NONLINEAR AND NONLOCAL DEGENERATE DIFFUSIONS ON BOUNDED DOMAINS

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We study quantitative properties of nonnegative solutions to a nonlinear and nonlocal diffusion equation posed in a bounded domain, with appropriate homogeneous Dirichlet boundary conditions. The diffusion is driven by a linear operator in a quite general class, that includes the three most common versions of the fractional Laplacians on a bounded domain with zero Dirichlet boundary conditions, as well as many other examples. The nonlinearity is allowed to be degenerate, the prototype being $|u|^{m-1}u$, with $m>1$

We will shortly present some recent results about existence, uniqueness and a priori estimates for a quite large class of very weak solutions, that we call weak dual solutions.

Then we will concentrate on the regularity theory: decay and positivity, boundary behavior, Harnack inequalities, interior and boundary regularity, and asymptotic behavior. All this is done in a quantitative way, based on sharp a priori estimates. Although our focus is on the fractional models, our techniques cover also the local case $s = 1$ and provide new results even in this setting. A surprising instance of this problem is the possible presence of nonmatching powers for the boundary behavior: this unexpected phenomenon is a completely new feature of the nonlocal nonlinear structure of this model, and it is not present in the semilinear elliptic case, for which we will shortly present the most recent results.

The above results are contained on a series of recent papers in collaboration with A. Figalli, Y. Sire, X. Ros-Oton and J. L. Vazquez.