Title: Interpolation by analytic functions in Sobolev spaces.

Abstract. Interpolation problems by analytic functions is a subject of more than a century old that is rich of interesting and deep results but also continues to stimulate new research. Interpolation has come a long way since the fundamental papers of Pick [5] and Nevanlinna [3, 4], but the spirit of the problems is invariant. One is given a subset of analytic functions in the unit disc $\varepsilon \subset \mathcal{O}(\mathbb{D})$, a sequence (finite of infinite) $\{z_i\} \subset \mathbb{D}$ of *interpolating nodes* and a *target space* $\mathcal{X}$, i.e. a set of sequences to be interpolated. The problem is then to determine whether or not for any data $\{w_i\} \in \mathcal{X}$ there exists a function $f \in \varepsilon$ such that

$$f(z_i) = w_i, \ \ \forall i.$$

Carleson's [2] work is a milestone in the theory of interpolation. Carleson gave a characterization of all sequences $\mathcal{Z} = \{z_i\} \subset \mathbb{D}$ such that for any bounded data $\{w_i\} \in \ell^\infty$ there exists a bounded analytic function $f \in H^\infty(\mathbb{D})$ such that $f(z_i) = w_i, i = 1, 2, \dots$

Carleson's result had a profound impact in the function theory of the Banach algebra $H^\infty(\mathbb{D})$ and in particular it lies in the heart of the proof of the Corona theorem [1] for $H^\infty(\mathbb{D})$. Later work of Shapiro and Shields [6] completed the picture by considering a related interpolation problem for the Hardy space $H^2(\mathbb{D})$. A sequence $\mathcal{Z} = \{z_i\}$ is called (simply) interpolating for $H^2(\mathbb{D})$ if for any $\alpha \in \ell^2$ there exists $f \in H^2(\mathbb{D})$ such that

$$f(z_i) = \alpha_i (1 - |z_i|^2)^{-1/2}, i = 1, 2, \dots$$

Shapiro and Shields [6] showed that the two notions coincide; a sequence is (simply) interpolating for $H^2(\mathbb{D})$ if and only if it is interpolating for $H^\infty(\mathbb{D})$.

In this talk we will focus primarily on interpolation by analytic functions in unit disc which belong to the Sobolev space $H_1(\mathbb{D})$. Equipped with the seminorm

$$\int_{\mathbb{D}} |f'|^2 dA,$$

it becomes a Hilbert space of analytic functions which is usually called the Dirichlet space. We shall present a characterization of simply interpolating sequences in the Dirichlet space. The same characterization is conjectured to hold in all complete Nevanlinna Pick spaces but the problem remains open despite recent progress. Finally, we are going to discuss some variants of the classical interpolation problem, such as random interpolation. This is a field where numerous questions remain open.

References

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