The Dirichlet space on the bi-disc: Carleson measures

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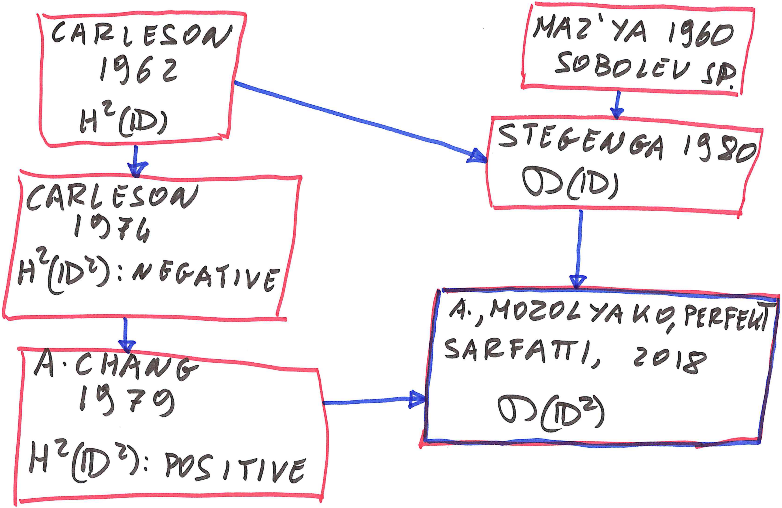
Geometric Analysis and Potential Theory

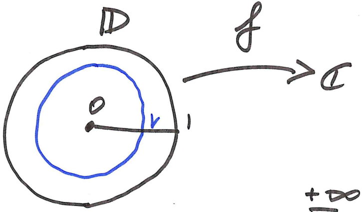
Plan

- a biased survey on Carleson measures;
- the Dirichlet space on the bi-disc;
- what lieas ahead.

Within context



CARLESON MEASURES FOR THE HARDY SPACE



$$\|f\|_{H^2}^2 := \sup_{r<1} \frac{1}{2\pi}\int_{-\pi}^{\pi} |f(re^{it})|^2 dt$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} |f(e^{it})|^2 dt$$

$$= \sum_{n=0}^{+\infty} |\hat{f}(n)|^2$$

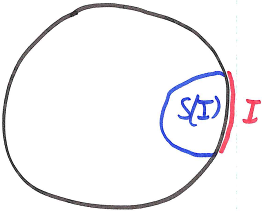
$\mu \geq 0$ on $\mathbb{D}$: $\int_{\mathbb{D}} |f|^2 d\mu \leq C(\mu)\cdot \|f\|_{H^2}^2$?

CARLESON 1962

TFAE: (A) $\int_{\mathbb{D}} |f|^2 d\mu \leq C(\mu) \cdot \|f\|^2_{H^2}$

μ "is CARLESON" FOR $H^2(\mathbb{D})$

(B) $\mu(S(I)) \leq C'(\mu) \cdot |I|$



WHY CARE?

- INTERPOLATING SEQUENCES
- BMO AND $(H^1)^* = BMO$
- HANKEL FORMS
- CORONA THEOREM

ETC.

Bi-disc

HARDY SPACE ON THE BI-DISC

$$H^2(\mathbb{D}^2) = H^2(\mathbb{D}) \otimes H^2(\mathbb{D})$$

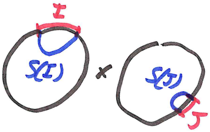
$$\|f\|^2_{H^2(\mathbb{D}^2)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2\pi}\int_{-\pi}^{\pi} |f(e^{is}, e^{it})|^2 \, ds\, dt$$

$\mu \geq 0$ ON $\mathbb{D}^2$ IS CARLESON IF:

$$\iint_{\mathbb{D}^2} |f|^2 \, d\mu \leq C(\mu) \cdot \|f\|^2_{H^2(\mathbb{D}^2)}$$

Carleson's counterexample

HARDY / BI-DISC: 1974 COUNTEREXAMPLE BY CARLESON



THERE IS $\mu \geq 0$ S. T.

$$\mu(S(I) \times S(J)) \leq |I \times J|$$

YET:

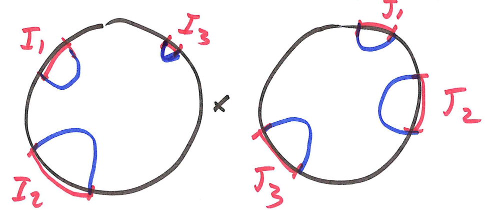
$$\cancel{\iint_{\mathbb{D}^2} |f|^2 d\mu \leq C(\mu) \cdot \|f\|^2_{H^2(\mathbb{D}^2)}}$$

- M-L REPORT
- EXPANDED BY T. TAO
- EXPANDED BY B. WICK

...

Chang's Theorem

HARDY SPACE ON THE BI-DISC
ALICE CHANG 1979



THM. μ IS CARLESON FOR $H^2(\mathbb{D}^2)$

$$\Leftrightarrow \mu\Big(\bigcup_j S(I_j)\times S(J_j)\Big) \le C(\mu)\cdot\Big|\bigcup_j I_j\times J_j\Big|$$

SHARP TRACE INEQUALITIES
MAZ'YA ~1960

THM. TFAE

(A) $$\int_{\mathbb{R}^n} |f|^2 \, d\mu \leq C(\mu) \cdot \int_{\mathbb{R}^n} \left(|\nabla f|^2 + |f|^2 \right) dx$$

(B) $$E \subseteq \mathbb{R}^n \Rightarrow \mu(E) \leq C'(\mu) \cdot \mathrm{CAP}_R(E)$$

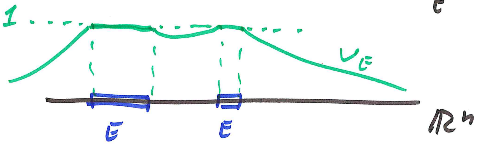
ONTOLOGY: (A) MEANS THAT f "EXISTS" ON SUPP(μ).

SUPP(μ)

Capacity by gradients

CAPACITY BY GRADIENTS

$$\mathrm{CAP}(E) = \inf\left\{ \int_{\mathbb{R}^n} |\nabla v|^2\,dx : v|_E \geq 1;\ v(\infty) = 0 \right\}$$



v_E : EXTREMAL

$v_E \in W^{1,2}$

DIRICHLET SPACE ON THE DISC

$$\mathbb{D} \xrightarrow{f} \mathbb{C} \qquad \mathbb{T} = \partial\mathbb{D}$$

$$\|f\|^2_{\mathcal{D}(\mathbb{D})} = \frac{1}{\pi}\int_{\mathbb{D}} |f'(z)|^2\,dx\,dy + \|f\|^2_{H^2} = \sum_{n=0}^{+\infty} \underbrace{(n+1)}_{\sqrt{n+1}^{\,2}} |\hat{f}(n)|^2$$

$\sqrt{n+1}^{\,2}$: $f \in W^{1/2,2}(\mathbb{T})$

$\mu \geq 0$ ON $\overline{\mathbb{D}}$ IS CARLESON FOR $\mathcal{D}(\mathbb{D})$

IF: $\int_{\overline{\mathbb{D}}} |f|^2\,d\mu \leq C(\mu)\cdot \|f\|^2_{\mathcal{D}}$

Carleson measure for the Dirichlet space

DIRICHLET SPACE ON THE DISC

$$\mathbb{D} \xrightarrow{f} \mathbb{C} \qquad \|f\|^2_{\mathcal{D}(\mathbb{D})} = \frac{1}{\pi}\int_{\mathbb{D}} |f'(z)|^2\,dx\,dy + \|f\|^2_{H^2}$$

$$\mathbb{T} = \partial\mathbb{D} \qquad = \sum_{n=0}^{+\infty} \underbrace{(n+1)}_{\sqrt{n+1}^{\,2}}|\hat{f}(n)|^2$$

$\sqrt{n+1}^{\,2}$: $f \in W^{1/2,2}(\mathbb{T})$

$\mu \geq 0$ ON $\overline{\mathbb{D}}$ IS CARLESON FOR $\mathcal{D}(\mathbb{D})$

IF: $\displaystyle\int_{\overline{\mathbb{D}}} |f|^2\,d\mu \leq C(\mu)\cdot \|f\|^2_{\mathcal{D}}$

i.e.

$\forall r<1$: $\displaystyle\int_{\overline{\mathbb{D}}} |f(rz)|^2\,d\mu(z) \leq C(\mu)\cdot \|f\|^2_{\mathcal{D}}$

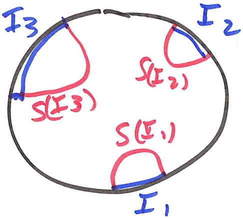
INDEP. OF $r \in (0,1)$

STEGENGA 1980

Thm. $\int_{\overline{\mathbb{D}}} |f|^2 d\mu \leq C(\mu) \cdot \|f\|^2_{\mathcal{D}(\mathbb{D})}$

$\Updownarrow$

$\mu\left(\bigcup_{j=1}^{n} S(I_j)\right) \leq C'(\mu) \cdot \mathrm{Cap}_{\mathcal{D}}\left(\bigcup_{j=1}^{n} I_j\right)$



$$\boxed{\mu(S(I)) \lesssim \frac{1}{\log \frac{1}{|I|}} \approx \mathrm{Cap}_{\mathcal{D}}(I)}$$

Capacity by kernels

CAPACITY BY KERNELS

$$\underset{\text{METRIC}}{X} \times \underset{\text{MEASURE}}{M} \xrightarrow{k} \mathbb{R}^+ \cup \{+\infty\}$$

$$M \xrightarrow{f} \mathbb{R}^+ \rightsquigarrow Kf(x) = \int_M k(x,y)\, f(y)\, dy$$

$$\mu \ge 0 \text{ ON } X \rightsquigarrow \check{K}\mu(y) = \int_X k(x,y)\, d\mu(x)$$

$$E \subseteq X \longmapsto \boxed{\mathrm{CAP}(E)} = \inf\left\{ \|f\|^2_{L^2(M)} : Kf \ge 1 \text{ ON } E \right\}$$

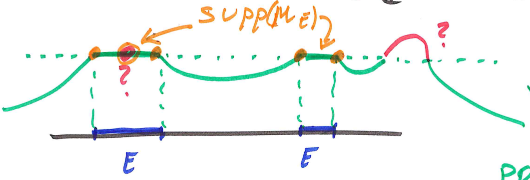
$$= \sup\left\{ \frac{\mu(E)^2}{\mathcal{E}(\mu)} : \mathrm{supp}(\mu) \subseteq E \right\}$$

$$\mathcal{E}(\mu) = \int_M \check{K}\mu(y)^2\, dy$$

Capacity by kernels

EXTREMALS $\quad f^E = \check{K}\mu_E$

$$\mu^E(E) = \mathrm{CAP}(E) = \mathcal{E}(\mu^E) = \|f^E\|^2_{L^2(M)}$$



$$V^{\mu^E} = K\check{K}\mu^E$$

POTENZIALE DI EQUILIBRIO

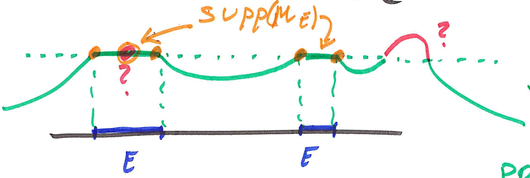
COMPARISON WITH NEWTONIAN SETTING

$$X = M = \mathbb{R}^n$$

$$K(x,y) = c_n \cdot \frac{1}{|x-y|^{n-2}}$$

EXTREMALS $f^E = \check{K}\mu_E$

$$\mu^E(E) = \mathrm{CAP}(E) = \mathcal{E}(\mu^E) = \|f^E\|^2_{L^2(\mu)}$$



$V^{\mu^E} = K\check{K}\mu^E$

POTENZIALE DI EQUILIBRIO

COMPARISON WITH NEWTONIAN SETTING

↓

CAP IS AS IN MAZ'YA

$$X = M = \mathbb{R}^n$$

$$k(x,y) = c_n \cdot \frac{1}{|x-y|^{n-2}}$$

Capacity by kernels

WHAT'S $CAP_{\mathcal{D}}$ IN STEGENGA'S?

$\mathbb{T} \equiv [0,1]$: $k(x,y) = \dfrac{1}{|x-y|^{1/2}}$

$CAP_{\mathcal{D}} \approx CAP_{W^{1/2,2}(\mathbb{T})}$

Sketch of Stegenga's: necessity

WHAT'S $CAP_{\mathcal{D}}$ IN STEGENGA'S?

$\mathbb{T} \equiv [0,1]$: $k(x,y) = \frac{1}{|x-y|^{1/2}}$

$Cap_{\mathcal{D}} \approx Cap_{W^{1/2,2}(\mathbb{T})}$

SKETCH OF STEGENGA'S ARGUMENT

NECESSITY $\int |f|^2 d\mu \leq \int_{\mathbb{D}} |f'|^2 + \int_{\mathbb{T}} |f|^2$

$\quad \| f \|_{\mathcal{D}}^2$

$\Rightarrow \mu(\cup I_j) \leq \inf\{\|f\|_{\mathcal{D}}^2 : Re f \geq 1$ on $\cup I_j\}$

$\approx Cap(\cup I_j)$

SOME CONS. FNCTN ENTERS HERE

$\circledast Re(f) \geq 1$ on $I \Rightarrow Re(f) \geq c$ on $S(I)$

Sufficiency: Strong Capacitary Inequality

SKETCH OF STEGENGA'S SUFFICIENCY.

(A) STRONG CAPACITARY INEQUALITY MAZ'YA ADAMS

$$\sum_{k=-\infty}^{+\infty} 2^{2k} \mathrm{Cap}(\alpha \in \mathbb{T} : \mathrm{Re} f(\alpha) \geq 2^k) \lesssim \|f\|_{\mathcal{D}}^2$$

OBS $2^{2k} \mathrm{Cap}(\alpha : \mathrm{Re} f(\alpha) \geq 2^k) \lesssim \|f\|_{\mathcal{D}}^2$

IS TAUTOLOGICAL

SKETCH OF STEGENGA'S

SUFFICIENCY.

(A) STRONG CAPACITARY INEQUALITY MAZ'YA ADAMS

$$\sum_{k=-\infty}^{+\infty} 2^{2k}\,\mathrm{Cap}(\alpha\in T:\ \mathrm{Re}\, f(\alpha)\geq 2^k)\leq \|f\|_{\mathcal{D}}^2$$

OBS $\cancel{2^{2k}}\,\mathrm{Cap}(\alpha:\ \mathrm{Re}\,\frac{f(\alpha)}{2^k}\geq \cancel{2^k}\,1)\leq \left\|\frac{f}{2^k}\right\|_{\mathcal{D}}^2$

IS TAUTOLOGICAL

SKETCH OF STEGENGA'S SUFFICIENCY.

(A) STRONG CAPACITARY INEQUALITY MAZ'YA ADAMS

$$\sum_{k=-\infty}^{+\infty} 2^{2k}\,\mathrm{Cap}(\alpha\in\mathbb{T}^2:\ \mathrm{Re}\,f(\alpha)\ge 2^k)\lesssim \|f\|_{\mathcal{D}}^2$$

OBS $\cancel{2^{2k}}\,\mathrm{Cap}\left(\alpha:\ \mathrm{Re}\,\frac{f(\alpha)}{2^k}\ge \cancel{2^k}\,1\right)\lesssim \left\|\frac{f}{2^k}\right\|_{\mathcal{D}}^2$

IS TAUTOLOGICAL

(B) IF SUPP$(\mu)\subseteq\mathbb{T}^2$: $\int |f|^2 d\mu \le$

$$\le \sum_k 2^{2k}\mu(|f|\ge 2^k)$$

$$\lesssim \sum_k 2^{2k}\,\mathrm{Cap}(|f|\ge 2^k) \overset{\text{S.C.I.}+\dots}{\lesssim} \|f\|_{\mathcal{D}}^2$$

DIRICHLET SPACE ON THE BI-DISC

$$\mathcal{D}(\mathbb{D}^2) = \mathcal{D}(\mathbb{D}) \otimes \mathcal{D}(\mathbb{D})$$

$$\|f\|^2_{\mathcal{D}(\mathbb{D}^2)} = \left(\frac{1}{\pi}\int_{\mathbb{D}} |(\cdot)'|^2 dA + \frac{1}{2\pi}\int_{-\pi}^{\pi} |(\cdot)|^2 ds\right) \otimes \left(\frac{1}{\pi}\int_{\mathbb{D}} |(\cdot)'|^2 dA + \frac{1}{2\pi}\int_{-\pi}^{\pi} |(\cdot)|^2 dt\right)(f)$$

$$= \frac{1}{\pi}\int_{\mathbb{D}} \frac{1}{\pi}\int_{\mathbb{D}} |\partial_{zw} f(z,w)|^2 dA(z)\, dA(w)$$

$$+ \frac{1}{\pi}\int_{\mathbb{D}} \frac{1}{2\pi}\int_{-\pi}^{\pi} |\partial_z f(z, e^{it})|^2 dt\, dA(z)$$

$$+ \cdots$$

PROBLEM: CHARACTERIZE ITS CARLESON μ

$$\iint_{\mathbb{D}^2} |f(z,w)|^2 d\mu(z,w) \le C(\mu) \cdot \|f\|^2_{\mathcal{D}(\mathbb{D}^2)}$$

Dirichlet space on the bi-disc: Carleson measures

The characterization

Thm. $\mu \geq 0$ IS CARLESON FOR $\mathcal{D}(\mathbb{D}^2)$

$$\iff \mu\left(\bigcup S(I_j)\times S(J_j)\right) \leq C'(\mu)\cdot \mathrm{Cap}_{\mathcal{D}(\mathbb{D}^2)}(I_j \times J_j)$$

[AMPS 2018]

SAME AS STEGENGA !

The characterization

<u>Thm</u>. $\mu \geq 0$ IS CARLESON FOR $\mathcal{D}(\mathbb{D}^2)$

$\iff \mu\left(\bigcup S(I_j)\times S(J_j)\right) \leq C(\mu)\cdot \mathrm{Cap}_{\mathcal{D}(\mathbb{D}^2)}(I_j \times J_j)$

[AMPS 2018]

SAME AS STEGENGA !

IN FACT:

Theorem $\Leftarrow$ Capacitary Strong Inequality

THE 1-PARAMETER (MAZ'YA, ADAMS, STEGENGA) WORLD

$$X \times M \xrightarrow{k} \mathbb{R}^+ \cup \{+\infty\}$$

(SCI) $$\boxed{\sum_{j=-\infty}^{+\infty} 2^{2j} \cdot \mathrm{Cap}(kf \geq 2^k) \lesssim \|f\|^2_{L^2(M)}}$$

$$\Uparrow$$

(MAX PRINC) $E \subseteq X$; $\mu = \mu^E$:

$$\sup_{x \in X} k\check{k}\mu^E(x) \leq C$$

$$\| \quad V^{\mu^E}(x)$$

E $\quad V^{\mu^E} = 1$ $\quad V(x) \leq C$ $\quad x$

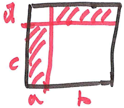
$$\Uparrow$$

(RIESZ-LIKE POTENTIALS) $d(x,y) := \dfrac{1}{k(x,y)}$ BEHAVES LIKE A DISTANCE:

$$d(x,y) \approx d(y,x) \lesssim d(x,z) + d(z,y)$$

THE 2-PARAMETER WORLD

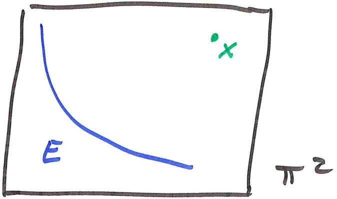
(RIESZ-LIKE) FAILS!



$$ac + bd << (a+b)\cdot(c+d)$$

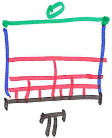
(MAX-PRINC) FAILS!

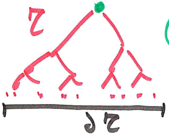
$$V^{\mu^E}(x) >> 1 = V^{\mu^E}\big|_E$$



(SCI) HOLDS!

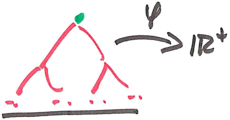
1 - PARAMETER CARICATURE

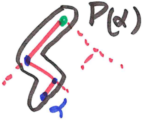




DYADICIZATION

$f(0)=0$: $\quad f(z) = \int_0^z f'(w)\,dw \quad$ in $\mathbb{D}$





$$I\varphi(\alpha) = \sum_{\beta \geq \alpha} \varphi(\beta)$$

$\mathbb{D}$	T
f'	φ
f	$I\varphi$

$\mathbb{D}$	T
$\mu \geq 0$ ON $\overline{\mathbb{D}}$	$\mu \geq 0$ ON $\overline{T}$
$\int_{\mathbb{D}} \lvert f' \rvert^2$	$\sum_{\alpha \in T} \lvert \varphi(\alpha) \rvert^2$

Dyadicization: a one parameter caricature

CARICATURE OF CARLESON (m) FOR $\mathcal{D}(\mathbb{D})$

(DISC) $\int_{\overline{T}} |I\varphi|^2 d\tilde{\mu} \le C(\tilde{\mu}) \cdot \sum_{\alpha \in T} |\varphi(\alpha)|^2$

IS THE CARICATURE OF

(CONT) $\int_{\mathbb{D}} |f|^2 d\mu \le C(\mu) \cdot \|f\|^2_{\mathcal{D}(\mathbb{D})}$

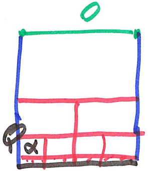
CARICATURE OF CARLESON m FOR $\mathcal{D}(\mathbb{D})$

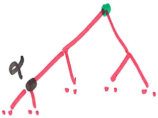
(DISC) $\int_{T} |I\varphi|^2 \, d\tilde{\mu} \le C(\tilde{\mu}) \cdot \sum_{\alpha \in T} |\varphi(\alpha)|^2$

IS THE CARICATURE OF

(CONT) $\int_{\mathbb{D}} |f|^2 \, d\mu \le C(\mu) \cdot \|f\|^2_{\mathcal{D}(\mathbb{D})}$

Thm. (DISC) $\Longleftrightarrow$ (CONT)





$\tilde{\mu}(\alpha) := \mu(Q_\alpha)$

+ BOUNDARY STUFF

2-PARAMETER CARICATURE

$$T \times T \xrightarrow{\varphi} \mathbb{R}^+$$

$$\mathbb{I}\varphi(\alpha,\beta) = (I \otimes I)\varphi(\alpha,\beta) := \sum_{\gamma \in P(\alpha)} \sum_{\delta \in P(\beta)} \varphi(\gamma,\delta)$$

$\tilde{\mu} \geq 0$ ON $\overline{T \times T}$

$$\boxed{(\mathrm{DISC}) \quad \iint_{\overline{T\times T}} |\mathbb{I}\varphi|^2 \, d\tilde{\mu} \leq C(\tilde{\mu}) \sum_{T \times T} \varphi(\alpha,\beta)^2}$$

$$\Updownarrow$$

$$\boxed{(\mathrm{CONT}) \quad \iint_{\overline{\mathbb{D}}^2} |f|^2 \, d\mu \leq C(\mu) \cdot \|f\|^2_{\mathcal{D}(\mathbb{D}^2)}}$$

where $\mu \longleftrightarrow \tilde{\mu}$ ...

2-PARAMETER CAPACITY AND S.C.I.

(CAP) $E \subseteq (\partial T)^2$: $CAP(E) = \inf\{\sum_{\alpha \in T^2} \varphi(\alpha)^2 : I\varphi(x) \geq 1 \text{ on } E\}$

i.e. $k(x, \alpha) = \chi(\alpha \in \mathbb{P}(x) := P(x_1) \times P(x_2))$

THE MAXIMUM PRINCIPLE FAILS!

YET ...

LEMMA IF $\mu \geq 0$ ON $(\partial T)^2$ ← IT SUFFICES THIS CASE

THEN $2^{2k} \cdot Cap(I I^* \mu \geq 2^k) \lesssim \frac{\|I^* \mu\|_{\ell^2}^2}{2^k}$

$I^* \mu(\alpha) = \check{I} \mu(\alpha) = \mu(x : \alpha \in \mathbb{P}(x))$

WHAT'S THE LEMMA GOOD FOR ?

(A) IT MEANS THAT ($k \geq 0$) WE HAVE A GAIN OVER TAUTOLOGICAL:

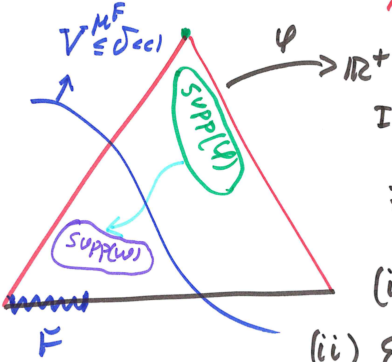
$$2^{2k} \cdot \mathrm{Cap}(\mathbb{I}\mathbb{I}^*\mu \geq 2^k) \leq \|\mathbb{I}^*\mu\|^2_{\ell^2}$$

(B) INSERTED IN (S.C.I.)'S L.H.S. ($k \geq 0$):

$$\sum_{k \geq 0} 2^{2k} \cdot \mathrm{Cap}(\mathbb{I}\mathbb{I}^*\mu \geq 2^k) \lesssim \underbrace{\sum_{k \geq 0} 1/2^k \cdot \|\mathbb{I}^*\mu\|^2_{\ell^2}}_{\text{R.H.S. OF (S.C.I.) FOR } \varphi = \mathbb{I}^*\mu}$$

(C) THE REST IS ROUTINE...

Sketch of the proof



ALL BOILS DOWN TO
A 1-PARAMETER
LEMMA

If $V^{\mu F} \leq \delta \ll 1$ ON supp(φ)

$\Downarrow$

$\exists\, \omega \geq 0$ (measure)
s.t.

(i) $V^{\omega} \geq If$ ON F

(ii) $\mathcal{E}(\omega) \lesssim \delta \cdot \|f\|^2_{\ell^2}$

We couldn't do otherwise...

OTHER WAYS TO CHARACTERIZE C.M. FOR $\mathcal{D}(\mathbb{D}^2)$?

- GOOD-λ ARGUMENTS OF Kerman-Sawyer OR A.-Rochberg-Sawyer? FAILED
- MAXIMAL FUNCTION TRICK? FAILED
- BELLMAN FUNCTION À LA NAZAROV-TREIL-VOLBERG (STOCH. OPT. CONTROL)? FAILED
- SAWYER'S COMBINATORICS FOR 2-PARAM. HARDY OPERATOR? FAILED
- CHALMOUKIS TAUTOLOGICAL TRICK? FAILED

More and less open questions

WHAT LIES AHEAD ?

- NON LINEAR CASE ($p \neq 2$)
  $\hookrightarrow$ IT BOILS DOWN TO WOLFF'S INEQUALITY
  IT WORKS
- 3 - PARAMETE CASE? WIDE OPEN
- 2 - PARAMETER POTENTIAL THEORY
  ($\mathbb{R}^n$? METRIC SPACES?): WORKING ON THAT
- APPLICATIONS TO FUNCTION THEORY
  FOR $\mathcal{D}(\mathbb{D}^2)$: WIDE OPEN but for characterizing multipliers
- CAN THIS SAY SOMETHING MORE ON
  2 - PAR. MAXIMAL FUNCTIONS ?
  WIDE OPEN
- APPLICATIONS TO "BROWNIAN SHEET"?
  WIDE OPEN

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Auguri Massimo!