$\varepsilon_0 = 1$

WE COULD AS WELL DEVELOP THE THEORY STARTING FROM ($\mu \geq 0$ IF NEEDED):

FOR $\mu$: $\quad I_1\mu(x) = \frac{1}{2\pi^2}\int_{\mathbb{R}^3} \frac{d\mu(y)}{|x-y|^2}$

FOR $f \geq 0$: $\quad I_1\mu(x) = \frac{1}{2\pi^2}\int_{\mathbb{R}^3} \frac{d\mu(y)}{|x-y|^2}$

DEFINE: $V^{\mu}(x) = I_1 I_1 \mu(x)$; potential of $\mu$

DEFINE: $\mathcal{E}(\mu) = \int (I_1\mu)^2 \, dVol$

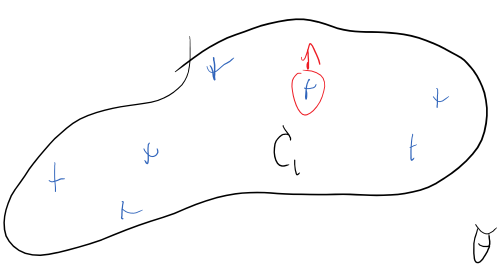
INGREDIENTS: • KERNEL $\quad g(x,y) = \frac{1}{2\pi^2}\frac{1}{|x-y|^2}$

•• BACKGROUND MEASURE: $dVol$

••• TOPOLOGY TO DEFINE BOREL MEASURES $\mu$'s.

MAIN OBJECT OF INTEREST: SET CAPACITY FOR US

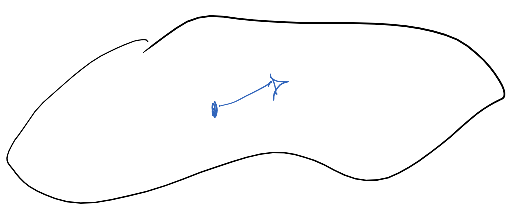
GAUSS' VIEWPOINT. $\quad C_1 \subseteq \mathbb{R}^3$ : CONDUCTOR




$\mu \geq 0$ : CHARGE DISTRIBUTION

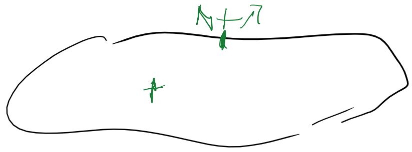
THE CHARGES MOVE UNTIL THEY REACH AN EQUILIBRIUM DISTRIBUTION $\mu^{C_1}$

• $E^{\mu^{C_1}}$: electric field: $E^{\mu^{C_1}} = 0$ in $C_1$ (at least on supp($\mu^{C}$))



$\mu^{C_1}$ ... on supp($\mu^{C}$))

- $V^{\mu^G}$ CONSTANT ON $G$ (AT LEAST ON SUPP($\mu^G$))



- $\mathrm{SUPP}(\mu^G) \subseteq \partial G$

CAPACITY: $\quad \mathrm{CAP}(G) = \mathrm{MAX}\{ \underset{\mu(G)}{\underset{\shortparallel}{\|\mu\|}} : V^{\mu^G} = 1 \text{ ON } G \}.$

<u>GAUSS</u>: $\quad \mu(G) = \mu^G(G)$

$$\mathcal{E}(\mu^G) \leq \mathcal{E}(\mu)$$

$V^{\mu^G}$ IS CONSTANT ON $G$

$V^{\mu^G} \leq 1$ ON $\mathbb{R}^3$ IF $V^{\mu^G} = 1$ ON $G$

MAXIMUM PRINCIPLE