Quantum circuits for quantum walks with position-dependent coin operator

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Quantum walks

Quantum Walks (QW) constitute the quantum counterpart to classical random walks.
Random walks are stochastic processes with many applications (Brownian motion, search algorithms, stock market...)

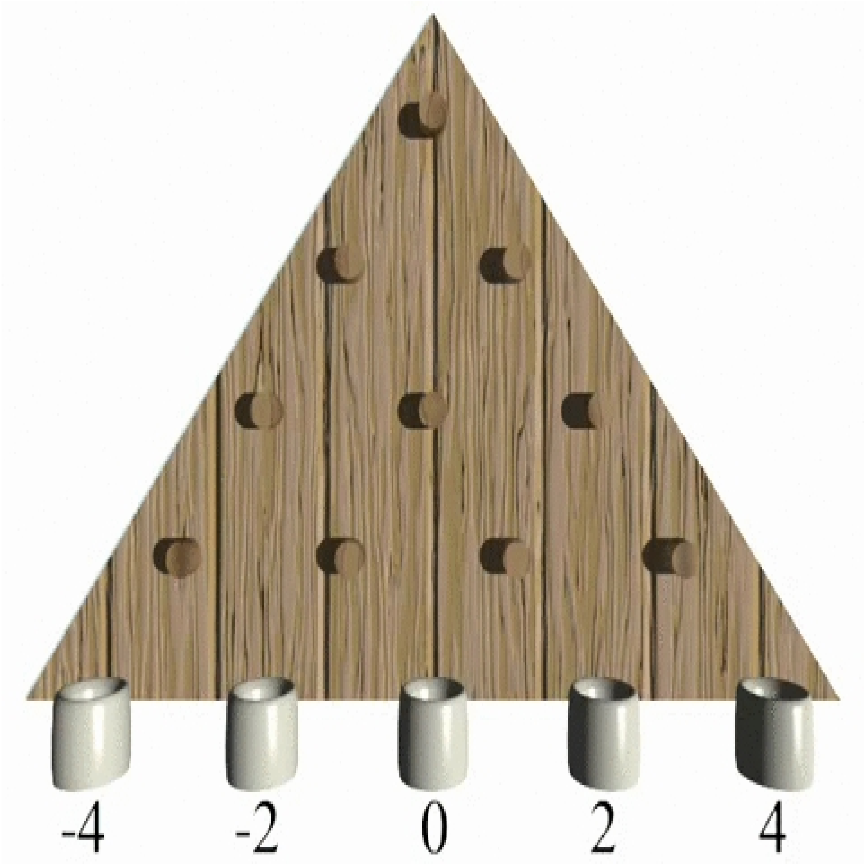
Two implementations. Given a lattice:

- Discrete time (DQW) This talk. In addition of the spatial Hilbert space they need an additional d.o.f. (coin). Repetition of a unitary operator acting on the total Hilbert space.

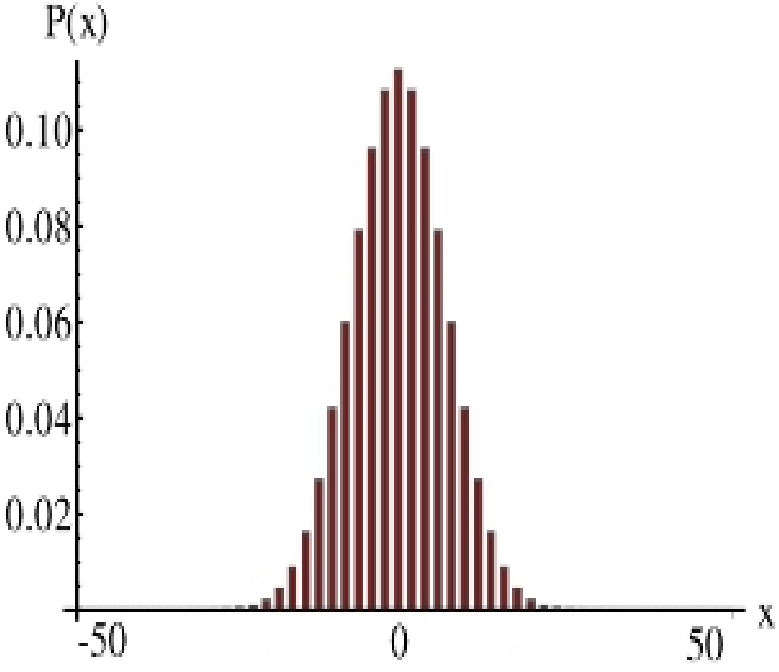
- Continuous time (CQW). Defined by a Hamiltonian (matrix). Schrödinger equation.

Galton Board

Probability distribution

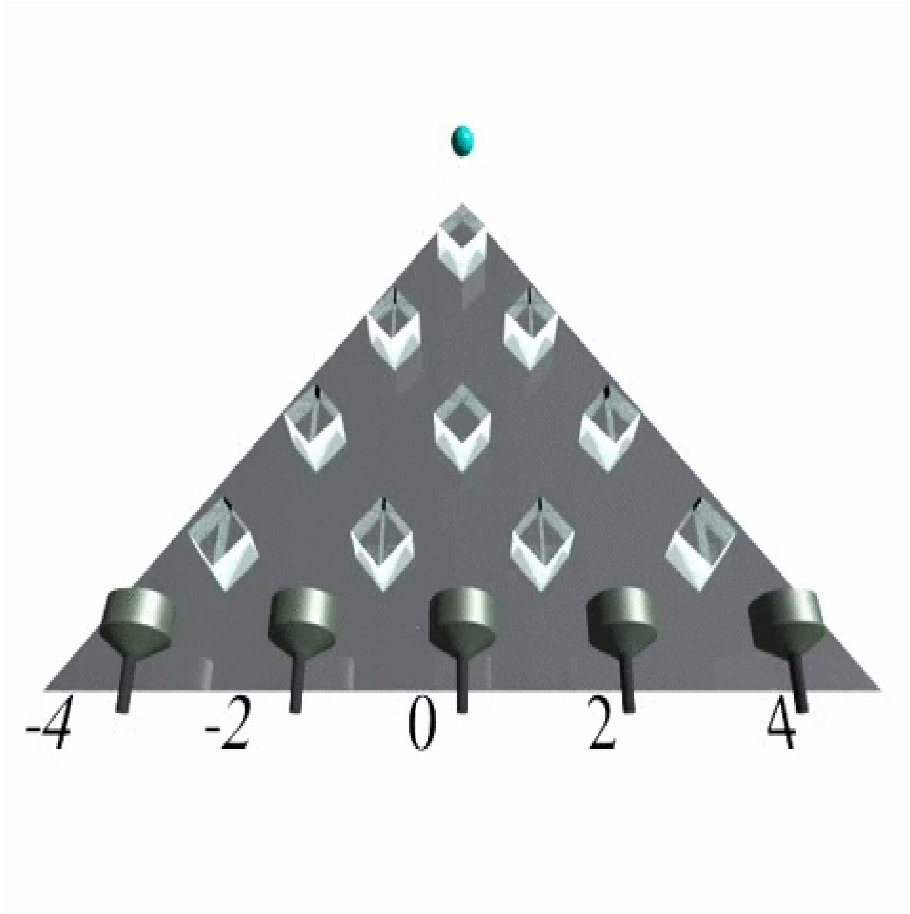


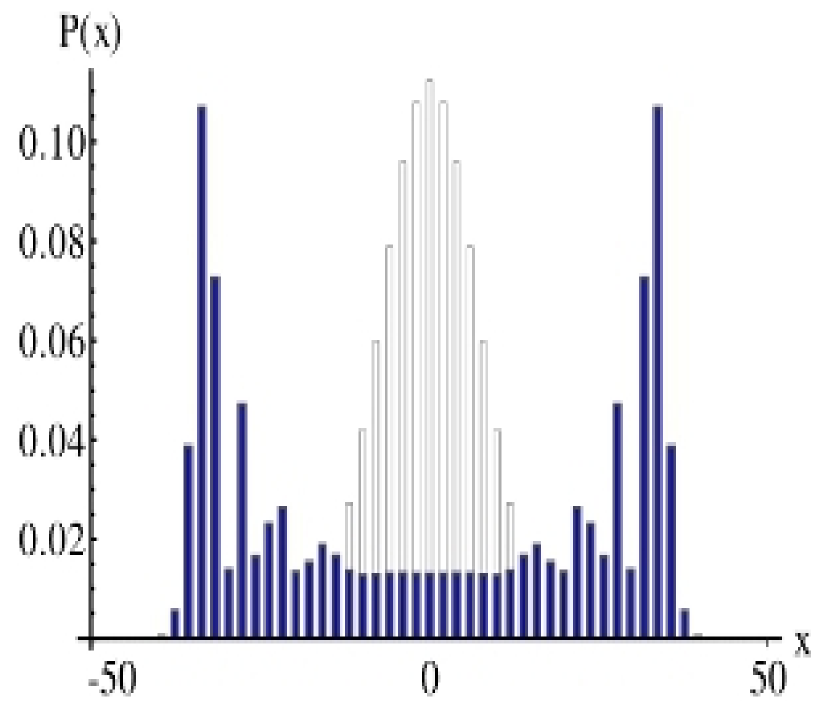
Binomial distribution (tends to Gaussian)



From https://physik.uni-paderborn.de/silberhorn/forschung/quantum-networking/quantum-walks

Quantum generalization (Discrete time Quantum Walk)





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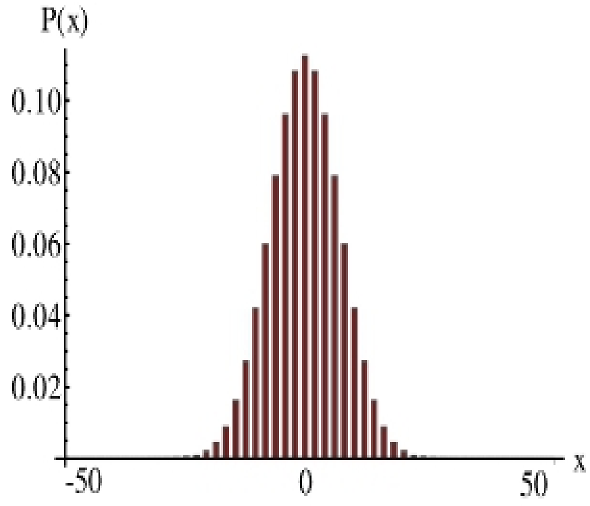
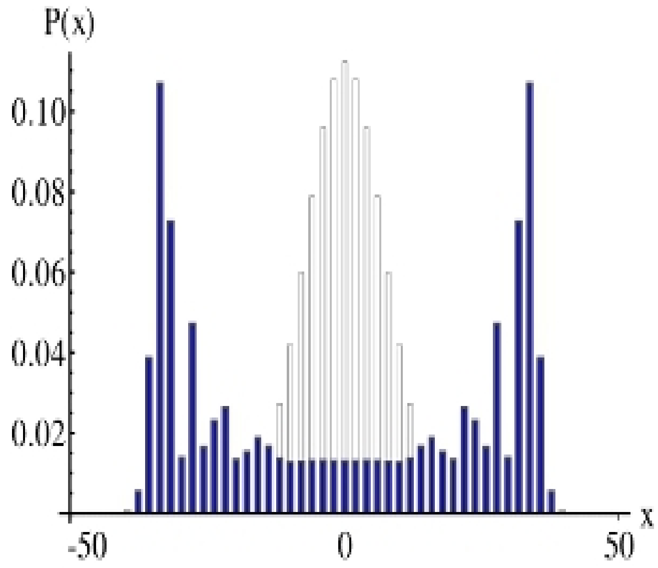
Classical



$$\sigma_{CW} \propto \sqrt{t}$$

Quantum



$$\sigma_{QW} \propto t$$

The QW spreads Quadratically Faster!

DQW time evolution (1D)

$$|\psi_{j+1}\rangle = W\,|\psi_j\rangle$$ Time step j

with $W = SC$ where $C \in U(2)$ (coin operator)

$$S = |\uparrow\rangle\langle\uparrow| \otimes \sum_p |p+1\rangle\langle p| + |\downarrow\rangle\langle\downarrow| \otimes \sum_p |p-1\rangle\langle p|$$

(displacement operator)

Applications in algorithmics

- Quantum search can be described as a QW: A. M. Childs and J. Goldstone, “Spatial search by quantum walk,” Phys. Rev. A 70, 022314 (2004).

- Element distinctness (determining whether all the elements of a list are distinct). A. Ambainis, “Quantum walk algorithm for element distinctness,” SIAM J. Comput. 37, 210–239 (2007).

Simulation of physical phenomena

Many DQWs have as continuum limit the Dirac equation.

Simulate spin-1/2 particles in an external gauge field.
Physica A 443, 179–191 (2016), Phys. Rev. A 93, 052301 (2016),
Phys. Rev. A 94, 012335 (2016), Phys. Rev. A 98, 032333 (2018),
J. Math. Phys. 60, 012107 (2019), ...

Also in a gravitational potential
Phys. Rev. A 88, 042301 (2013), Physica A 397, 157–168 (2014),
Ann. Phys. (N. Y.) 383, 645–661 (2017), Quantum Inf. Process. 15,
3467–3486 (2016), Quantum Inf. Comput. 17, 810–824 (2017),
Phys. Rev. A 97, 062111 (2018), ...

Models with extra dimensions
Phys. Rev. A 95, 042112 (2017), Sci. Rep. 12, 1926 (2022)

Neutrino oscillations: New J. Phys. 18, 103038 (2016),
Eur. Phys. J. C 77, 85 (2017).

These simulations need a position-dependent coin operator. Our goal is the simulation of these processes on a quantum computer.

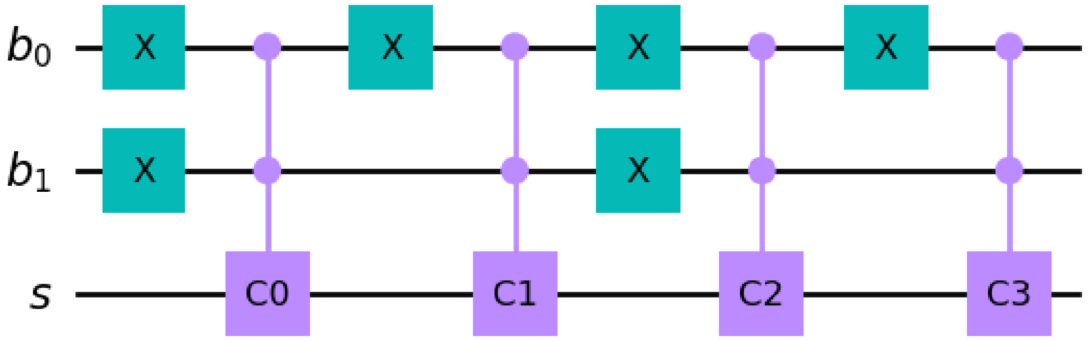
We consider n qubits (position) + 1 (coin). $N=2^n$ positions with periodic boundary conditions.
arXiv:2211.05271 (Quant. Inf. Process.)

$$C = \sum_{k=0}^{N-1} |k\rangle \langle k| \otimes C_k$$

Three proposals:

1) Naive quantum circuit
2) Linear-depth quantum circuit
3) Walsh decomposition

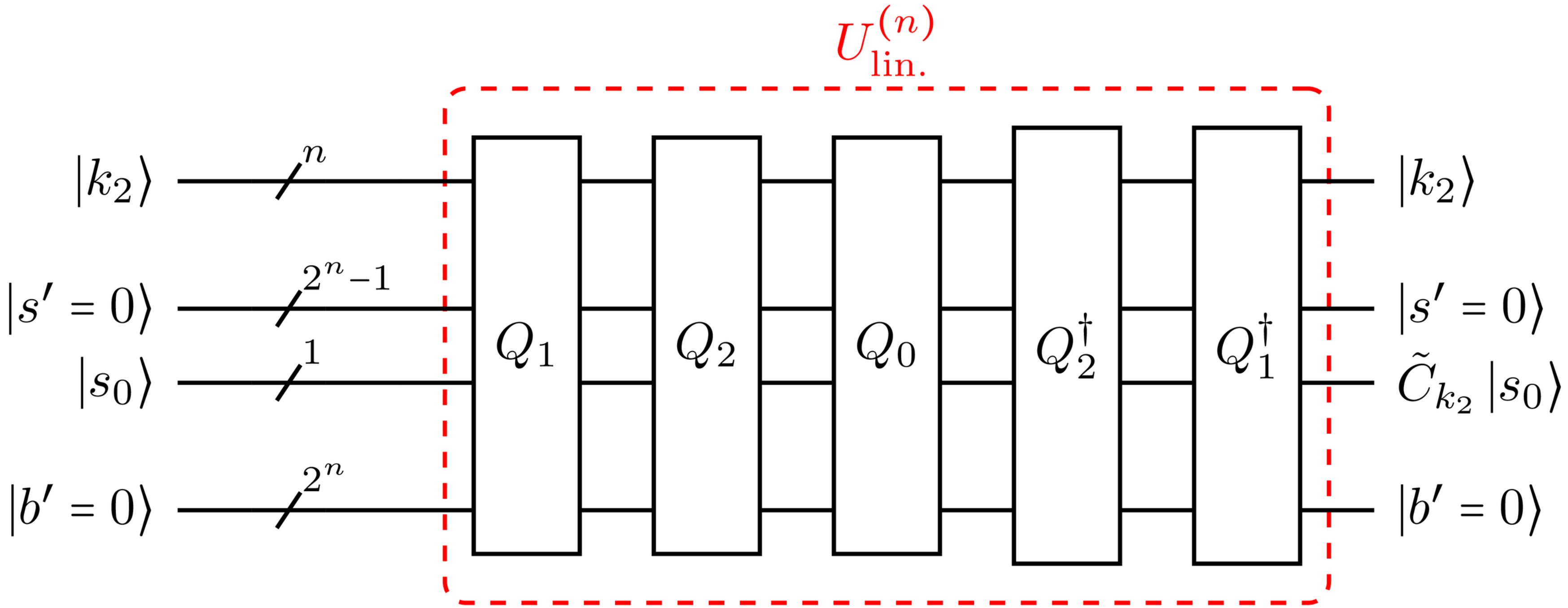
Displacement operator: based on QFT
Quantum Inf Process 19, 323 (2020).



Naive circuit

- Simple to implement
- Exponential (with n) circuit depth.

Linear depth circuit: we add (an exponential number of) auxiliary coin and space qubits, such that all C_k gates are applied in parallel.



Walsh decomposition. We first rewrite C_k as

$$C_k = e^{\mathrm{i}F_0(k)} e^{\mathrm{i}F_1(k)\sigma^3} e^{\mathrm{i}F_2(k)\sigma^2} e^{\mathrm{i}F_3(k)\sigma^3}$$

Using Walsh series, one can decompose

$$e^{\mathrm{i}F(k)\sigma} = \prod_{j=0}^{2^n-1} U_j$$

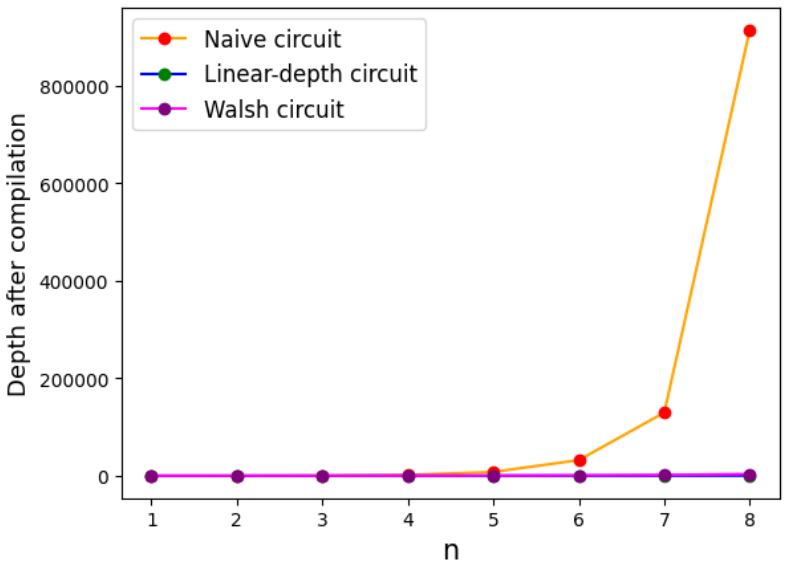
U_j : CNOTS, single qubit rotations and CZ Gates.

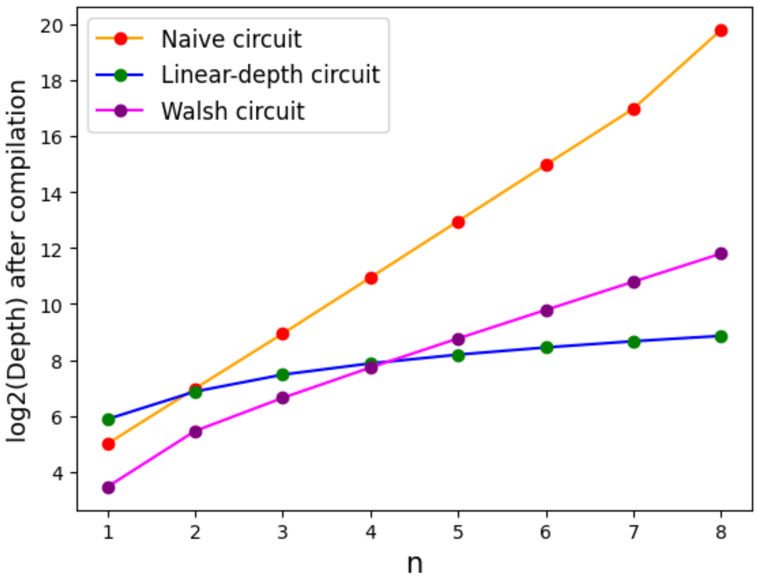
If F(k) is smooth enough, this decomposition can be truncated up to some

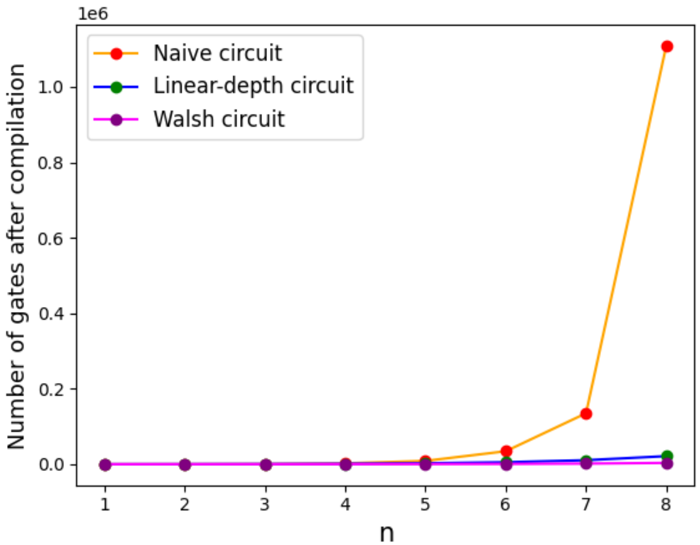
$m \ll n$ (smooth fields)

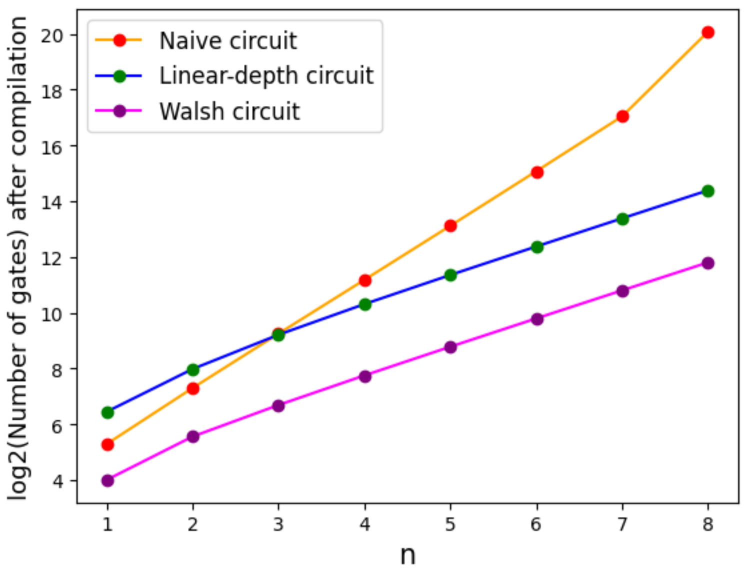
Particular case: linear F(k). Only n qubit gates.

Gate and depth counting after QASM compilation (random angles for coin operator)









Conclusions:

1) Qws with position-dependent coin operators are fundamental to simulate many Physical phenomena. We examined three proposals to implement a 1D QW with a position-dependent Coin operator:

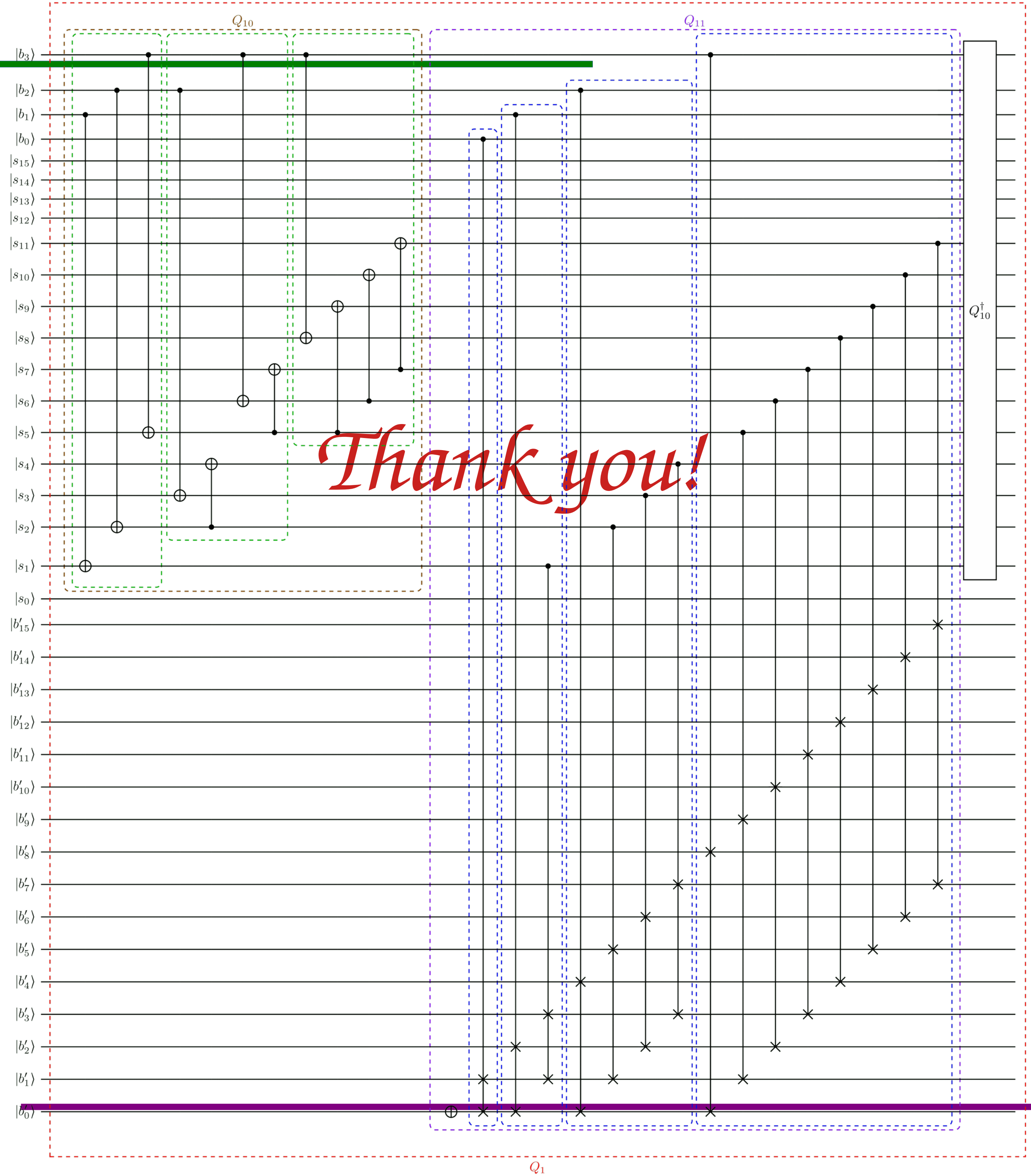
1) Naive implementation implies an exponential depth in n.

2) Linear-depth circuit requires an exponential number of auxiliary qubits.

3) Walsh decomposition can be truncated for smooth functions. Gauge fields.

4) Qws are very demanding on quantum computers.

Example: N=8 and 3 time steps. If we require a final error of the order 0.01, Gate errors need to be of the order 10^{-6}, which is still far from present status. However, maybe errors will dramatically decrease in next years.



Q_{10}
Q_{11}
$|b_3\rangle$
$|b_2\rangle$
$|b_1\rangle$
$|b_0\rangle$
$|s_{15}\rangle$
$|s_{14}\rangle$
$|s_{13}\rangle$
$|s_{12}\rangle$
$|s_{11}\rangle$
$|s_{10}\rangle$
$|s_9\rangle$
$|s_8\rangle$
$|s_7\rangle$
$|s_6\rangle$
$|s_5\rangle$
$|s_4\rangle$
$|s_3\rangle$
$|s_2\rangle$
$|s_1\rangle$
$|s_0\rangle$
$|b'_{15}\rangle$
$|b'_{14}\rangle$
$|b'_{13}\rangle$
$|b'_{12}\rangle$
$|b'_{11}\rangle$
$|b'_{10}\rangle$
$|b'_9\rangle$
$|b'_8\rangle$
$|b'_7\rangle$
$|b'_6\rangle$
$|b'_5\rangle$
$|b'_4\rangle$
$|b'_3\rangle$
$|b'_2\rangle$
$|b'_1\rangle$
$|b'_0\rangle$
$Q_{10}^\dagger$
Thank you!
Q_1