer uses a bidding strategy designed to achieve in the market that optimal schedule. Finally, the market operator uses a market-clearing procedure to determine the actual production of each producer.

The day-ahead price quota curves of a price maker can be obtained 1) by market simulation or 2) using forecasting procedures [1]. Both techniques are outside the scope of this paper, and therefore, the hourly price quota curves of the price maker considered are assumed to be known data.

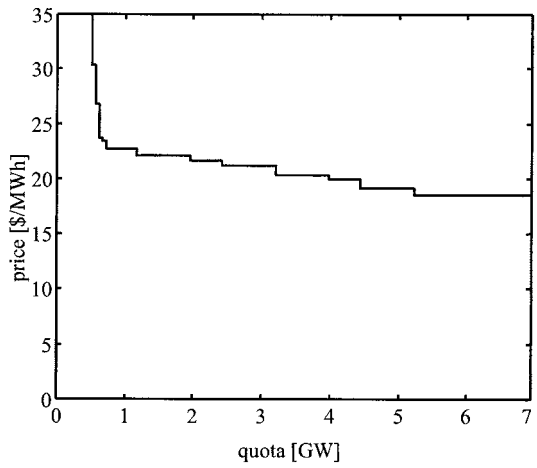

Fig. 1. Price quota curve.

For the sake of illustration, Fig. 1 shows a typical price quota curve.

B. Nonlinear Formulation

The natural formulation of the optimization problem that a price maker has to face is nonlinear, due to the products between the variables that appear in the objective function [6].

This formulation is

$$\underset{p_{t,i},\,q_t}{\text{maximize}} \quad \sum_{t=1}^{T}\left[\lambda_t(q_t)\,q_t - \sum_{i=1}^{m} c_{t,i}\right] \tag{1}$$

$$\text{subject to} \quad p_{t,i} \in \Pi_i, \quad i=1,\ldots,m, t=1,\ldots,T \tag{2}$$

$$q_t = \sum_{i=1}^{m} p_{t,i}, \quad t=1,\ldots,T. \tag{3}$$

The objective function (1) expresses the profit of the price maker over the planning horizon. The first term is the total revenue, and the second one is the total production cost, as formulated in the Appendix and in [12].

The set of constraints (2) enforces that every unit works within its feasible operating region over the whole planning horizon. A precise mixed-integer linear description of this feasibility region can be found in [12], [13], and in the Appendix.

The set of constraints (3) expresses for every hour the price maker quota as the sum of the power production of its units.

C. Linear Formulation

An alternative equivalent formulation of problem (1)–(3) that is linear is provided as follows:

$$\underset{p_{t,i},\,q_t,\,b_{t,s},\,u_{t,s}}{\text{maximize}} \quad \sum_{t=1}^{T}\left[\sum_{s=1}^{n_t}\lambda_{t,s}\left(b_{t,s}+u_{t,s}q_{t,s}^{\min}\right) - \sum_{i=1}^{m} c_{t,i}\right] \tag{4}$$

$$\text{subject to} \quad p_{t,i} \in \Pi_i, \quad i=1,\ldots,m,\ t=1,\ldots,T \tag{5}$$

$$q_t = \sum_{i=1}^{m} p_{t,i}, \quad t=1,\ldots,T \tag{6}$$

$$q_t = \sum_{s=1}^{n_t}\left(b_{t,s}+u_{t,s}q_{t,s}^{\min}\right), \quad t=1,\ldots,T \tag{7}$$

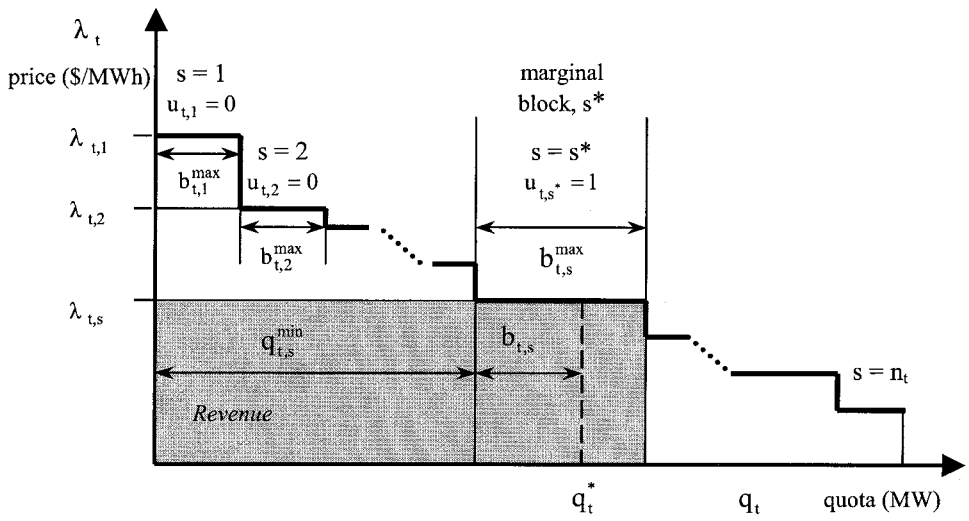

Fig. 2. Price quota curve: illustration of the linear formulation.

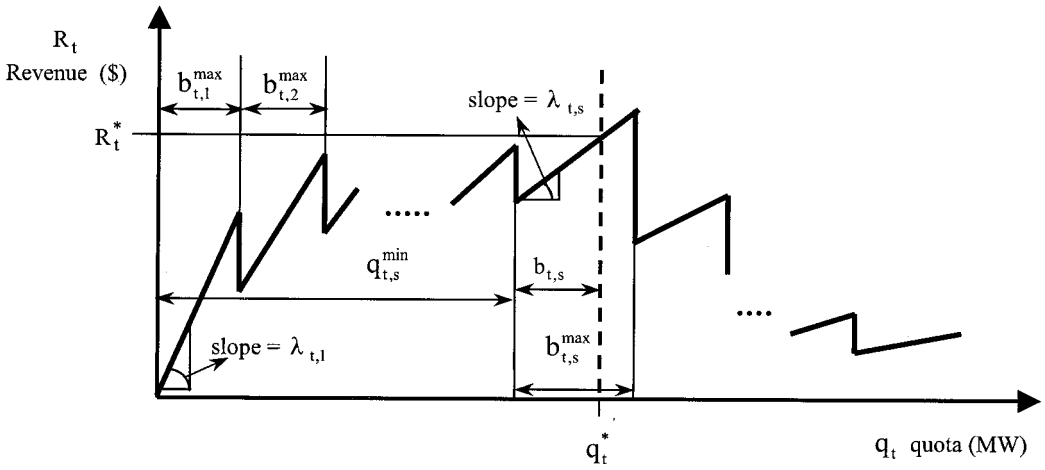

Fig. 3. Revenue versus quota.

$$0 \le b_{t,s} \le u_{t,s} b_{t,s}^{\max}, \quad s = 1, \ldots, n_t,\ t = 1, \ldots, T \quad (8)$$

$$\sum_{s=1}^{n_t} u_{t,s} = 1, \quad t = 1, \ldots, T. \quad (9)$$

The objective function (4) expresses the profit of the price maker: total revenue minus total costs. Figs. 2 and 3 illustrate the variables and constants needed to express linearly the price maker revenue as a function of its quota. Taking advantage of the stepwise nature of the price quota curve in every hour, the total revenue is expressed linearly using real variables $b_{t,s}$ and binary variables $u_{t,s}$, as illustrated in Fig. 2. Note that the revenue is the shadowed area in that figure. Fig. 3 also illustrates the above linear formulation. This figure expresses the price maker total revenue as a function of its quota. Observe the nonconvex and discontinuous nature of this function. Quota values that originate discontinuities on revenues do not lead to an ambiguous formulation, because the maximization of the objective function always leads to the highest revenue values.

The sets of constraints (5) and (6) are identical to sets (2) and (3), respectively. The set of constraints (7) expresses linearly the price maker quota in every hour as a function of variables $b_{t,s}$ and $u_{t,s}$, as illustrated in Fig. 2. The block of equations (8) expresses that the megawatt blocks of the price quota curve of every hour are nonnegative values, bounded above. The block of equations (9) states that only one variable $u_{t,s}$ is different from 0 in every hour. Thus, sets of equations (8) and (9) together enforce that only one variable $b_{t,s}$ is different from 0 in every hour. It should be noted that both formulations (1)–(3) and (4)–(9) are fully equivalent, and this allows saying that the linear formulation (4)–(9) is exact

III. Case Study

The considered electricity market includes one price maker producer owning 40 thermal units and different competitive fringe producers comprising 120 thermal units. It should be noted that mixed hydrothermal price makers can be analyzed in a similar way as thermal price makers. The differences among these analyses are simply related to the description of the feasible operating regions of the hydroelectric units. The market time horizon is 24 hours. Data for all units are based on the 1996 IEEE RTS [14] and are detailed in Table I. In this table, Type indicates the unit type (A, B, C, D, E, F, or G); PM/CF indicates the number of units corresponding to the price maker and the competitive fringe producers, respectively; $\overline{P}$ and

TABLE I
GENERATING UNITS DATA

Type	A	B	C	D	E	F	G
PM / CF	6 / 18	6 / 18	6 / 18	6 / 18	6 / 16	6 / 16	4 / 16
$\overline{P}$	12	76	100	155	197	350	400
$\underline{P}$	2.4	15.2	25	54.25	68.95	140	100
$C_1^{(*)}$	23.41	11.46	18.60	9.92	19.20	10.08	5.31
$C_2^{(*)}$	23.76	11.96	20.03	10.25	20.32	10.68	5.38
$C_3^{(*)}$	26.84	13.89	21.67	10.68	21.22	11.09	5.53
$C_4^{(*)}$	30.40	15.97	22.72	11.26	22.13	11.72	5.66
RR[MW/h]	12	76	100	155	180	120	400
SC($)	196	1353	1635	2173	2239	10190	NA$^{(**)}$
MUT (h)	4	8	8	8	12	24	NA$^{(**)}$
MDT (h)	2	4	8	8	10	48	NA$^{(**)}$

$^{(*)}$ Units: [$/MWh]. $^{(**)}$ NA: Not Applicable.

$\underline{P}$ indicate, respectively, the maximum and minimum power output; every C_j value provides the production cost of the block j of the unit (four-block piecewise convex cost curves are considered); RR gives both ramp-up and ramp-down maximum values; SC is the constant start-up cost; and MUT and MDT represent the minimum up and down times, respectively.

Price quota curves for the price maker are obtained simulating the market behavior as stated in [6]. They can also be obtained through forecasting procedures. However, it should be noted that they are exogenous data for the problem addressed in this paper. For the sake of illustration, the price quota curve faced by the price maker in hour 21 is shown in Fig. 1.

For this case study, problem (4)–(9) is solved using CPLEX 7.0 under the General Algebraic Modeling System (GAMS) [15] on an SGI R12000 (400 MHz) processor with 500 MB of RAM. The required CPU time is about 15 min.

Optimal profit for the price maker producer is US $869,122. The production self-schedule for the price maker is illustrated in Figs. 4–6. Fig. 4 provides for the 24-hour time horizon: 1) the price maker hourly production (gigawatts), 2) the hourly total served demand (gigawatts), and 3) the hourly market-clearing price ($ per megawatt hour). The total hourly served demand is obtained after evaluating the optimal self-scheduling of every market participant. It can be observed that the market-clearing price is highly correlated with the served demand. However, that price is not correlated with the price maker production, and this is an indication that market power is being exercised. This lack of correlation can be used as a monitoring variable to assess market power.

Fig. 5 shows the quota of the price maker in percentage with respect to the total served power in every hour, and from Fig. 5 it is apparent that the price maker quota in percentage is higher in off-peak hours than in high-demand hours. This is again an indication that power is withheld in high-demand hours to keep the price high in these hours. This quota variation can also be used as a monitoring variable to appraise market power.

Fig. 6 provides specific information on the manner in which the price maker exercises its market power. The upper and lower solid plots represent the power range for which the price does not change. The $\times$ plot provides the price maker production in the marginal block of the price quota curve, i.e., the last block of this curve used by the price maker. Fig. 6 shows that, for some hours, (mostly high-demand ones) the marginal production (the production in the marginal block) of the price maker is in the upper limit of the range for the final price. This may mean that the price maker is withholding its production in those hours not to force the price down, and this is again a clear indication that the price maker is exercising its market power. Fig. 6 can be used as a monitoring tool to assess the exercise of market power.

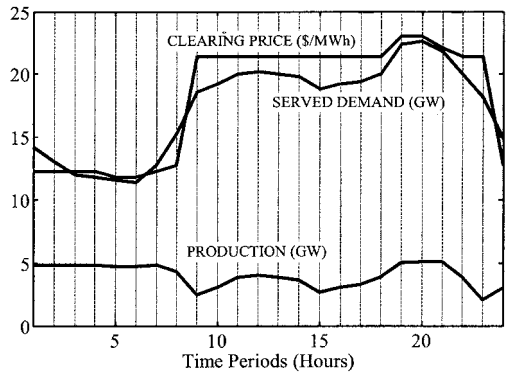

Fig. 4. Price maker production, served demand, and market-clearing price.

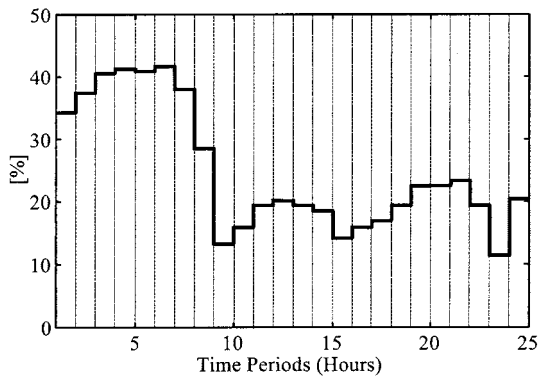

Fig. 5. Percentage of served power assigned to the price maker.

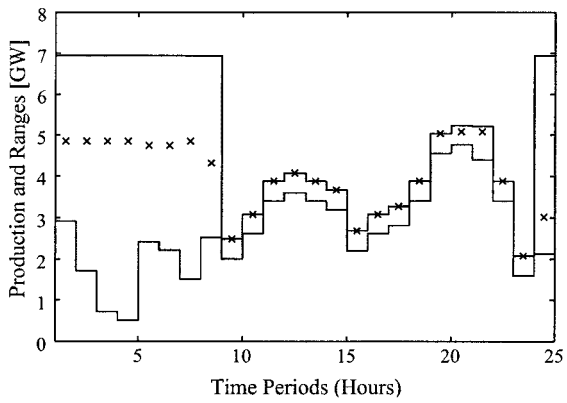

Fig. 6. Range of constant optimal price and price maker production.

On the other hand, during other hours, (i.e., hours 1–8, 21, 22, and 24) the optimal production is located somewhere between the lower limit and the upper limit that preserve the market-clearing price. There are three different reasons for that. In hour 8, this is so to meet the ramp-down constraint that becomes active between hours 8 and 9; it should be noted that the total production of the price maker begins to decrease in hour 8, when the demand rises. Secondly, in hour 24, the reason is to meet

the ramp-up limit between hours 23 and 24. Finally, in hours 1–7, 21, and 22, the reason is that a relevant change in the price maker marginal cost occurs within the range of constant optimal price; note that hours 1–7 present a very wide range of constant optimal price. For example, in hour 4, the optimal price is US \$12.3/MWh, but the price maker only owns 4.858 GW of power cheaper than that price (see Table I); therefore, the price maker is only willing to offer that amount, because, if it offers more than 4.858 GW, its profit decreases.

IV. Conclusion

This paper provides a mixed-integer LP formulation for the self-scheduling problem faced by a price maker in a pool-based electricity market. This formulation is exact in the sense that it is equivalent to a "natural" nonlinear formulation of the problem. This linear formulation allows an efficient solution using a standard branch-and-cut solver. Extensive computational analyses based on realistic price makers in realistic markets have shown the appropriate functioning of the proposed formulation.

Appendix

The running cost of a thermal unit and its technical constraints are described in this Appendix. The running cost $c_{t,i}$ is expressed as

$$c_{t,i} = A_i v_{t,i} + vc_{t,i} + UC_i y_{t,i} + DC_i z_{t,i} \quad i=1,\ldots,m, \quad t=1,\ldots,T \tag{A1}$$

where A_i represents the fixed cost of unit i; $vc_{t,i}$ is the piecewise linear variable cost of unit i in hour t; UC_i denotes the startup cost of unit i; DC_i is the shutdown cost of unit i; $v_{t,i}$ is a 0/1 variable that is equal to 1 if unit i is online in period t; $y_{t,i}$ denotes the 0/1 variable that is equal to 1 if unit i is started up at the beginning of period t; and $z_{t,i}$ is the 0/1 variable that is equal to 1 if unit i is shutdown at the beginning of period t. Equations (A1) express the running cost of unit i in period t as the sum of a fixed term, different from zero if the unit is online, plus the variable cost, the startup cost, and the shutdown cost.

The piecewise linear variable cost $vc_{t,i}$ is formulated as

$$vc_{t,i} = \sum_{n=1}^{N} F_n(i) b_n(i,t), \quad i=1,\ldots,m, t=1,\ldots,T \tag{A2}$$

$$p_{t,i} = \underline{P}_i v_{t,i} + \sum_{n=1}^{N} b_n(i,t), \quad i=1,\ldots,m, t=1,\ldots,T \tag{A3}$$

$$0 \le b_n(i,t) \le \bar{b}_n(i), \quad n=1,\ldots,N \quad i=1,\ldots,m, t=1,\ldots,T \tag{A4}$$

where N is the number of blocks of the piecewise linear variable cost function; $b_n(i,t)$ represents the power produced by unit i in period t using the nth power block; $\bar{b}_n(i)$ is the size of the nth power block of unit i; $\underline{P}_i$ is the minimum power output of unit i; and $F_n(i)$ denotes the slope of block n of the variable cost of unit i.

Constraints (A2) express the variable cost of unit i in period t as the sum of the corresponding terms of the piecewise linearization. Constraints (A3) state that the power output of unit i in period t is the sum of the power generated using each block plus the minimum power output. Constraints (A4) set the limits of the power generated in each block. This power should be greater than zero and less than the size (in megawatts) of each block. This formulation assumes that the cost is monotonically increasing. Nonconvex costs can be easily modeled by using additional binary variables [12].

The feasible operating region Π_i is formulated through the following linear constraints, which are further described in [12]:

$$p_{t,i} \ge \underline{P}_i v_{t,i}, \quad i=1,\ldots,m, t=1,\ldots,T \tag{A5}$$

$$p_{t,i} \le \bar{P}_i (v_{t,i} - z_{t+1,i}) + z_{t+1,i}\mathrm{SD}_i, \quad i=1,\ldots,m, \quad t=1,\ldots,T \tag{A6}$$

$$p_{t,i} \le p_{t-1,i} + \mathrm{RU}_i v_{t-1,i} + \mathrm{SU}_i y_{t,i}, \quad i=1,\ldots,m, \quad t=1,\ldots,T \tag{A7}$$

$$p_{t-1,i} - p_{t,i} \le \mathrm{RD}_i v_{t,i} + \mathrm{SD}_i z_{t,i}, \quad i=1,\ldots,m, \quad t=1,\ldots,T \tag{A8}$$

$$\sum_{t=1}^{G_i} (1 - v_{t,i}) = 0, \quad i=1,\ldots,m \tag{A9}$$

$$\sum_{\ell=t}^{t+\mathrm{UT}_i-1} v_{\ell,i} \ge \mathrm{UT}_i y_{t,i}, \quad i=1,\ldots,m, \quad t=G_i+1\ldots T-\mathrm{UT}_i+1 \tag{A10}$$

$$\sum_{\ell=t}^{T} (v_{\ell,i} - y_{t,i}) \ge 0, \quad i=1,\ldots,m, \quad t=T-\mathrm{UT}_i+2,\ldots,T \tag{A11}$$

$$\sum_{t=1}^{F_i} v_{t,i} = 0, \quad i=1,\ldots,m \tag{A12}$$

$$\sum_{\ell=t}^{t+\mathrm{DT}_i-1} (1 - v_{\ell,i}) \ge \mathrm{DT}_i z_{t,i}, \quad i=1,\ldots,m, \quad t=F_i+1,\ldots,T-\mathrm{DT}_i+1 \tag{A13}$$

$$\sum_{\ell=t}^{T} (1 - v_{\ell,i} - z_{t,i}) \ge 0, \quad i=1,\ldots,m, \quad t=T-\mathrm{DT}_i+2,\ldots,T \tag{A14}$$

$$y_{t,i} - z_{t,i} = v_{t,i} - v_{t-1,i}, \quad i=1,\ldots,m, t=1,\ldots,T \tag{A15}$$

$$y_{t,i} + z_{t,i} \le 1, \quad i=1,\ldots,m, t=1,\ldots,T. \tag{A16}$$

where

$$G_i = \min\left[T, \left(\mathrm{UT}_i - U_i^0\right) v_{0,i}\right]; \tag{A17}$$

$$F_i = \min\left[T, (\mathrm{DT}_i - S_{0,i})(1 - v_{0,i})\right]. \tag{A18}$$

In the expressions above, $\bar{P}_i$ is the capacity of unit i; SD_i is the shutdown ramp limit of unit i; RU_i represents the ramp-up limit of unit i; SU_i is the startup ramp limit of unit i; RD_i is the ramp-down limit of unit i; UT_i is the minimum up time of unit i; DT_i is the minimum down time of unit i; U_i^0 expresses the time periods unit i has been online at the beginning of the market horizon (end of period 0); $v_{0,i}$ provides the initial commitment status of unit i (1 if it is online, 0 otherwise); and $S_{0,i}$ represents the time periods unit i has been offline at the beginning of the

market horizon (end of period 0). For unit consistency, it should be noted that time periods of one hour are considered.

Constraints (A5) and (A6) set the lower and upper limits of the power output, respectively. The upper limit is restricted by the maximum capacity of the unit in normal operation and by the shutdown ramp rate if the unit is shutdown in the next period. It should be noted that the power output becomes zero if the unit is offline, i.e., if binary variable $v_{t,i}$ is equal to zero.

The set of constraints (A7) imposes the ramp-up rate limit as well as the startup ramp rate limit. Analogously, ramp-down and shutdown ramp rate limits are enforced by constraints (A8).

Equations (A9)–(A11) represent the linear expressions of minimum up-time constraints. The set of equations (A9) is related to the initial status of the units. G_i is the number of initial periods during which unit i must be online to meet the minimum up-time requirement. If unit i does not declare any initial status, G_i is assumed to be equal to zero. The set of equations (A10) is used for the periods following G_i, and it ensures the satisfaction of the minimum up-time constraint during all the possible sets of consecutive periods of size UT_i. Finally, the set of equations (A11) is needed for the last $\mathrm{UT}_i - 1$ periods, i.e., if a unit is started up in one of these periods, it remains online during the remaining periods. Similarly, (A12)–(A14) provide the formulation of the minimum down-time constraints. Equations (A12)–(A14) are identical to (A9)–(A11) just by changing $v_{t,i}$, $z_{t,i}$, DT_i, and $S_{0,i}$ by $(1 - v_{t,i})$, $y_{t,i}$, UT_i, and U_i^0, respectively.

Finally, constraints (A15) and (A16) are necessary to model the startup and shutdown status of the units and to avoid the simultaneous commitment and decommitment of a unit [13].

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2. Bidding tools for producers – Stochastic programming

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Price-Taker Bidding Strategy Under Price Uncertainty

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***Abstract*—This paper provides a framework to obtain the optimal bidding strategy of a price-taker producer. An appropriate forecasting tool is used to estimate the probability density functions of next-day hourly market-clearing prices. This probabilistic information is used to formulate a self-scheduling profit maximization problem that is solved taking advantage of its particular structure. The solution of this problem allows deriving a simple yet informed bidding rule. Results from a realistic case study are discussed in detail.**

***Index Terms*—Bidding strategy, forecasting, MILP, pool-based electricity market, price-taker producer, self-scheduling.**

NOMENCLATURE

The notation used throughout the paper is stated below.

a_t	Constant used to obtain the upper bound of the confidence interval for hour t.
b_t	Constant used to obtain the lower bound of the confidence interval for hour t.
B	Random variable describing the total profit of the thermal generator (all hours) in \$.
B^{avg}	Average value of the total profit of the thermal generator (all hours) in \$.
B_t	Random variable describing the profit of the thermal generator at hour t in \$.
B_t^{avg}	Average value of the profit of the thermal generator at hour t in \$.
c_t	Operating cost function of the thermal generator at hour t in \$/h. It is precisely described in the Appendix.
c_t^*	Optimal value of the operating cost function of the thermal generator at hour t in \$/h.
n	Number of scenarios.
p_t	Power produced by the thermal generator at hour t in MW.
p_t^*	Optimal power produced by the thermal generator at hour t in MW.
$\bar{P}$	Maximum power output of the thermal generator in MW.
R	Covariance matrix of random variables $\lambda_1, \ldots, \lambda_T$.
s	Scenario index.
t	Hour index.
T	Time span in hours.
$\gamma_{\alpha\%}$	α-percentage point of the $N(0,1)$ distribution.
λ_t	Random variable describing the market-clearing price at hour t in \$/MWh.
λ_t^{avg}	Average value of random variable λ_t in \$/MWh.
λ_t^{est}	Estimate of the market-clearing price at hour t in \$/MWh.
λ_t^{true}	True market-clearing price at hour t in \$/MWh.
λ_{ts}	Market-clearing price at hour t and scenario s in \$/MWh.
Π	Feasible operating region of the thermal generator. It is precisely described in the Appendix.
ω_s	Probability of scenario s.
σ_t^{est}	Estimate of the standard deviation of random variable λ_t in \$/MWh.
$E_{\lambda_1,\ldots,\lambda_T}\{\cdot\}$	Expected value operator with respect to random variables $\lambda_1, \ldots, \lambda_T$.
$E_{\lambda_t}\{\cdot\}$	Expected value operator with respect to random variable λ_t.
$\text{Var}_{B_t}\{\cdot\}$	Variance operator with respect to random variable B_t.
$\text{Var}_{B_1,\ldots,B_T}\{\cdot\}$	Variance operator with respect to random variables $B_1, \ldots, B_T$.

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I. INTRODUCTION

THIS paper addresses the bidding problem faced by a thermal price-taker producer in a pool-based electric energy market.

It is assumed that price uncertainty is high and that an appropriate forecasting tool is available to forecast next-day hourly prices and to estimate their associated probability density functions.

This paper provides a simple yet informed bidding rule that allows a price-taker producer to obtain optimal bidding decisions.

The analysis consists of three steps:

1) An appropriate price-forecasting tool is used to estimate the probability density functions corresponding to next-day hourly energy prices.
2) A self-scheduling problem is formulated using the probabilistic price information derived in 1). This problem is efficiently solved taking advantage of its singular structure, which is described in Section III.
3) The solution of the problem formulated and solved in 2) allows deriving a simple yet informed bidding rule for the price-taker.

The framework for the analysis above is a pool-based electricity market [1]–[3]. It is assumed that the market is cleared one day in advance on an hourly basis, and that producers and

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consumers submit hourly bidding curves consisting of blocks of energy and their corresponding prices. It is considered that, in each hour, every generator must bid all its available power in one or several power blocks at increasing prices of its choice [1]. No particular assumption is made on the market structure and its participants. The market-clearing price is used to pay any accepted production bid, and it is also the price paid by any accepted demand bid.

The analysis in this paper is restricted to considering a price-taker producer, i.e., a producer with no capability of altering market-clearing prices. In such situation, the profit maximization problem faced by the producer decomposes into independent subproblems. Each of these subproblems corresponds to the profit maximization of each generator owned by the producer [4]. Therefore, for the sake of simplicity, a single generator is considered henceforth. The problem faced by a price-maker producer, i.e., a producer with capability of altering market-clearing prices is outside the scope of this paper.

This paper extends the model reported in [4], providing a probabilistic framework for the treatment of uncertain market-clearing prices, and deriving a simple yet informed bidding rule.

Quite a few bidding methods addressing the strategic bidding problem have been published so far [5]. Pioneering paper [6] solves the optimal bidding problem for a single time period using dynamic programming. In [7], an analytical formulation for building the optimal bidding strategy in the former pool-based electricity market of England & Wales was developed under the assumption of a perfectly competitive market. However, this assumption does not seem reasonable for most electricity markets. In [8], a bidding strategy was proposed for the situation of two buyers competing for a single block of energy. Huse *et al.* [9] proposed a simple bidding strategy based on heuristics without taking into consideration the effect of intertemporal constraints. In [10], a simple bidding model is derived after estimating the probability of winning below and on the margin. In [11], a bidding strategy is developed to maximize the profit obtained by a supplier by adjusting its submitted operational parameters such as the declared minimum power output.

In [12]–[15], game theory is applied to find an equilibrium state (Nash equilibrium) of the bidding game, corresponding to the optimal bidding strategies achieved by the participants. Contreras *et al.* [16] propose an iterative Cournot model to find the optimal bidding policy of a generating company. In [17], Nash equilibrium is applied under the framework of bilateral based electricity markets. These methods are more suitable for analyzing strategic behavior rather than for proposing a tool to develop bidding strategies.

Under the framework of multi-round auctions, several bidding strategies are proposed [18]–[20]. In [21], [22], evolutionary and artificial intelligence techniques such as genetic algorithms, genetic programming and finite state automata are used to develop adaptive and evolutionary bidding strategies. Unfortunately, iterative auctions are not implemented in most electricity markets.

Other methods such as ordinal optimization [23], Lagrangian relaxation [24], stochastic optimization [25], and Markov decision process [26] have also been applied to solve the optimal bidding strategy problem. In [23], an ordinal optimization based bidding strategy is used for seeking good enough bids with high probabilities. An innovative model and a Lagrangian relaxation-based method are presented in [24] to solve the bidding and self-scheduling problem. In [25], the optimal bidding problem is modeled as a stochastic optimization problem taking into account the effect of competitors through Monte Carlo simulation. In [26], this problem is represented as a multiple stage probabilistic decision-making problem and a Markov decision process was applied to calculate bidding decisions.

The remainder of this paper is organized as follows. Section II provides the proposed modeling framework to deal with price uncertainty. In Section III, the price-taker self-scheduling problem under price uncertainty is formulated, analyzed and solved. Section IV provides a probabilistic description of the profit achieved by the price-taker. Section V presents the simple yet informed bidding rule derived from the solution of the self-scheduling problem. In Section VI, the results of a realistic case study are analyzed in detail. Section VII presents some relevant conclusions. Finally, in the Appendix, a mathematical linear description of the cost function and the feasible operating region of a generator is provided.

II. Price Uncertainty Modeling

A model for the uncertainty in the hourly market-clearing prices is proposed in this section. This model provides a probabilistic characterization of these prices that has a definitive impact on the self-scheduling problem presented in Section III. Moreover, the model also provides a probabilistic characterization of the profit for the price-taker, which is described in Section IV.

The model is based on the probability density functions of forecast prices. Several techniques to forecast electricity prices can be found in the technical literature. For instance, jump diffusion/mean reversion models have been proposed in [27]. Neural networks are used to predict prices in the England & Wales pool [28], in California [29], and in the Victorian market [30]. Techniques based on Fourier and Hartley transforms have been studied in [31]. Recently, in [32], two models based on time series analysis have been proposed. These models, which produce accurate predictions, relate actual prices to demands and past prices.

All the aforementioned forecasting procedures assume that the market-clearing price at hour $t(t = 1, \ldots, T)$ is a random variable denominated λ_t, which has to be forecast. It should be noted that random variables $\lambda_t(t = 1, \ldots, T)$ depend on the actual price values of the time series that is used for forecasting. From a statistical point of view, they are random variables "conditioned" to the actual price values of the time series used for forecasting. This time series spans from an arbitrary origin up to hour 24 of the day preceding the one whose prices have to be forecast. Under the above assumption, and using a time series forecasting procedure [33], the expected value of random variable λ_t is the actual price prediction at hour t, λ_t^{est}, that is

$$\lambda_t^{\mathrm{est}} = \lambda_t^{\mathrm{avg}} = E_{\lambda_t}\{\lambda_t\}. \tag{1}$$

This is a key fact that is used in Section III.

The estimate of the standard deviation of the random variable λ_t is readily available from the forecasting procedure, and it is denominated σ_t^{est}. Moreover, it can be shown that the distribution of random variable λ_t is approximately Lognormal [32], i.e.,

$$\lambda_t \equiv \text{Lognormal}\left(\lambda_t^{\text{est}}, \sigma_t^{\text{est}}\right). \tag{2}$$

Upper and lower bounds of the confidence interval are computed respectively as

$$\begin{aligned} &\lambda_t^{\text{est}} + a_t \sigma_t^{\text{est}} \\ &\lambda_t^{\text{est}} - b_t \sigma_t^{\text{est}}. \end{aligned} \tag{3}$$

It should be noted that parameters a_t and b_t are obtained directly from the forecasting procedure and depend on the considered level of confidence to be guaranteed, e.g., 99% or 95% [33].

They are computed so as to cover 99% or 95% of the total area under the Lognormal distribution. Formulae to compute parameters a_t and b_t are provided below [34]:

$$a_t = \frac{\exp\left(\frac{1}{2}\left(\sigma_t^{\text{est}}\right)^2 + \lambda_t^{\text{est}} + \gamma_{\alpha\%}\sigma_t^{\text{est}}\right) - \lambda_t^{\text{est}}}{\sigma_t^{\text{est}}} \tag{4}$$

$$b_t = \frac{\lambda_t^{\text{est}} - \exp\left(\frac{1}{2}\left(\sigma_t^{\text{est}}\right)^2 + \lambda_t^{\text{est}} - \gamma_{\alpha\%}\sigma_t^{\text{est}}\right)}{\sigma_t^{\text{est}}} \tag{5}$$

where $\gamma_{\alpha\%}$ depends on the desired level of confidence.

For a level of confidence of 99%, $\gamma_{99\%}$ is obtained from Probability $(N(0,1) > \gamma_{99\%}) = 0.01/2$, which results in $\gamma_{99\%} = 2.5758$. Analogously, $\gamma_{95\%} = 1.9600$.

III. Self-Scheduling

Under price uncertainty, the profit maximization problem of a price-taker generator can be formulated as

$$\begin{aligned} &\underset{p_t}{\text{maximize}}\ E_{\lambda_1,\ldots,\lambda_T}\left\{\sum_{t=1}^{T}\lambda_t p_t\right\} - \sum_{t=1}^{T} c_t \\ &\text{subject to } p_t \in \Pi. \end{aligned} \tag{6}$$

The objective function of the problem above is the expected value of profit for selling energy, i.e., expected revenues minus incurred operating costs (as described in the Appendix). Note that 1 hour time intervals are considered. The only constraint of this problem states that the generator must operate within its feasible operating region (power output limits, ramp-rate constraints, and minimum up and down time constraints). This feasible operating region is also precisely described in the Appendix.

It should be noted that the random variable describing market-clearing prices only affects the objective function, and particularly, the term corresponding to revenues.

Formulation (6) suggests a reformulation to allow a scenario-based solution approach, i.e.,

$$\begin{aligned} &\underset{p_t}{\text{maximize}} \sum_{s=1}^{n}\left(\omega_s \sum_{t=1}^{T}\lambda_{ts} p_t\right) - \sum_{t=1}^{T} c_t \\ &\text{subject to } p_t \in \Pi \end{aligned} \tag{7}$$

where $\sum_{s=1}^{n}\omega_s = 1$.

Note, however, that this scenario formulation leads to an intractable problem. For instance, considering 24 hours and 3 price values per hour results in 3^{24} scenarios which is a number higher than 2.8×10^{11}, and this number constitutes an excessive number of scenarios. An alternative approach is therefore needed, and it is developed in what follows.

Using basic probability theory [35], expectation and summation operators can be swapped in (6), resulting in

$$\begin{aligned} &\underset{p_t}{\text{maximize}} \sum_{t=1}^{T} E_{\lambda_t}\{\lambda_t\} p_t - \sum_{t=1}^{T} c_t \\ &\text{subject to } p_t \in \Pi. \end{aligned} \tag{8}$$

And using the expected value λ_t^{est} defined in (1), problem (8) becomes

$$\begin{aligned} &\underset{p_t}{\text{maximize}} \sum_{t=1}^{T}\left(\lambda_t^{\text{est}} p_t - c_t\right) \\ &\text{subject to } p_t \in \Pi. \end{aligned} \tag{9}$$

This problem is mixed-integer and linear (see the Appendix) and its size is moderate [4]. It can be easily solved using a standard branch and cut solver such as CPLEX under GAMS [36].

The solution of problem (9) provides the best possible production decision under price uncertainty: p_t^*, $t = 1, \ldots, T$. Note that all information available on prices (probability density functions) is used to reach the above optimal production decision. Note, also, that no additional information is available before the bidding procedure is carried out and the market cleared. Therefore, no additional information alters the optimal production decision.

The next step is to establish bidding rules to ensure that the generator gets allocated its optimal self-scheduled production, i.e., p_t^*, $t = 1, \ldots, T$. This is done in Section V.

IV. Profit

The profit achieved by the generator at hour t, B_t, is also a random variable. Using basic statistics theory [35], if the actual price is within the confidence interval, the average value of B_t is computed as

$$B_t^{\text{avg}} = \lambda_t^{\text{est}} p_t^* - c_t^* \tag{10}$$

and its variance is estimated as

$$\text{Var}_{B_t}(B_t) = \left(\sigma_t^{\text{est}}\right)^2 \left(p_t^*\right)^2. \tag{11}$$

The profit achieved by the generator during the 24 hours of the day, B, is also a random variable. Its average value is easily computed as [35]

$$B^{\text{avg}} = \sum_{t=1}^{T}\lambda_t^{\text{est}} p_t^* - c_t^*. \tag{12}$$

The variance of B can be estimated as follows

$$\begin{aligned}\mathrm{Var}_{B_1,\ldots,B_T}(B) &= \mathrm{Var}_{B_1,\ldots,B_T}\left(\sum_{t=1}^{T} B_t\right)\\ &= [p_1^* \quad \ldots \quad p_T^*]\, R \begin{bmatrix} p_1^* \\ \vdots \\ p_T^* \end{bmatrix}. \end{aligned} \tag{13}$$

The diagonal elements of the $T \times T$ covariance matrix R are the estimates of the variances of the random variables λ_t, i.e., $(\sigma_t^{\mathrm{est}})^2$, $t = 1, \ldots, T$. The off-diagonal elements are the covariances between each couple of random variables λ_t. That is, element $R_{i,j} (i \neq j)$ is the covariance of random variables λ_i and λ_j. The computation of matrix R is somehow involved but it is precisely described in [37].

V. Bidding Strategy

Using the results obtained in Section III, the proposed bidding strategy is stated below. The generator should submit to the market operator a bidding curve for each hour of the market horizon. Each one of these hourly bidding curves consists of a set of blocks of power and their corresponding increasing prices. For example, a 42-MW unit in hour 21 may bid powers 10, 20 and 12 MW at prices 20, 25, and 30 $/MWh. It should be noted that a convex bidding curve is required, i.e., prices have to be associated with the power blocks bid.

The bidding rule formulated below to determine the hourly bidding curve of the generator only requires up to two blocks of power and their corresponding prices. If, as a result of the market rules, each hourly bidding curve should have a number of blocks larger than two, the rule below can be modified straightforwardly.

In the rule below, recall that σ_t^{est} is the estimate of the standard deviation of the probability density function describing the market-clearing price at hour t. Additionally, it is assumed that parameters a_t and b_t have been obtained for a level of confidence of 99%.

The bidding curve for hour t is formulated as a function of the optimal self-scheduled production in that hour, p_t^*. Three cases are possible and are analyzed below.

Case 1) If $p_t^* = 0$, the bidding curve consists of a single block of power $\bar{P}$ at price $\lambda_t^{\mathrm{est}} + a_t \sigma_t^{\mathrm{est}}$. See Fig. 1(a).

It should be noted that this bidding curve guarantees with a level of confidence of 99% that the power accepted in this situation is 0, which is the optimal self-scheduled power for this case.

Case 2) If p_t^* is such that $0 < p_t^* < \bar{P}$, the bidding curve consists of two blocks of power and their corresponding prices. These two blocks of powers are p_t^*, and $\bar{P} - p_t^*$, and their prices are $\lambda_t^{\mathrm{est}} - b_t \sigma_t^{\mathrm{est}}$ and $\lambda_t^{\mathrm{est}} + a_t \sigma_t^{\mathrm{est}}$, respectively. See Fig. 1(b).

Note that this bidding curve guarantees with a level of confidence of 99% that the power accepted in this situation is p_t^*, which is the optimal self-scheduled power for this case.

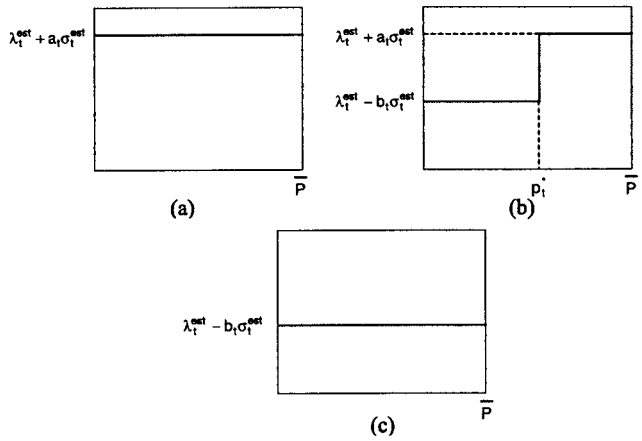

Fig. 1. Bid curves for the proposed bidding strategy.

TABLE I
Variable Cost

	Cost ($/MWh)	Power (MW)
Block 1	25.84	130
Block 2	26.52	148
Block 3	27.83	166
Block 4	29.00	184
Block 5	30.62	202
Block 6	34.97	220
Block 7	33.14	238
Block 8	36.37	256
Block 9	39.00	274
Block 10	41.27	294

Case 3) If $p_t^* = \bar{P}$, the bidding curve consists of a single block of power $\bar{P}$ at price $\lambda_t^{\mathrm{est}} - b_t \sigma_t^{\mathrm{est}}$. See Fig. 1(c). It should be noted that this bidding curve guarantees with a level of confidence of 99% that the power accepted in this situation is $\bar{P}$, which is the optimal self-scheduled power for this case.

If the required level of confidence has to be larger than 99%, parameters a_t and b_t should be computed accordingly. However, note that a low profile bidding behavior is convenient in many markets, and this requires bidding prices not far away from the actual market-clearing prices.

VI. Case Study

This section provides a comparison in terms of profit and power schedule obtained by a generator under price uncertainty and under perfect knowledge of true energy prices.

Data for the considered power unit as well as price values are given below. The characteristics of the unit are based on the data of [38]. The shut-down cost is considered constant and equal to $56 and the fixed cost is $700. The start-up cost is considered constant and equal to $1038. Reference [4] provides a model to consider the exponential variation of the start-up cost that can be easily integrated in the framework provided in this paper.

A ten-block nonconvex variable cost is considered and given in Table I. A nonconvex cost has been selected to illustrate the capability of the proposed formulation to handle this type of costs.

TABLE II
FEASIBLE OPERATING REGION

$\underline{P}$ [MW]	$\overline{P}$ [MW]	SU [MW/h]	SD [MW/h]	RU [MW/h]	RD [MW/h]	UT [h]	DT [h]
112	294	170	160	60	50	4	4

TABLE III
ENERGY PRICE DATA ($/MWh)

t	λ_t^{true}	λ_t^{est}	Lower bound	Upper bound	σ_t^{est}
1	28.52	33.30	27.22	40.75	2.61
2	25.23	26.52	21.63	32.51	2.10
3	23.45	22.15	18.04	27.20	1.76
4	22.93	23.10	18.81	28.36	1.84
5	22.79	22.59	18.40	27.74	1.80
6	22.93	23.15	18.85	28.43	1.84
7	23.29	24.64	20.07	30.26	1.96
8	25.46	24.75	20.15	30.39	1.97
9	22.93	25.50	20.76	31.31	2.03
10	28.28	27.58	22.46	33.86	2.20
11	31.08	31.59	25.73	38.79	2.52
12	39.93	35.59	28.99	43.70	2.84
13	40.24	41.05	33.43	50.40	3.27
14	40.55	41.60	33.88	51.08	3.32
15	39.93	38.98	31.74	47.86	3.11
16	39.93	39.73	32.36	48.79	3.17
17	40.75	42.02	34.22	51.59	3.35
18	41.56	42.09	34.28	51.68	3.36
19	40.75	40.74	33.18	50.02	3.25
20	39.74	38.80	31.60	47.64	3.09
21	39.54	39.63	32.27	48.66	3.16
22	42.77	46.14	37.58	56.66	3.68
23	40.55	39.03	31.79	47.93	3.11
24	29.23	33.67	27.42	41.35	2.68

Table II shows the limits that constrain the feasible operating region, Π, of the generator: minimum power output, maximum capacity, start-up ramp rate limit, shut-down ramp rate limit, ramp-up rate limit, ramp-down rate limit, minimum up time, and minimum down time.

Finally, in the hour before the market horizon the unit has been running for 11 hours and producing 170 MW.

It is assumed that the power output of the thermal unit is constant throughout each hour. However, a linear variation of the power output during each hour can be modeled as stated in [39]. For clarity, this model is not considered in this paper.

Price data are provided in Table III. The second column corresponds to the actual prices obtained in the electricity market of mainland Spain on Wednesday August 29th, 2001 [1]. The third column shows the estimates of the energy prices using the forecasting method proposed in [32] with a level of confidence equal to 99%. Lower and upper bounds of the estimate of each price, as well as an estimate of its standard deviation are also shown in this table. Fig. 2 depicts the actual energy prices, the forecast energy prices and their bounds.

Firstly, the self-scheduling problem is solved with the forecast prices. The hourly power output can be found in Table IV. For this production schedule, the profit that the generator would have obtained can be computed using the true prices (settlement procedure). In this case, the actual settlement profit, computed using optimal self-scheduled powers and true prices is

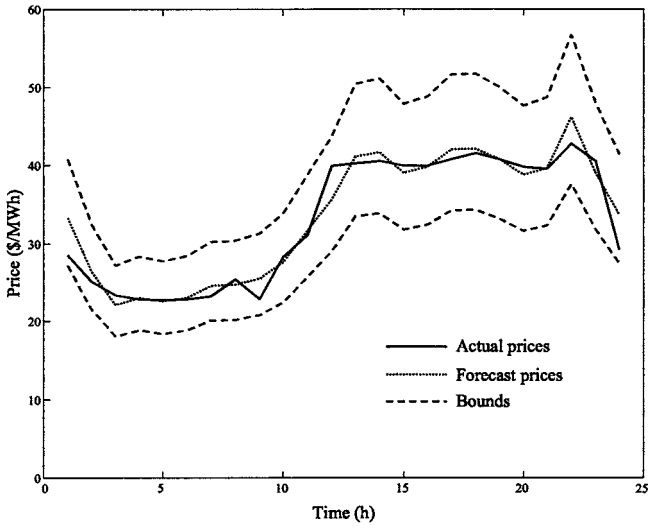

Fig. 2. Price forecast versus actual prices.

U.S. $27 207.70. Note that the optimal self-scheduled powers have been obtained with forecast prices.

In order to assess the actual loss in profit due to price uncertainty, it is assumed that the generator is a perfect fortune-teller, i.e., it has perfect knowledge of the true prices in advance. Note, however, that this information is not available in the real world. If the self-scheduling problem is solved with the true price profile, the settlement profit is equal to U.S. $27 268.95, which represents the maximum profit the generator can make. Note that there is only a 0.22% difference between the maximum attainable profit and the actual profit obtained with the forecast schedule. This slight difference shows that both, the forecasting technique and the bidding strategy, are efficient. Power output in each hour is shown in Table IV for the two price profiles. As it can be noted, both schedules meet the technical constraints presented in Table II. Moreover, the production schedules obtained with both price profiles are different (hours 14, 15, 17, 20, 23, and 24) implying that several blocks of energy are bid at different prices in one case versus the other; however, the commitment status is identical for both cases.

Finally, Table V presents the structure of the hourly bids that the generator should submit to the market operator. For the sake of simplicity, it is assumed that the generator bids one or two blocks in every hour, as proposed in Section V. In hours of scheduled power between 0 and maximum power output (hours 1, 11–13, 15, 16, 19–21, 23, and 24), the actual scheduled energy is bid at a price smaller than the true one (the lower bound of the confidence interval) and the remaining power at a price greater than the true one (the upper bound of the confidence interval). In those hours of scheduled power equal to maximum power output (hours 14, 17, 18, and 22) price bids are smaller than the corresponding true prices. On the other hand, in those hours where the scheduled power is equal to 0 (hours 2–10) the corresponding price bid is greater than the true price. Note that with the above bidding strategy the desired schedule does become the actual one.

The model has been implemented on a SGI R12000, 400 MHz based processor with 500 MB of RAM using CPLEX 7.5 under

TABLE IV
SELF-SCHEDULE IN MW

t	With forecast prices	With true prices
1	160.0	160.0
2	0.0	0.0
3	0.0	0.0
4	0.0	0.0
5	0.0	0.0
6	0.0	0.0
7	0.0	0.0
8	0.0	0.0
9	0.0	0.0
10	0.0	0.0
11	170.0	170.0
12	230.0	230.0
13	274.0	274.0
14	294.0	274.0
15	256.0	274.0
16	274.0	274.0
17	294.0	274.0
18	294.0	294.0
19	274.0	274.0
20	256.0	274.0
21	274.0	274.0
22	294.0	294.5
23	256.0	252.0
24	206.0	202.0

TABLE V
BIDDING STRATEGY

t	Power bid (MW)		Price bid ($/MWh)		λ_t^{true}
	Block 1	Block 2	Block 1	Block 2	
1	160	134	27.22	40.75	28.52
2	294	-	32.51	-	25.23
3	294	-	27.20	-	23.45
4	294	-	28.36	-	22.93
5	294	-	27.74	-	22.79
6	294	-	28.43	-	22.93
7	294	-	30.26	-	23.29
8	294	-	30.39	-	25.46
9	294	-	31.31	-	22.93
10	294	-	33.86	-	28.28
11	170	124	25.73	38.79	31.08
12	230	64	28.99	43.70	39.93
13	274	20	33.43	50.40	40.24
14	294	-	33.88	-	40.55
15	256	38	31.74	47.86	39.93
16	274	20	32.36	48.79	39.93
17	294	-	34.22	-	40.75
18	294	-	34.28	-	41.56
19	274	20	33.18	50.02	40.75
20	256	38	31.60	47.64	39.74
21	274	20	32.27	48.66	39.54
22	294	-	37.58	-	42.77
23	256	38	31.79	47.93	40.55
24	206	88	27.42	41.35	29.23

GAMS [36]. The optimal solutions to both cases (with forecast and with actual energy prices) were achieved in 2.0 seconds of computing time.

VII. CONCLUSIONS

This paper provides a bidding rule that allows a price-taker producer to achieve, under price uncertainty, its optimal self-schedule. An appropriate probability description of hourly market-clearing prices is provided. It is used to formulate and solve an expected maximum profit self-scheduling problem. The solution of this problem allows determining a simple yet informed bidding rule to achieve the actual optimal self-schedule. The analysis of multiple case studies allows concluding that the rule developed is effective in achieving the optimal (and feasible) schedule of the price-taker producer.

APPENDIX

The operating cost and the set of operating constraints, Π, are presented in this Appendix.

The nonlinear and nonconvex operating cost, c_t, can be formulated as

$$c_t = Cz(t) + Av(t) + d(p_t) + Sy(t) \quad \forall\, t = 1, \ldots, T \tag{A1}$$

where C is the shut-down cost [$/h], $z(t)$ is a 0/1 variable which is equal to 1 if the unit is shut-down at the beginning of hour t, A is the fixed cost [$/h], $v(t)$ is a 0/1 variable which is equal to 1 if the unit is on-line at hour t, $d(p_t)$ expresses the variable production cost at hour t [$/h], which is a nonlinear function of the power output at that hour. Finally, $y(t)$ is a 0/1 variable which is equal to 1 if the unit is started-up at the beginning of hour t, and S is the start-up cost [$/h].

A mixed-integer linear formulation of the nonconvex and nondifferentiable variable production cost is provided in this Appendix and can also be found in [4]

$$\tilde{d}(t) = \sum_{\ell=1}^{\mathrm{NL}} \left[F_\ell \delta_\ell(t) + t_\ell(t) \sum_{m=1}^{\ell-1} F_m \left(T_m - T_{m-1}\right) \right] \quad \forall\, t = 1, \ldots, T \tag{A2}$$

$$p_t = \sum_{\ell=1}^{\mathrm{NL}} \left[\delta_\ell(t) + t_\ell(t) T_{\ell-1}\right] \quad \forall\, t = 1, \ldots, T \tag{A3}$$

$$\sum_{\ell=1}^{\mathrm{NL}} t_\ell(t) = 1 \quad \forall\, t = 1, \ldots, T \tag{A4}$$

$$\delta_\ell(t) \le (T_\ell - T_{\ell-1})\, t_\ell(t) \quad \forall\, \ell = 1, \ldots, NL,\ \forall\, t = 1, \ldots, T \tag{A5}$$

$$\delta_\ell(t) \ge 0 \quad \forall\, \ell = 1, \ldots, NL,\ \forall\, t = 1, \ldots, T \tag{A6}$$

$$t_\ell(t) \in \{0, 1\} \quad \forall\, \ell = 1, \ldots, NL,\ \forall\, t = 1, \ldots, T \tag{A7}$$

where $\tilde{d}(t)$ is the piecewise linear variable cost at hour t [$/h] which replaces the nonlinear variable cost, $d(\cdot)$, in (A1), F_ℓ is the slope of block ℓ of the variable cost [$/MWh], NL is the number of blocks of the variable cost, $\delta_\ell(t)$ is the power produced in the block ℓ at hour t [MW], $t_\ell(t)$ is a 0/1 variable which is equal to 1 if block ℓ determines the power at hour t, and T_ℓ is the upper limit of block ℓ [MW].

Finally, the following set of linear constraints formulates the feasible operating region, Π, comprising power limits, ramp rate limits and minimum up and down time constraints [4]

$$p_t \ge \underline{P} v(t) \quad \forall\, t = 1, \ldots, T \tag{A8}$$

$$p_t \le \bar{P}\left[v(t) - z(t+1)\right] + z(t+1)\mathrm{SD} \quad \forall\, t = 1, \ldots, T \tag{A9}$$

$$p_t \le p_{t-1} + \mathrm{RU} v(t-1) + \mathrm{SU} y(t) \quad \forall\, t = 1, \ldots, T \tag{A10}$$

$$p_{t-1} - p_t \leq \mathrm{RD}v(t) + \mathrm{SD}z(t) \quad \forall\, t = 1, \ldots, T \tag{A11}$$

$$\sum_{t=1}^{G} [1 - v(t)] = 0 \tag{A12}$$

$$\sum_{j=t}^{t+\mathrm{UT}-1} v(j) \geq \mathrm{UT}y(t) \quad \forall\, t = G+1, \ldots, T - \mathrm{UT} + 1 \tag{A13}$$

$$\sum_{j=t}^{T} [v(j) - y(t)] \geq 0 \quad \forall\, t = T - \mathrm{UT} + 2, \ldots, T \tag{A14}$$

$$\sum_{t=1}^{F} v(t) = 0 \tag{A15}$$

$$\sum_{j=t}^{t+\mathrm{DT}-1} [1 - v(j)] \geq \mathrm{DT}z(t) \quad \forall\, t = F+1, \ldots, T - \mathrm{DT} + 1 \tag{A16}$$

$$\sum_{j=t}^{T} [1 - v(j) - z(t)] \geq 0 \quad \forall\, t = T - \mathrm{DT} + 2, \ldots, T \tag{A17}$$

$$y(t) - z(t) = v(t) - v(t-1) \quad \forall\, t = 1, \ldots, T \tag{A18}$$

$$y(t) + z(t) \leq 1 \quad \forall\, t = 1, \ldots, T \tag{A19}$$

$$z(t) \in \{0, 1\} \quad \forall\, t = 1, \ldots, T \tag{A20}$$

where

F $\quad \mathrm{Min}\,\{T, [\mathrm{DT} - s(0)]\,[1 - v(0)]\}$;

G $\quad \mathrm{Min}\left[T, \left(\mathrm{UT} - U^0\right) v(0)\right]$.

In the above formulation, $\underline{P}$ is the minimum power output [MW], SD is the shut-down ramp rate limit [MW/h], RU is the ramp-up rate limit [MW/h], SU is the start-up ramp rate limit [MW/h], RD is the ramp-down rate limit [MW/h], G is the number of intervals the unit must be initially on-line due to the minimum up time constraint [h], UT is the minimum up time [h], F is the number of intervals the unit must be initially off-line due to the minimum down time constraint [h], DT is the minimum down time [h], $s(0)$ is the number of periods the unit has been off-line at the beginning of the market horizon (end of hour 0) [h], and U^0 is the number of periods the unit has been on-line at the beginning of the market horizon (end of hour 0) [h].

Constraints (A8) and (A9) set the limits on the power output. Ramp rate limits (ramp-up, start-up, ramp-down and shut-down) are imposed by constraints (A10) and (A11). Constraints (A12)–(A14) and (A15)–(A17) enforce the minimum up and down time constraints respectively. Constraints (A18) and (A19) preserve the logic of the variables representing running, start-up, and shut-down status changes [40]. Finally, variables $z(t)$ are stated as binary in constraints (A20).

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Multimarket Optimal Bidding for a Power Producer

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***Abstract*—This paper considers a profit-maximizing thermal producer that participates in a sequence of spot markets, namely, day-ahead, automatic generation control (AGC), and balancing markets. The producer behaves as a price-taker in both the day-ahead market and the AGC market but as a potential price-maker in the volatile balancing market. The paper provides a stochastic programming methodology to determine the optimal bidding strategies for the day-ahead market. Uncertainty sources include prices for the day-ahead and AGC markets and balancing market linear price variations with the production of the thermal producer. Results from a realistic case study are reported and analyzed. Conclusions are duly drawn.**

***Index Terms*—Electricity spot markets, market power, optimal bidding strategies, stochastic programming.**

NOTATION

The notation used throughout the paper is reproduced below for quick reference.

Sets:

H Set of hours.
J Set of generating units.
K Set of scenarios.
M Set of blocks of the revenue function in the balancing market.

Numbers:

B Number of blocks of the revenue function in the balancing market.
N^e Number of scenarios for market e, where $e = \{$ D (day-ahead), G (AGC), A (balancing)$\}$.
N Total number of scenarios.

Constants:

C_j Variable operating cost for unit j (Euro/MWh).
P_{j0} Power output of unit j at the beginning of period 1 (MW).
P_j^{AGC} AGC capacity for unit j (MW).
$P_j^{\max}$ Capacity of unit j (MW).
$P_j^{\min}$ Minimum power output of unit j (MW).
R_j^{dw} Ramp-down limit for unit j (MW/h).
R_j^{up} Ramp-up limit for unit j (MW/h).
$T_h^{\max}$ Maximum energy to be traded in the balancing market in hour h (MWh).
$\lambda^{\max}$ Upper bound for random variable price (Euro/MWh).
π_k Probability of scenario k.

Random variables:

α_h Intercept of the inverse demand curve corresponding to hour h in the balancing market.
β_h Slope of the inverse demand curve corresponding to hour h in the balancing market.
λ_h^e Price in hour h and market e, where $e = \{$ D (day-ahead), G (AGC)$\}$.

It should be noted that a subscript k affecting any of the above variables indicates actual uncertainty realization k.

Real variables:

p_{jhk}^e Power of unit j in market $e(e = \{D, G, A\})$, hour h, and scenario k (MW).
q_{jhk}^e Energy produced by unit j in market e and scenario k during hour h (MWh).
t_{hk}^e Total energy production (of the generating company) in hour h, scenario k, and market $e(e = \{D, G, A\})$ (MWh).
t_{mhk}^A Block m of the total energy production of the generating company in the balancing market in hour h and scenario k (MWh).

Binary variables:

w_{jhk} It is equal to 1 if unit j is engaged in AGC in hour h and scenario k and 0 otherwise.

Functions:

$g_{hk}^A(\cdot)$ Revenues for hour h and scenario k in the balancing market (Euro).
$E_K(\cdot)$ Expected value over K.
$\Psi_k^e(\cdot)$ Recourse function associate to stage $e(e = \{D, G, A\})$ and scenario k.
$O(\cdot)$ Function that provides the price ranking (increasing value) for all the considered scenarios.

I. INTRODUCTION

WE consider a power producer that owns several generating units and participates in a pool-based market framework that includes three independent and successive markets: the day-ahead market, the automatic generation control (AGC) market, and the balancing market. We consider that the producer has no market power capability in the day-ahead and AGC markets, but it can influence price in the volatile balancing market. Not modeling market power in this market is unrealistic and leads to unreasonable results. The objective of the producer is to maximize its expected profits from selling energy in the day-ahead and balancing markets and AGC in the AGC market.

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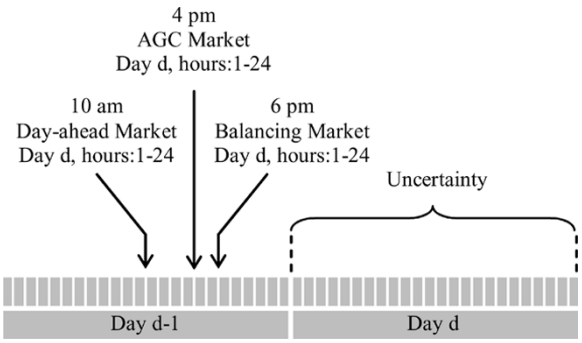

Fig. 1. Time sequence for market clearing.

For instance, in the electricity market of mainland Spain, AGC total revenues are typically 3% of the energy revenues, but prices range from 16 to 100 Euro/MW (2003) and there exist opportunities for profit spikes. The volume traded in the day-ahead market of mainland Spain in 2004 was Euro 5800 million and the volume negotiated in the AGC market was Euro 128 million. This amount represents only 2.2% of the energy traded in the day-ahead market but note that AGC is provided at almost no cost to the generating company.

The day-ahead market is cleared through an auction mechanism. That is, producers submit selling bids including energy production blocks and their corresponding minimum selling prices, while consumers submit buying bids consisting in energy consumption blocks and their corresponding maximum buying prices. In turn, the market operator clears the market using a suitable market-clearing procedure [1], [2].

The AGC and balancing markets are cleared using analogous auction procedures. For any of these three markets, market clearing provides hourly prices and accepted production and consumption bids.

The time framework for market clearing is as follows. The day-ahead market concerning the whole day d is cleared around 10 am of day $d-1$. Analogously, the AGC market concerning the whole day d is cleared around 4 pm of day $d-1$. The balancing market concerning the whole day d is cleared around 6 pm of day $d-1$. For these three markets, bids are sent to the market operator just before the closing (clearing) hour, and market-clearing results are known just after that closing hour. This time framework is illustrated in Fig. 1. It should be noted that additional balancing markets can be considered; however, they are generally less relevant than the first balancing market, and this is why they are not considered in this paper. The electricity market of mainland Spain motivates this modeling framework.

This paper provides a methodology that allows a producer to develop appropriate bidding strategies for the day-ahead market, thereby selling its production in the best possible manner. This methodology is based on stochastic programming [3].

It should be noted that the methodology developed can be straightforwardly applied to derive also bidding strategies for the AGC and balancing markets by moving the time window and repeating the procedure. That is, once the day-ahead market results are available, a two-stage stochastic programming problem can be solved to derive bidding stacks for the AGC market. Also, once the results for the AGC market are also available, a one-stage stochastic programming problem can be solved to derive bidding stacks for the balancing market.

TABLE I
STOCHASTIC PROCESSES FOR PRICES AND PRICE FUNCTIONS

Variable	Weekdays
λ_h^D	ARIMA(1,0,1)(2,0,1)$_{24}$
λ_h^G	ARIMA(5,0,1)(1,0,0)$_{24}$
$\alpha_h - \lambda_h^D$	ARIMA(1,0,2)(1,0,1)$_{24}$
β_h	ARIMA(2,0,1)(1,0,1)$_{24}$

Uncertainty sources include prices for the day-ahead and AGC markets and linear functions describing the way the market price in the balancing market changes with the total production of the producer. We characterize those uncertainties using stochastic models based on historical information.

We consider that self-scheduling is carried out on a weekly basis and therefore the on/off status of the units is known *a priori* at the time of constructing bidding stacks. Therefore, no start-up and shut-down decisions are considered in this paper.

Relevant references on stochastic programming and electric power are [4]–[8]. Within an electricity market framework, [9] models in detail the day-ahead market but not other markets, and [10] focuses on hydrodominated systems.

Background material on electricity markets can be found in [11]–[15], while background material for stochastic programming is covered in [3], [16], and [17].

This paper is organized as follows. Section II describes the stochastic programming optimization framework that allows deriving optimal bidding strategies for the day-ahead market. In Section III, the stochastic processes involving day-ahead market prices and AGC market prices are characterized. Additionally, the two processes that characterize linearly the price variation with production in the balancing market are characterized. Section IV is a detailed case study where results are reported and analyzed. Some relevant conclusions are drawn in Section V. Section A of the Appendix provides useful definitions, while Section B provides a description of the scenario reduction algorithm used in this paper.

II. DECISION FRAMEWORK

This section describes in detail the proposed methodology to derive bidding stacks for the day-ahead market.

A. Scenario Tree

Prices in the day-ahead market, prices in the AGC market, and price slopes and intercepts in the balancing market are characterized using seasonal ARIMA models. These ARIMA models are provided in Table I. Once those stochastic models have been identified, and their parameters properly fitted using time series data, they can be used to generate price scenarios using a tree format (see Fig. 2). The tree constitutes a discrete and finite approximation of the probability distribution of the stochastic process. The probability associated to a given scenario is the

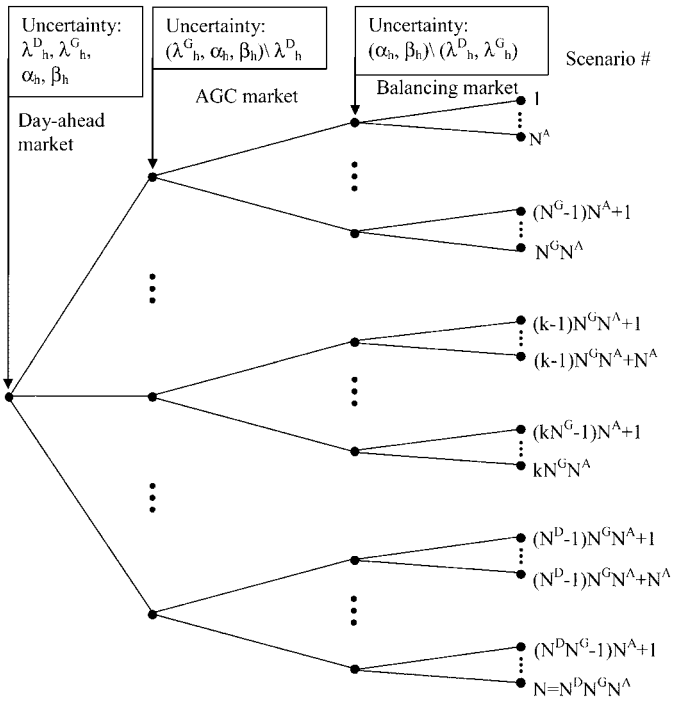

Fig. 2. Scenario tree.

product of the probabilities of occurrence of the prices that constitute the scenario. Note also that those stochastic models capture the dependency of prices across hours.

Therefore, the uncertainty framework is modeled through a symmetric scenario tree [8]. That is, different realizations of prices for the day-ahead market are considered. Then, for each realization of day-ahead market prices, different realizations of the AGC market prices are simulated. Finally, for each realization of the AGC market, different realizations of the two parameters describing the price-production variation of the balancing market are considered.

The number of scenarios for the day ahead market is N^D (typically 100) and for the AGC and balancing markets, N^G (typically 100) and N^A (typically 100), respectively. The total number of scenarios is therefore $N = N^D N^G N^A$ (typically 10^6). The scenario tree considered is shown in Fig. 2.

The scenario tree represented in Fig. 2 can be interpreted as a multistage stochastic process $\xi = \{\xi_e\}_{e=\{D,G,A\}}$, whose probability distribution P is approximated through a finite number of scenarios $\xi^k = \{\xi_e^k\}_{e=\{D,G,A\}}, k = 1, \ldots, N$ with probability p_k, $\sum_{k=1}^{N} p_k = 1$. A scenario reduction technique intended to reduce drastically the number of scenarios while maintaining the statistic properties of the considered stochastic process is carried out [18]. Through this reduction, a subset of scenarios $\{k^{[1]}, \ldots, k^{[\ell]}, \ldots, k^{[L]}, L \leq N\}$ denoted by $K^{[L]}$ is selected, such that their corresponding probability distribution $P^{[L]}$ is the closest to the original distribution in terms of a certain probability distance between P and $P^{[L]}$.

The technique used to reduce the number of scenarios determines a scenario subset of prescribed cardinality and assigns new probabilities to the preserved scenarios, such that their corresponding probability distribution is the closest to the original distribution in terms of the following probability distance

$$D(P, P^{[L]}) = \sum_{\tilde{k} \in K \setminus K^{[L]}} p_{\tilde{k}} \min_{k \in K^{[L]}} \sum_{e} \left| \xi_e^k - \xi_e^{\tilde{k}} \right| \quad (1)$$

where $e = \{D, G, A\}$ and $|\cdot|$ represents a norm in $\Re^n$. Note that the method proposed in this paper uses the scenario-based approximation, which means discrete probability distributions, and considers scenario probabilities as elements of a linear space. Section B of the Appendix provides a heuristic algorithm that approaches the optimal selection of the subset $K^{[L]}$ and assigns new probabilities to the preserved scenarios. Further details can be found in [18]–[20].

For the problem considered in this paper, the scenario reduction technique is first applied to each stage, resulting in N_{red}^D, N_{red}^G, and N_{red}^A scenarios, respectively, and then to the resulting tree comprising $N_{\text{red}}^D N_{\text{red}}^G N_{\text{red}}^A$ scenarios. This second reduction results in a small number (around 25) of scenarios. For computing expression (1), the quadratic norm 2 is used.

B. Optimization Framework

The mathematical programming models to be solved to determine the optimal bidding strategies for the power producer in the different spot markets are stated below.

The day-ahead bidding strategy is determined through the following stochastic programming model:

$$\begin{aligned} &\text{Maximize} \quad p_{jhk}^D, q_{jhk}^D, t_{hk}^D \\ &E_K \left\{ \sum_h \left(t_{hk}^D \lambda_{hk}^D - \sum_j C_j q_{jhk}^D \right) \right. \\ &\qquad \left. + E_{K \setminus \lambda_{hk}^D} \left\{ \Psi_k^G \left(p_{jhk}^D, q_{jhk}^D, t_{hk}^D \right) \right\} \right\} \end{aligned} \quad (2)$$

subject to

$$q_{jhk}^D = \frac{1}{2} \left(p_{j(h-1)k}^D + p_{jhk}^D \right) \ \forall j, \ \forall h, \ \forall k \quad (3)$$

$$t_{hk}^D = \sum_j q_{jhk}^D \ \forall h, \ \forall k \quad (4)$$

$$q_{jhk}^D - q_{jh\tilde{k}}^D \leq 0 \ \forall k, \tilde{k} : \left(O\left(\lambda_{hk}^D\right) + 1 = O\left(\lambda_{h\tilde{k}}^D\right) \right), \forall h, \ \forall j \quad (5)$$

$$p_{jhk}^D = p_{jh\tilde{k}}^D \ \forall k, \tilde{k} : \left(\lambda_{hk}^D = \lambda_{h\tilde{k}}^D \ \forall h \right), \ \forall j. \quad (6)$$

The objective function (2) includes revenues obtained from the day-ahead market plus expected revenues from AGC and balancing markets. These expected revenues are computed through the problem (7)–(12) below. The expectation operator is taken over AGC and balancing random variables.

Equation (3) defines energies as average values of powers for every scenario. Equation (4) determines total hourly productions for each scenario. Equation (5) is explained below. Equation (6) is an nonanticipativity constraints.

The purpose of the technique proposed in this paper is not to compute the optimal quantities (in a stochastic programming sense) to be sold in the day-ahead market (e.g., 40 MWh in hour

1, 25 MWh in hour 2, etc.) but rather to derive optimal bidding curves for every hour of the day-ahead market (see Figs. 6 and 7 in Section IV). Therefore, to obtain such curves, variables $\left(p^D_{jhk}, q^D_{jhk}, t^D_{hk}\right)$ are made dependent on scenarios [9] and (5) are introduced to ensure higher productions at higher prices. These conditions make bidding curves nondecreasing, which is a bidding requirement in most markets. In order to obtain the optimal quantity to be sold in the day-ahead market in a standard stochastic programming framework, (6) should be replaced by the nonanticipativity constraint $p^D_{jhk} = p^D_{jh\tilde{k}}\ \forall\, k, \tilde{k} \in K$. It should be noted that this last constraint is more restrictive than constraint (6) because it enforces nonanticipativity for all markets, while (6) only enforces nonanticipativity for the AGC and balancing markets.

The AGC market strategy is determined through the problem below.

$$\begin{aligned}&\text{Maximize}\\ &\quad p^G_{jhk}, q^G_{jhk}, t^G_{hk}, w_{jhk}\\ &\sum_h t^G_{hk}\lambda^G_{hk} + E_{K\setminus(\lambda^D_{hk},\lambda^G_{hk})}\\ &\qquad \left\{\Psi^A_k\left(p^D_{jhk}, q^D_{jhk}, t^D_{hk}, p^G_{jhk}, q^G_{jhk}, t^G_{hk}, w_{jhk}\right)\right\}\end{aligned} \tag{7}$$

subject to

$$q^G_{jhk} = \frac{1}{2}\left(p^G_{j(h-1)k} + p^G_{jhk}\right)\ \ \forall\, j,\ \forall\, h,\ \forall\, k \tag{8}$$

$$q^G_{jhk} \le w_{jhk} P^{AGC}_j\ \ \forall\, j,\ \forall\, h,\ \forall\, k \tag{9}$$

$$t^G_{hk} = \sum_j q^G_{jhk}\ \ \forall\, h,\ \forall\, k \tag{10}$$

$$p^G_{jhk} = p^G_{jh\tilde{k}}\ \ \forall\, k, \tilde{k} : \left(\left(\lambda^D_{hk}, \lambda^G_{hk}\right) = \left(\lambda^D_{h\tilde{k}}, \lambda^G_{h\tilde{k}}\right)\ \forall\, h\right),\ \forall\, j \tag{11}$$

$$w_{jhk} \in \{0,1\}\ \ \forall\, j,\ \forall\, h,\ \forall\, k. \tag{12}$$

The objective function (7) includes revenues obtained from the AGC market plus expected revenues from the balancing market. These expected revenues are computed through the problem (13)–(22) below. The expectation operator is taken over the random variables of the balancing market.

Equation (8) expresses energy values as the average of the powers at the beginning and the end of any considered hour and scenario. Equation (9) limits the amount of power devoted to AGC during any hour and scenario by any generating unit. Equation (10) expresses the total energy associated to AGC activity for every hour and scenario. Equation (11) is an nonanticipativity constraint. Equation (12) declares binary variables that define which units are in AGC operation.

Note that using a binary variable per unit is necessary to differentiate whether the unit is contributing to AGC or ramping. These binary variables complicate the solution of the problem, but they are needed to reflect the actual functioning of generating units. It should be noted that a machine providing AGC cannot ramp up or down as its control is transferred to the ISO. This is common practice in the electricity market of mainland Spain, as stated in the corresponding operating policy enforced by the ISO and reported in [21]. Moreover, the gaps between the power output of a machine providing AGC and its upper and lower bounds, respectively, must allow both up and down control.

Similarly to the day-ahead market, it should be noted that variables $\left(p^G_{jhk}, q^G_{jhk}, t^G_{hk}\right)$ are made dependent on scenarios, which is not the case in a standard stochastic programming framework. This is consistent with using a two-stage model (AGC and balancing markets) similar to the one presented in this paper to generate AGC bidding curves. Equation (11) enforces nonanticipativity only with respect to the balancing market. In a conventional stochastic programming approach, nonanticipativity constraints would enforce nonanticipativity with respect to both the AGC and balancing markets.

The balancing market strategy is determined through the problem below.

$$\begin{aligned}&\text{Maximize}\quad p^A_{jhk}, q^A_{jhk}, t^A_{hk}\\ &\sum_h\left(g^A_{hk} - \sum_j C_j q^A_{jhk}\right)\end{aligned} \tag{13}$$

subject to

$$q^D_{jhk} + q^A_{jhk} + \frac{1}{2}q^G_{jhk} \le P^{\max}_j\ \ \forall\, j,\ \forall\, h,\ \forall\, k \tag{14}$$

$$q^D_{jhk} + q^A_{jhk} - \frac{1}{2}q^G_{jhk} \ge P^{\min}_j\ \ \forall\, j,\ \forall\, h,\ \forall\, k \tag{15}$$

$$p^D_{jhk} + p^A_{jhk} \le P^{\max}_j\ \ \forall\, j,\ \forall\, h,\ \forall\, k \tag{16}$$

$$p^D_{jhk} + p^A_{jhk} \ge P^{\min}_j\ \ \forall\, j,\ \forall\, h,\ \forall\, k \tag{17}$$

$$p^D_{jhk} + p^A_{jhk} - \left(p^D_{j(h-1)k} + p^A_{j(h-1)k}\right) \le (1 - w_{jhk})R^{up}_j\ \ \forall\, j,\ \forall\, h,\ \forall\, k \tag{18}$$

$$p^D_{j(h-1)k} + p^A_{j(h-1)k} - \left(p^D_{jhk} + p^A_{jhk}\right) \le (1 - w_{jhk})R^{dw}_j\ \ \forall\, j,\ \forall\, h,\ \forall\, k \tag{19}$$

$$q^A_{jhk} = \frac{1}{2}\left(p^A_{j(h-1)k} + p^A_{jhk}\right)\ \ \forall\, j,\ \forall\, h,\ \forall\, k \tag{20}$$

$$t^A_{hk} = \sum_j q^A_{jhk}\ \ \forall\, h,\ \forall\, k \tag{21}$$

$$g^A_{hk} = t^A_{hk}\left(\alpha_{hk} + \beta_{hk} t^A_{hk}\right)\ \ \forall\, h,\ \forall\, k. \tag{22}$$

The objective function (13) is the revenue obtained from the balancing market.

Equation (14)–(17) sets the maximum (capacity) and minimum power output of any generating unit at any hour and scenario, including AGC activity (if any). Equation (18)–(19) enforces, respectively, up and down ramping limits for generating units not engaged in AGC for every scenario. Equation (20) expresses energy values as an average of the powers at the beginning and the end of any considered hour for every scenario. Equation (21) expresses the total energy associated to the balancing market in every hour and scenario. Equation (22) expresses the balancing market revenue for each hour and scenario as a quadratic concave function. This function is linearized below.

It should be noted that variables $\left(p^A_{jhk}, q^A_{jhk}, t^A_{hk}\right)$ are made dependent on scenarios to achieve bidding curves, which is not the case in a standard stochastic programming framework. No nonanticipativity constraint is required in this last stage.

Note also that the variables q can be eliminated from the models above using (3), (8), and (20), thus reducing the number of variables required in the formulation.

C. Equivalent Formulation as a Mathematical Program With Recourse

The above stochastic programming problem is equivalent to the deterministic one stated below (mathematical program with recourse). This version is readily solvable using an appropriate mixed-integer linear programming solver.

$$\begin{aligned}&\text{maximize}\\&\sum_k\left[\sum_h\left(t^D_{hk}\lambda^D_{hk}-\sum_j C_j q^D_{jhk}\right)\right]\pi_k\\&\quad+\sum_k\left(\sum_h t^G_{hk}\lambda^G_{hk}\right)\pi_k\\&\quad+\sum_k\left[\sum_h\left(g^A_{hk}-\sum_j C_j q^A_{jhk}\right)\right]\pi_k.\end{aligned}\tag{23}$$

Conditions that ensure a monotonously increasing bidding stack for the day-ahead market are

$$q^D_{jhk}-q^D_{jh\tilde{k}}\le 0\,\forall\,k,\tilde{k}:\left(O\left(\lambda^D_{hk}\right)+1=O\left(\lambda^D_{h\tilde{k}}\right)\right)\forall\,h\,,\ \forall\,j.\tag{24}$$

Nonanticipativity constraints for the day-ahead market are

$$p^D_{jhk}=p^D_{jh\tilde{k}}\ \ \forall\,k,\tilde{k}:\left(\lambda^D_{hk}=\lambda^D_{h\tilde{k}}\,\forall\,h\right)\ \ \forall\,j.\tag{25}$$

Nonanticipativity constraints for the AGC market are

$$\begin{aligned}&p^G_{jhk}=p^G_{jh\tilde{k}}\\&\quad\forall\,k,\ \tilde{k}:\left(\left(\lambda^D_{hk},\lambda^G_{hk}\right)=\left(\lambda^D_{h\tilde{k}},\lambda^G_{h\tilde{k}}\right)\ \ \forall\,h\right)\ \ \forall\,j.\end{aligned}\tag{26}$$

Energy balance in the spot (day-ahead, AGC, and balancing) markets are

$$t^e_{hk}=\sum_j q^e_{jhk}\ \ \forall\,h,\ \ \forall\,k,e\in\{D,G,A\}\tag{27}$$

$$\begin{aligned}q^e_{jhk}=&\frac{1}{2}\left(p^e_{j(h-1)k}+p^e_{jhk}\right)\ \ \forall\,j,\ \ \forall\,h,\ \ \forall\,k,\\&e\in\{D,G,A\}.\end{aligned}\tag{28}$$

Generating unit functioning constraints are

$$q^D_{jhk}+q^A_{jhk}+\frac{1}{2}q^G_{jhk}\le P^{\max}_j\ \ \forall\,j,\ \ \forall\,h,\ \ \forall\,k\tag{29}$$

$$q^D_{jhk}+q^A_{jhk}-\frac{1}{2}q^G_{jhk}\ge P^{\min}_j\ \ \forall\,j,\ \ \forall\,h,\ \ \forall\,k\tag{30}$$

$$p^D_{jhk}+p^A_{jhk}\le P^{\max}_j\ \ \forall\,j,\ \ \forall\,h,\ \ \forall\,k\tag{31}$$

$$p^D_{jhk}+p^A_{jhk}\ge P^{\min}_j\ \ \forall\,j,\ \ \forall\,h,\ \ \forall\,k\tag{32}$$

$$q^G_{jhk}\le w_{jhk}P^{AGC}_j\ \ \forall\,j,\ \ \forall\,h,\ \ \forall\,k\tag{33}$$

$$\begin{aligned}&p^D_{jhk}+p^A_{jhk}-\left(p^D_{j(h-1)k}+p^A_{j(h-1)k}\right)\\&\quad\le(1-w_{jhk})R^{up}_j\ \ \forall\,j,\ \ \forall\,h,\ \ \forall\,k\end{aligned}\tag{34}$$

$$\begin{aligned}&p^D_{j(h-1)k}+p^A_{j(h-1)k}-\left(p^D_{jhk}+p^A_{jhk}\right)\\&\quad\le(1-w_{jhk})R^{dw}_j\ \ \forall\,j,\ \ \forall\,h,\ \ \forall\,k\end{aligned}\tag{35}$$

$$w_{jhk}\in\{0,1\}\ \ \forall\,j,\ \ \forall\,h,\ \ \forall\,k.\tag{36}$$

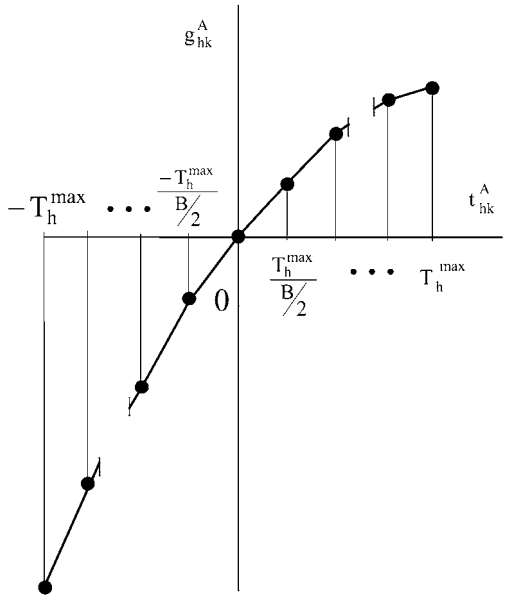

Fig. 3. Illustration of the linearization provided by (37)–(40).

The piece-wise linear (concave) representation of revenues in the balancing market is achieved through the following constraints:

$$g^A_{hk}=\sum_{m=1}^{B}\left(\alpha_{hk}+\frac{2T^{\max}_h(2m-(B+1))\beta_{hk}}{B}\right)t^A_{mhk}\quad\forall\,h,\ \ \forall\,k\tag{37}$$

$$\frac{-T^{\max}_h}{\frac{B}{2}}\le t^A_{mhk}\le 0,\ \ \forall\,m\le\frac{B}{2},\ \ \forall\,h,\ \ \forall\,k\tag{38}$$

$$0\le t^A_{mhk}\le\frac{T^{\max}_h}{\frac{B}{2}},\ \ \forall\,m>\frac{B}{2},\ \ \forall\,h,\ \ \forall\,k\tag{39}$$

$$t^A_{hk}=\sum_{m=1}^{B}t^A_{mhk},\ \ \forall t^A_{hk}\in\left(-T^{\max}_h,T^{\max}_h\right),\ \ \forall\,h,\ \ \forall\,k.\tag{40}$$

It should be noted that (37)–(40) constitute a straightforward linearization of (22). Fig. 3 illustrates the piece-wise concave linearization carried out through (37)–(40).

It should also be noted that we have experienced better numerical performance using a linear model than a quadratic one.

D. Solution Procedure

The problem formulated in the previous section is mixed-integer and linear and can be solved using an appropriate mixed-integer linear programming solver, such as CPLEX [22]. This procedure is used in the case study below.

III. Characterization of the Stochastic Processes

Market-clearing prices for the day-ahead and the AGC markets, respectively, are characterized through stochastic processes. Only weekdays are considered. The treatment for weekends is similar and is not reported below.

Analogously, the intercept and slope that describe the participation of the power producer in the balancing market are characterized using stochastic processes. To enhance numerical perfor-

TABLE II
CHARACTERISTICS OF THE THERMAL UNITS

Type	C_j^F (Euro/h)	C_j (Euro / MWh)	P_j^{max} (MW)	P_j^{min} (MW)	P_j^{AGC} (MW)	R_j^{up} (MW/h)	R_j^{dw} (MW/h)
Coal_1	126.0	19.81	140	75	20	65	65
CCGT_1	1097.2	25.17	380	160	50	220	220
CCGT_2	992.8	25.51	390	180	50	210	210
Coal_2	575.0	29.37	500	250	40	250	250
Oil_1	91.5	37.91	50	25	0	25	25
Oil_2	1800.0	33.91	300	200	20	100	100

mance, instead of characterizing the hourly intercept, the difference between this intercept and the day-ahead market clearing price is characterized.

It should be noted that the above stochastic processes should generate an adequate number of scenarios, which properly covers the spectrum of possible prices.

All stochastic processes are modeled using seasonal ARIMA models [23]–[25]. The fitted ARIMA models for the week considered in the case study are provided in Table I. Model fitting is ensured using the autocorrelation and partial autocorrelation functions of the error.

Historical data comprising 30 working days are used to fit the models in Table I.

IV. CASE STUDY

Results from a case study based on the electricity market of mainland Spain [26] are reported in this section. The considered market framework includes the day-ahead market, the AGC market, and the first balancing market [26]. The week under study is June 7–11, 2004 (just working days) in the electricity market of mainland Spain.

It should be noted that in the electricity market of mainland Spain, no virtual bidding is allowed. Settled transactions constitute firm obligations.

The considered producer includes six thermal units whose characteristics are provided in Table II.

Table II includes fixed costs (C_j^F). In order to compute daily profit, the constant $24 \times \sum_j C_j^F$ should be subtracted from the revenue of the producer.

The numbers of scenarios for the day-ahead, AGC, and balancing markets are, respectively, 100, 100, and 100. The total number of scenarios is therefore 10^6. The scenario reduction technique is first applied to each stage, resulting in 25, 10, and 10 scenarios, respectively, and then to the resulting tree comprising 2500 scenarios. This second reduction results in just 25 scenarios. Fig. 4 provides a measure of the relative distance between the tree comprising 2500 scenarios and trees containing a reduced number of scenarios for the case of June 7, 2004. The relative distance is defined as the distance measured between the reduced tree and the original tree, divided by the distance measured between a reduced tree that considers just one scenario and the original tree. For this case study, a tree with 25 scenarios provides a reasonable approximation as can be deduced from Fig. 5, which shows how the optimal objective function value changes as the number of scenarios increases.

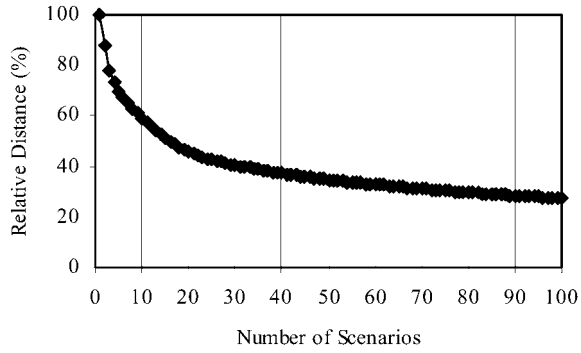

Fig. 4. Relative distance of the original tree to reduced scenario trees.

Fig. 5. Objective function value as a function of the number of scenarios considered.

TABLE III
SOLUTION TIME AND QUALITY

Monday, June 7, 2004	
CPU time (s)	100
Absolute gap (Euro)	834
Relative gap (%)	0.44

We have also analyzed in detail the best use of Benders decomposition to address the problem of interest, but our implementations have only produced frustrating results due to the size of the master problem. We decided not to try Lagrangian techniques because of their lack of robustness associated with their required tune-up of parameters.

The number of constraints, real variables, and binary variables in the resulting problem are, respectively, 44 833, 22 201, and 3600. It has been solved using CPLEX under GAMS [22]. The computational characteristics of the solution attained are illustrated in Table III. The CPU time needed to attain the solution using a Dell PowerEdge 6600 with two processors at 1.60 GHz and 2 Gb of RAM memory is 100 s.

The optimal profit values attained (for the three markets) using alternative approaches are reported in Table IV.

To obtain a measure of quality for the results attained if the technique proposed would have been applied to the actual electricity market situation of mainland Spain on June 7, 2004, and to compare these results with those obtained using other methods, the solutions below are considered.

Stochastic solution: Once day-ahead market bidding curves are obtained using the proposed model, energies for the day-ahead market are fixed considering actual price values (June 7,

TABLE IV
PROFIT (EURO THOUSAND)

Monday, June 7, 2004	
Stochastic	487.0
Deterministic	481.8
Classic	482.2

TABLE V
STOCHASTIC PROGRAMMING INDICATORS (EURO THOUSAND)

Monday, June 7, 2004	
RP	189.6
EEV	185.6
VSS	4.0

2004) for that market. Then, the proposed model is run considering the fixed energy values for the day-ahead market and only one scenario for the AGC and balancing markets comprising the actual price values (June 7, 2004). The result obtained constitutes a measure of the maximum benefit that the company can achieve in all markets if bidding is carried out according with the proposed model.

Deterministic solution: Using the expected values of all stochastic variables, the resulting deterministic model is run and optimal values for energy in the day-ahead market are determined. Then, energy for the day-ahead market is fixed considering the solution provided by the deterministic model and the proposed (stochastic) model is executed considering only one scenario comprising the actual price values (June 7, 2004) for the AGC and balancing markets. The result obtained constitutes a measure of the maximum benefit that the company can achieve in all markets if bidding is carried out using a deterministic model.

Classic solution: Using the expected values of the stochastic variables, a one-stage deterministic model comprising the day-ahead market is run and optimal values for energy in the day-ahead market are determined. Then, energy for the day-ahead market is fixed considering the solution provided by the classic model and the proposed (stochastic) model is executed considering only one scenario comprising the actual price values (June 7, 2004) for the AGC and balancing markets. The result obtained constitutes a measure of the maximum benefit that the company can achieve in all markets if bidding is carried out using a one-stage deterministic model.

Note that the difference between the deterministic and classic models is that the first model considers three stages to provide a solution for the day-ahead market while the second model only considers the day-ahead (one-stage) market.

Observe the profit advantage obtained by using a stochastic approach. The above comparison provides an appropriate measure of how the stochastic model used reflects the real world.

Several stochastic programming indicators are reported in Table V (as formulated in [3]). Once an optimal deterministic solution is obtained for the deterministic problem resulting from substituting all random variables for their corresponding expected values (EV solution), the expected result of using the EV solution (EEV indicator) is computed as the average value over scenarios of the objective function values corresponding to the optimal deterministic solution of the day-ahead market and all scenarios. Indicator RP is computed as the average value over scenarios of the objective function values corresponding to the optimal stochastic solution of the day-ahead market and all scenarios. Indicator VSS is computed as RP minus EEV. Note that VSS is a measure of the advantage obtained if a stochastic model is used rather than a naïve deterministic one. This indicator shows that the solution obtained is of a high quality from a stochastic programming viewpoint, and its value shows that it would be possible to attain additional savings of Euro 4000 by taking into account a stochastic solution to the problem.

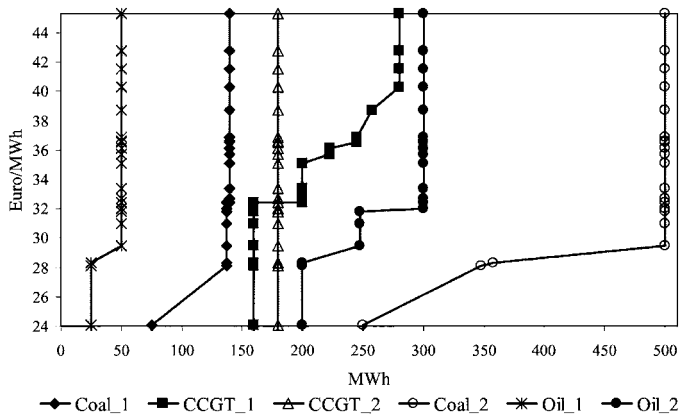

Fig. 6. Bidding stack at period 10 (9 A.M. to 10 A.M.) for the day-ahead market.

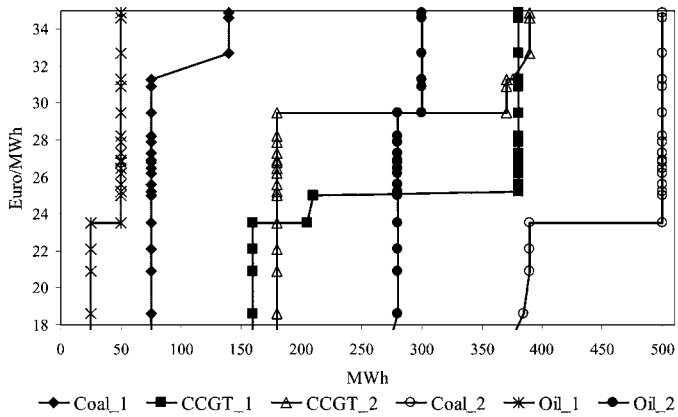

Fig. 7. Bidding stack at period 5 (4 A.M. to 5 A.M.) for the day-ahead market.

Fig. 5 shows how the objective function optimal value [(23)] changes as the number or scenarios increases. Note that this value coincides with indicator RP. Observe that a robust solution is attained for 25 or more scenarios and that the scenario reduction illustrated in Fig. 4 is appropriate. It should be noted that a critical issue when using a stochastic programming approach based on scenarios is the actual number of scenarios to consider. A low number of scenarios might result in inaccuracies while a large number implies high computational burden. An appropriate manner to resolve this tradeoff is to increase the number of scenarios until the objective function value stabilizes. This is the criterion used in this case study.

Figs. 6 and 7 provide the optimal bidding stacks for the day-ahead market at period 10 (from 9 am to 10 am) and 5 (4 am to 5 am), respectively. Bidding stacks for other hours are similar. It should be noted that for a market requiring step-wise bidding stacks (the case of the market in mainland Spain) the linear

TABLE VI
CHARACTERISTICS OF THE SOLUTION. JUNE 7–11, 2004
(FIGURES IN EURO THOUSAND)

June 7-11, 2004	7	8	9	10	11	Total
Stochastic	487.0	367.2	96.9	53.2	293.0	1297.2
Deterministic	481.8	358.0	97.9	56.7	291.0	1285.4
Classic	482.2	359.5	97.9	55.4	295.1	1290.2
RP	189.5	414.0	89.6	5.8	13.9	712.9
EEV	185.6	413.2	85.6	0.5	9.6	694.5
VSS	4.0	0.8	4.0	5.3	4.3	18.4

blocks of the stacks in Figs. 6 and 7 should be approximated by step-wise functions.

For instance, the solution obtained for period 5 (see Fig. 7) results in 85 MWh bought in the balancing market (average value for the solution obtained over the considered 25 scenarios). This value becomes 356 MWh if the same model is run without considering market power in the balancing market. Using the actual price quantity function for period 5, the actual payment is Euro 7798, while not considering market power would result in a payment of Euro 6657. Note the significant differences in energy traded and payments.

Only bidding stacks for the day-ahead market are reported in the paper, as the paper concentrates on the decisions pertaining to that market. Note that in multistage sequential markets, decisions in subsequent markets are made once the actual values of the previous-stage stochastic variables are known.

The solution obtained using the deterministic model for period 5 results in the three cheapest units plus one oil unit working close to their respective upper limits (Coal_1, CCGT_1, CCGT_2, and Oil_1). This solution implies buying back energy in the balancing market since the expected price for this market is comparatively low. However, the actual prices for period 5 in both the day-ahead and balancing markets are lower than the variable cost of most of the units, but buying back in the balancing market the whole surplus of energy is unprofitable due to the influence that this energy would have on the resulting price in the balancing market. On the other hand, considering the bidding curves provided by the stochastic model, the energy sold for the actual day-ahead price in period 5 of 20.4 Euro/MWh is smaller than the energy sold using the deterministic model. Consequently, one concludes that the stochastic solution is a better solution in this particular case.

The whole procedure is repeated for June 7–June 11, 2004 (Monday to Friday). Results (for the three markets) are reported in Table VI, where the advantage of using a multistage stochastic programming approach as the one presented in this paper should be readily apparent. For the working days of the week under study, 0.91% profit advantage (Euro 11.8 thousand) is obtained with respect to the deterministic approach and 0.54% (Euro 7.0 thousand) with respect to the classic one. Indicators EEV, RP, and VSS clearly show the advantage of using a multistage stochastic programming approach. Indicator VSS is Euro 18.4 thousand, a significant amount of money.

It can be concluded from Table VI that a stochastic approach clearly outperforms the deterministic and classic ones. Only on June 9 and 10, when forecast price values are close to actual prices (periods 49 to 96 in Fig. 8), is a deterministic approach more advantageous.

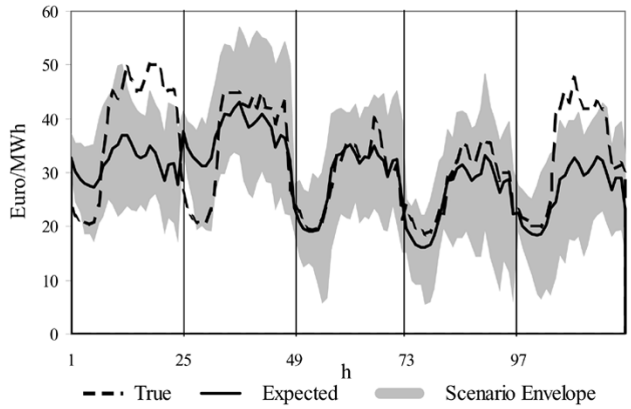

Fig. 8. True and forecast prices and price scenario envelope. June 7–11, 2004.

V. CONCLUSIONS

This paper provides a procedure to derive optimal bidding stacks for a power producer that participates in the spot electricity markets, including day-ahead, AGC, and balancing. The objective of the producer is to maximize the expected profit from its involvement in the spot markets. The proposed procedure targets a producer with no market power in the day-ahead and AGC markets but with the capability of influencing market-clearing prices in the volatile balancing market. The proposed technique is built within the versatile decision framework provided by the stochastic programming methodology. A robust yet efficient solution technique is used. A real-world case study based on the electricity market of mainland Spain is used to illustrate the appropriate functioning of the procedure developed.

APPENDIX

This Appendix provides definitions for acronyms and some specific stochastic programming expressions and a mathematical description of the scenario reduction algorithm used [18].

A. Acronyms and Other Definitions

1) ARIMA: Stochastic model to characterize price behavior.
2) RP: Indicator RP is computed as the average value over scenarios of the objective function values corresponding to the optimal stochastic solution of the day-ahead market and all scenarios.
3) EEV: Indicator EEV is computed as the average value over scenarios of the objective function values corresponding to the optimal deterministic solution of the day-ahead market and all scenarios.
4) VSS: Indicator VSS is computed as RP minus EEV.
5) Nonanticipativity: Nonanticipativity conditions enforce that decisions depend only on temporal information up to the time at which decisions are made.
6) Relative distance: The distance measured between the reduced tree and the original tree, divided by the distance measured between a reduced tree that considers just one scenario and the original tree.

B. Scenario Reduction Algorithm

The optimal choice of an index set $K^{[L]}$ for scenario reduction problem with fixed cardinality $\#K^{[L]}$ is given by the solution of the problem

$$\text{Minimize} \quad \{D(P, P^{[L]}) : \#K^{[L]} = L\}.$$

Since this problem is difficult to solve, a heuristic algorithm that approaches the optimal selection of the subset $K^{[L]}$ and assigns new probabilities to the preserved scenarios has been developed in [19]. This algorithm is reproduced below for the reader's convenience. Further details can be found in [18]–[20]. Note that the expression $D(P, P^{[L]})$ denotes the value $\min_{k \in K \backslash K^{[\ell-1]}} D_k^{[L]}$ computed by the algorithm at step L.

The algorithm proceeds as follows.

Step 1)
Compute

$$d_{\tilde{k}k}^{[1]} = \sum_e \left| \xi_e^k - \xi_e^{\tilde{k}} \right| \ \ \forall \ \tilde{k}, \ \ k \in \{1, \ldots, N\}$$

and

$$D_k^{[1]} = \sum_{\tilde{k} \neq k} \pi_{\tilde{k}} d_{\tilde{k}k}^{[1]} \ \ \forall \ k \in \{1, \ldots, N\}.$$

Set

$$k^{[1]} : k^{[1]} \in \arg \min_{k \in \{1, \ldots, N\}} D_k^{[1]}$$

and

$$K^{[1]} = \{k^{[1]}\}.$$

Step ℓ)
Compute

$$d_{\tilde{k}k}^{[\ell]} = \min \left\{ d_{\tilde{k}k}^{[\ell-1]}, d_{\tilde{k}k^{[\ell-1]}}^{[\ell-1]} \right\} \ \ \forall \ \tilde{k}, \ \ k \in K \backslash K^{[\ell-1]}$$

and

$$D_k^{[\ell]} = \sum_{\tilde{k} \in K \backslash \{K^{[\ell-1]} \cup k\}} \pi_{\tilde{k}} d_{\tilde{k}k}^{[\ell]} \ \ \forall \ k \in \{1, \ldots, N\}.$$

Set

$$k^{[\ell]} : k^{[\ell]} \in \arg \min_{k \in K \backslash K^{[\ell-1]}} D_k^{[\ell]}$$

and

$$K^{[\ell]} = \{K^{[\ell-1]} \cup k^{[\ell]}\}$$

Step L)
Compute

$$d_{\tilde{k}k}^{[L]} = \min \left\{ d_{\tilde{k}k}^{[L-1]}, d_{\tilde{k}k^{[L-1]}}^{[L-1]} \right\} \ \ \forall \ \tilde{k}, \ \ k \in K \backslash K^{[L-1]}$$

and

$$D_k^{[L]} = \sum_{\tilde{k} \in K \backslash \{K^{[L-1]} \cup k\}} \pi_{\tilde{k}} d_{\tilde{k}k}^{[L]} \ \ \forall \ k \in \{1, \ldots, N\}.$$

Set

$$k^{[L]} : k^{[L]} \in \arg \min_{k \in K \backslash K^{[L-1]}} D_k^{[L]}$$

and

$$K^{[L]} = K^{[L-1]} \cup k^{[L]}.$$

The subset $\{k^{[1]}, \ldots, k^{[\ell]}, \ldots, k^{[L]}\}$ constitutes an approximation to the optimal selection of the set $K^{[L]}$ for a scenario reduction problem with fixed cardinality $\#K^{[L]}$.
Step $L+1$)
For every scenario $k \in K \backslash K^{[L]}$, compute

$$\text{closestto}(k) = k^{[\ell]} : k^{[\ell]} \in \arg \min_{k^{[\ell]} \in k^{[L]}} \sum_e \left| \xi_e^{k^{[\ell]}} - \xi_e^k \right|.$$

Set the resulting probability of the scenarios included in $K^{[L]}$ as

$$\pi_{k^{[\ell]}}^{[L]} = \pi_{k^{[\ell]}} + \sum_{k \in K \backslash K^{[L]} : \text{closestto}(k) = k^{[\ell]}} \pi_k.$$

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