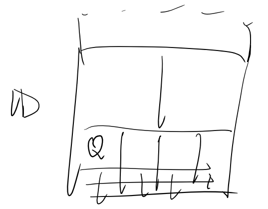
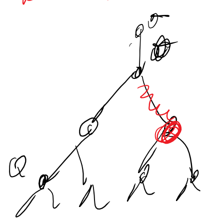
DISUGUAGLIANZE PESATE PER L'OPERATORE DI HARDY SULL'ALBERO DIADICO.





$T \xrightarrow{\varphi} \mathbb{R}^+$

$\mu(Q) = \int_Q d\mu$

$\mu \geq 0$ su D $\qquad$ $\mu \geq 0$ su T

$\varphi = \delta_\sigma \; ; \; I\delta_\sigma \equiv 1$
$\mu(T) \leq [[\mu]] \cdot 1$

$[\mu] \approx [[\mu]]$; (*) $\boxed{\sum_{x \in T} I\varphi(x)^2 \mu(x) \leq [[\mu]] \cdot \sum_{\alpha \in T} \varphi(\alpha)^2}$

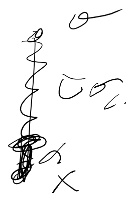
STIME PER $[[\mu]]$

DOVE $I\varphi(x) = \sum_{\alpha \in [o,x]} \varphi(\alpha)$

- TESTING CONDITION
- CAPACITÀ (ELETTROSTATICA ~~→~~ SOBOLEV)

(*) $\ell^2(T) \xrightarrow{I} \ell^2(\mu)$ $\quad \ell^2(\mu)$

$\ell^2(T)$ $\quad \varphi \in \ell^2(T) \xleftarrow[\text{sse}]{I^*_\mu} \ell^2(\mu) \ni g$

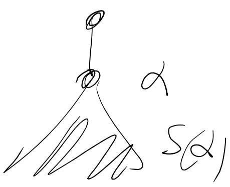


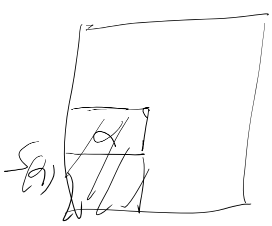
$\langle I^*_\mu g(\alpha), \varphi \rangle_{\ell^2} = \langle g, I\varphi \rangle_{\ell^2(\mu)} =$

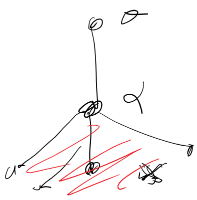
$= \sum_x g(x) \sum_{\alpha \in [o,x]} \varphi(\alpha) \mu(x)$

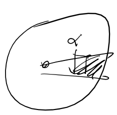
$x \in S(\alpha)$ sse $\alpha \in P(x)$

$= \sum_\alpha \varphi(\alpha) \sum_{x \in S(\alpha)} g(x) \mu(x)$









Regioni di Carleson.

$$\boxed{I^*_\mu g(\alpha) = \sum_{x \in S(\alpha)} g(x) \mu(x)}$$

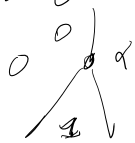
Condizioni Necessarie

$$\sum \left(I^*_\mu g(\alpha) \right)^2 \leq \sum g(x)^2 \mu(x) \cdot [[\mu]]$$

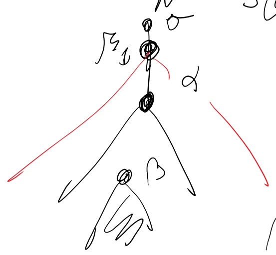
$$\sum_\alpha \left(\mathcal{J}_\mu^\sigma g(\alpha) \right)^2 \le \sum_x g(x)^2 \mu(x) \cdot [\![\mu]\!]$$

$\Rightarrow$ Test su una famiglia naturale di $\{g\}$: $g = \chi_E$: quali E?

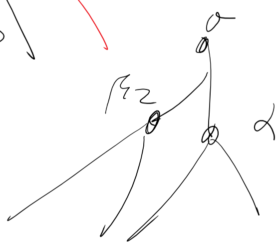
$$g = \chi_{S(\alpha)}$$



$$\mathcal{J}_\mu^\sigma \chi_{S(\alpha)}(\beta) = \int_{S(\beta)} \chi_{S(\alpha)} \, d\mu = \mu(S(\alpha) \cap S(\beta))$$



$$= \begin{cases} \mu(S(\beta_1)) & \text{se } \beta \in S(\alpha) \\ \mu(S(\alpha)) & \text{se } \beta_1 \in [\sigma, \alpha] \\ 0 & \text{altrove} \end{cases}$$



$$\boxed{\mu(T) < +\infty \text{!!!}}$$

$S(\beta_2) \cap S(\alpha) = \emptyset$

$g = \chi_{S(\alpha)}$

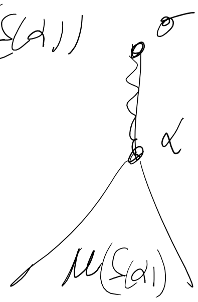
$$\sum \left(\mathcal{J}_\mu^\sigma \chi_{S(\alpha)}(\beta) \right)^2 \le [\![\mu]\!] \cdot \sum \left(\chi_{S(\alpha)}^{(x)} \right)^2 \mu(x)$$

$$\sum_{[\sigma, \alpha]} \mu(S(\alpha))^2 + \sum_{S(\alpha) \ni \beta} \mu(S(\beta))^2 \;\Big\|\; [\![\mu]\!] \cdot \mu(S(\alpha))$$

$$d(\sigma, \alpha) \cdot \underbrace{\mu(S(\alpha))^2}_{\mathrm{II}} + \underbrace{\sum_{S(\alpha) \ni \beta} \mu(S(\beta))^2}_{\mathrm{I}}$$

(II) $d(\sigma, \alpha) \cdot \mu(S(\alpha))^2 \le [\![\mu]\!] \cdot \mu(S(\alpha))$

$$\boxed{\mu(S(\alpha)) \le [\![\mu]\!] \cdot \frac{1}{d(\alpha, \sigma)}}$$



(SIMPLE CONDITION)

TROPPO DEBOLE.

(I) $\displaystyle\sum_{\beta \in S(\alpha)} \mu(S(\beta))^2 \le [\![\mu]\!] \cdot \mu(S(\alpha))$ MASSA

$\forall$ MASS/ENERGY

$$\text{(I)} \quad \sum_{\beta \in S(\alpha)} \mu(S(\beta)) \le [\![M]\!] \cdot \mu(S(\alpha))$$

ENERGIA — MASS/ENERGY CONDITION

TEOREMA: SE (I) VALE P.O. $\alpha \in T$ ALLORA VALE (II) CON $C \cdot [\![M]\!]$

$[\![M]\!] = 1 \cdot (w)$ (I)

$$\boxed{\sum (J\varphi)^2 \mu \le C \sum \varphi^2}$$

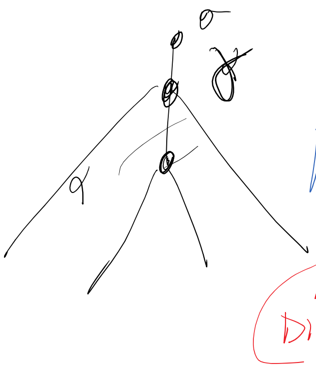
DIM. $\sum_\alpha J^*_\mu g(\alpha)^2 \le C \cdot \sum g(x)^2 \mu(x)$ VOGLIO

$$\sum_\alpha \left(\frac{\sum_{S(\alpha)\ni y} g(y)\,\mu(y)}{\mu(S(\alpha))}\right)^2 \mu(S(\alpha))^2$$

MEDIA INTEG DI g SU $S(\alpha)$

$\Uparrow$

$$\boxed{\sum_\alpha \max_{\gamma\in[0,\alpha]} \left(\frac{\sum_{S(\gamma)\ni y} g(y)\,\mu(y)}{\mu(S(\gamma))}\right)^2 \mu(S(\alpha))^2 \le C \cdot \sum g^2 \mu}$$



VANTAGGIO: $M_\mu g(\alpha)$ CRESCE CON α CHE SCENDE

$M_\mu g(\alpha)$ → FUNZ. MASS. DI g

$M_\mu g(\alpha_2) \ge M_\mu g(\alpha_1)$

$$\boxed{\sum_\alpha M_\mu g(\alpha)^2 \cdot \underbrace{\mu(S(\alpha))^2}_{\sigma(\alpha)} \le C \cdot \sum g^2 \mu}$$

DISUGUAGLIANZE PER FUNZIONI MASSIMALI: HARDY-LITTLEWOOD giorni nostri

VOGLIAMO:

$$g \in \ell^2(\mu) \xrightarrow{M_\mu} M_\mu g \in \ell^2(\sigma)$$

$$\sigma(\alpha) = \mu(S(\alpha))^2$$

SAPENDO: $\sum_{\beta \in S(\alpha)} \mu(S(\beta))^2 \le \mu(S(\alpha))$

CIOÈ: $\sigma(S(\beta)) \le \mu(S(\beta))$

ATTENZIONE!! M_μ NON È LINEARE, ESERCIZIO.

MA È SOTTOLINEARE: $g, h \ge 0$:

$$M_\mu(g+h) \le M_\mu g + M_\mu h$$

E OMOGENEO: $M_\mu(\lambda g) = \lambda \cdot M_\mu g$ se $\lambda > 0$ cost

$$\cancel{\text{MAX}} \quad \frac{\sum_{S(\gamma)} (g(\alpha) + h(\alpha))\mu(\alpha)}{\mu(S(\gamma))} = \frac{\sum_{S(\gamma)} g(\alpha)\mu(\alpha)}{\mu(S(\gamma))} + \frac{\sum_{S(\gamma)} h(\alpha)\mu(\alpha)}{\mu(S(\gamma))}$$

$$\wedge|$$

MAX .. g. + MAX .. h..
$\gamma \in [0,\alpha]$ $\quad \gamma \in [0,\alpha]$

$$\Longrightarrow M_\mu(g+h) \le M_\mu g + M_\mu h.$$

TEO. MARCINKIEVIC. SE $\ell(\mu) \xrightarrow{T} \ell(\sigma)$, $\ell(X_\mu, \mu) \xrightarrow{T} \ell(X_\sigma, \sigma)$

È ~~SOTTOLINEARE~~ SUBADDITIVO E $f \ge 0 \Rightarrow Tf \ge 0$

(0) È LIMITATO SU ℓ^∞:

$$\sup_{x \in X_\sigma} |Tf(x)| \le C_\infty \cdot \sup_{x \in X_\mu} |f(x)|$$

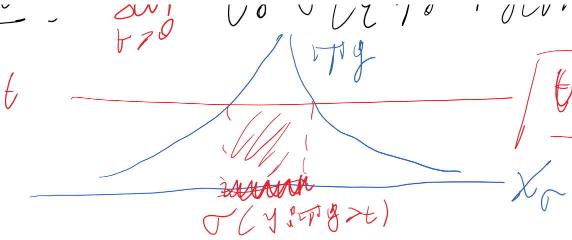
(00) È DEBOLMENTE LIMITATO SU ℓ^1

CIOÈ: $\sup_{t>0}$ $t \cdot \sigma(\{y : Tg(y) > t\}) \le C_1 \cdot \int g \, d\mu$

$\wedge$ Tg

t, ———— $\int t \cdot \mathbb{1}_{\{y : Tg > t\}} \le \int Tg \cdot d\sigma$

$\forall t>0$



$$\boxed{t\cdot\sigma(y: Tg>t)\le\int Tg\, d\sigma}$$

O.K. TROPPO GROSSO

ALLORA: T È LIMITATO SU L^p $\forall p\in(1,\infty)$:

$$\boxed{\int (Tg)^p\, d\sigma \le C(p)\cdot\int g^p\, d\mu}$$

$p=2$ CI INTERESSA

CI BASTA VERIFICARE AP. MARC.

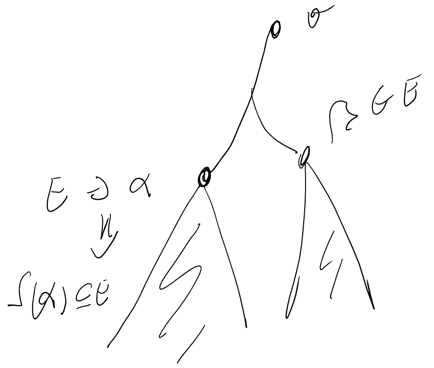
PER $T=\mathcal{M}_\mu$

- $\mathcal{M}_\mu$ è positivo, omogeneo, subadditivo

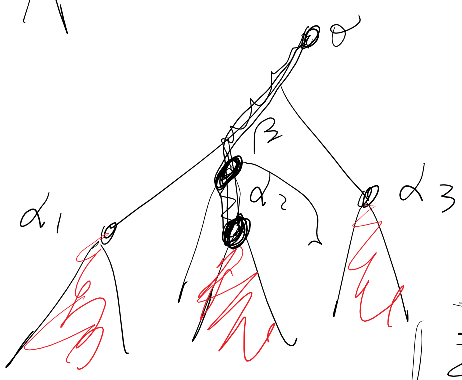
- (∞) $$\underbrace{\max_{\gamma\in\Gamma(o,\alpha]}\frac{\sum_{x\in S(\gamma)} g(x)\mu(x)}{\mu(S(\gamma))}}_{=\ \mathcal{M}_\mu g(\alpha)} \le \underbrace{\cancel{\max_{\gamma\in\Gamma(o,\alpha]}}\|g\|_{\ell^\infty}\cdot\cancel{\frac{\mu(S(\gamma))}{\mu(S(\gamma))}}}_{=\ \|g\|_{\ell^\infty}}$$

$C_\infty=1$

- (1) $t>0$: $$E=\{\alpha : \mathcal{M}_\mu g(\alpha)>t\} = \coprod_{j=1}^{\infty} S(\alpha_j)$$



DOVE α_j SONO MINIMALI CON LA PROP. $\mathcal{M}_\mu g(\alpha)>t$



FINE VERIFICA:

$$\boxed{\frac{\sum_{S(\alpha_j)} g\mu}{\mu(S(\alpha_j))}>t}$$

FINE VERIFICA:

$$t \cdot \sigma(E) = t \cdot \sum_j \sigma(S(\alpha_j))$$

$$\boxed{\frac{\ldots}{\mu(S(\alpha_j))} > t}$$

PERCHÉ...

IPOTESI MASSA/ENERGIA

$$\leq t \cdot \sum_j \mu(S(\alpha_j)) \qquad M_\mu g(\alpha_j) > t$$

$$< \sum_j \sum_{S(\alpha_j)} g\,\mu$$

e $M_\mu g(\beta) \leq t$ se $\beta \neq \alpha_j$, $\beta \in [\pi, \alpha_j]$

PER MASSIMALITÀ DI α_j (PAVEL)

$$\leq \sum g\,\mu$$

Cioè: $t \cdot \sigma(M_\mu g > t) \leq \sum g\,\mu = \|g\|_{L^1(\mu)}$