ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

Solving Mathematical puzzles

the viewpoints of Artificial Intelligence and Psychological Cognitive Science

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Premises...

AI has witnessed several achievements in the last years

To cite few of the most popular:

- ImageNet competition: in 2011 the margin of error of the AI systems in the recognition and classification of images was 26%, today is 3%. Notice: humans are less efficient and make more mistakes… on average 5%
- IBM Watson: in 2011 the software Watson competed against former human winners and won in the Jeopardy game.
- AlphaGO: in March 2016, the AlphaGO prototype by Google won against Lee Sedol, one of the best world players of GO

Why?

Premises...

Why?

Many reasons… at least two key factors:

1. Increased availability of **computational power** (memory, number of processors, high parallel algorithms, bandwith, etc.)

2. The **Big Data** era: huge quantity of data are every day collected and stored, and possibly analyzed.

Notice: Deep Neural Networks is one of the approaches that highly gained from the factors above…

… but other AI techniques have taken advantages.

However, some problems are still difficult

Let us consider the following problem:

Jacob, Lucy and Frank are three friends whose ages add up to 28 years. How many years later will their ages total 37 years?

- A 9 years old child can easily solve it.
- From the mathematical viewpoint, the problem is quite simple
- From an AI-researcher viewpoint, the problem is **trivial**

The problem in its original form (natural language), cannot be solved by current AI systems.

What is missing?

However, some problems are still difficult

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- From the mathematical viewpoint, the problem is quite simple
- From an AI-researcher viewpoint, the problem is trivial
    1. Model three friends' ages as three variables
    2. Variables must be integers; their sum is 28; their sum+3X is 37; find X.
    3. Give to a constraint solver, and find solution(s)

However, these steps ask for a substantial human intervention.

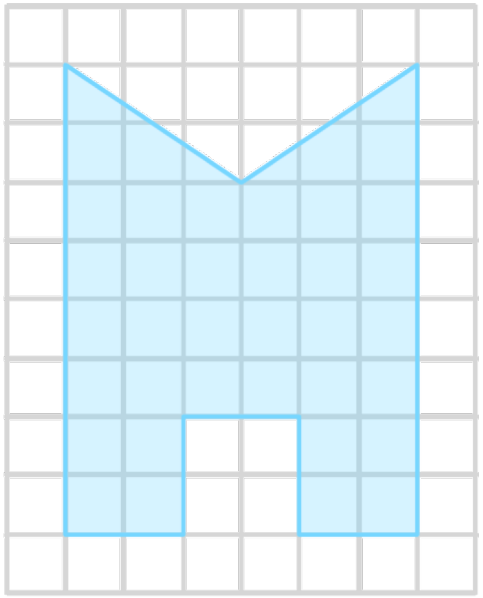
AI currently provides efficient yet specialized tools… but **human intervention is still required to comprehend a problem, and model it in a suitable, computer-understandable way.**

Mathematical puzzles as a challenge

Mathematical puzzles are recreative mathematical quizzes, with several distinct characteristics:

- Aimed to human entertainment
- They can be very different in nature
- They are described in text and/or diagrams

Mathematical puzzles as a challenge - examples



Which is biggest?

The blue area or the white?

There is an island where every inhabitants is either a knight or a knave (exclusive or).

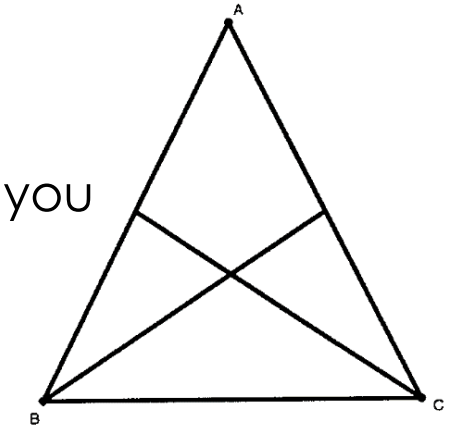
Knights always tell the truth, while knaves always lie.

You are a tourist just arrived in the island, and you met two inhabitant A and B.

A says "I am a knave, or B is a knight."

What are A and B?

How many triangles can you see?



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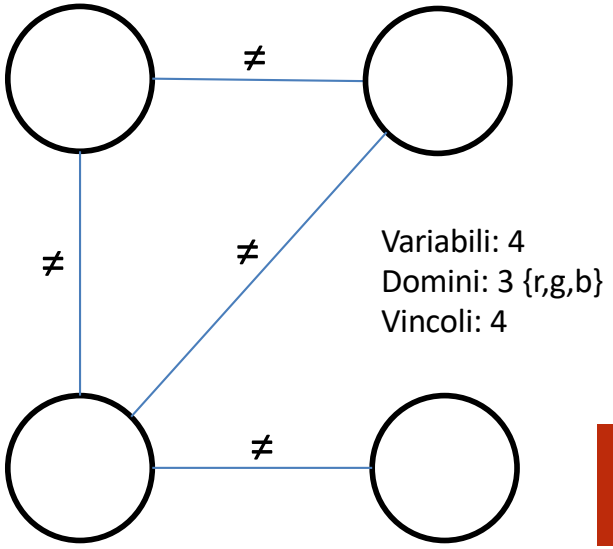
Mathematical puzzles as a challenge - examples

Uno zoo ha appena ricevuto in regalo quattro nuovi animali: un leone, un elefante, un topolino, e un gatto. Purtroppo le gabbie sono solo tre. Il problema è che il leone si mangerebbe volentieri sia il topo, che l'elefante. L'elefante a sua volta ha paura del topo. E il gatto? Beh, il gatto se potesse farebbe banchetto del topo. I guardiani dello zoo non sanno quali animali mettere in quali gabbie. Riesci a suggerire un'idea di chi potrebbe andare in quale gabbia?

From the AI viewpoint, a constraint problem with:

- 4 variables
- 3 domain values
- 4 constraints



Mathematical puzzles as a challenge

To be solved, they call for:

- Natural Language understanding (reading skills and semantic processing)
- Diagram comprehension
- Basic common, default knowledge
- Working, short- and long- term memories
- Basic inductive and deductive reasoning capabilities
- Quantitative knowledge (basic math skills)

To be solved, they DO NOT call for:

- Advanced mathematical skills
- Deep knowledge on a specific domain
- Huge data sets for training

A provocative slogan: they do no ask for big data, but rather for **big reasoning**

The ASIA-GiM project

- Internally funded by the University of Bologna, under the AlmaIDEA internal funding scheme, 2017
- Small team: it involves three informatics engineers, and two psychologist

Goals:

- Investigate and understand recent advancements in cognitive and psychological science, towards the goal of autonomous, fully fledged agents able to solve mathematical puzzles.
- Identify/collect/create a dataset of mathematical puzzles, to support analsys and research.
- Investigate and possibly develop a software prototype able to comprehend a mathematical puzzle, with a current restriction to CSP problems.

The ASIA-GiM project – many research questions

- Which problems are we going to consider?
- Problems, problem types and AI: any classification possible?
- Problems, problem types and psychological/cognitive sciences: any classification possible?

- Dealing with NLP and diagrams to provide a unified problem understanding – how?

- Which reasoning process? How a child select a solution method rather than another?
- From the problem description to the problem understanding to a problem representation – how?

- AI terms, and psychological/cognitive terms… any possible encounter?

Which problems are we going to consider?

Starting point: the International Competition of Mathematical Puzzles

- Target: different student ages, from 9 years old, up to 19 years old, and grouped by their ages

- International organizer: Fédération Française des Jeux Mathématiques https://www.ffjm.org/
- Italian organizer: Bocconi University and the MATEPristem initiative https://giochimatematici.unibocconi.it/
  A (copyrighted) database of games and solution available on their site

Our choice:

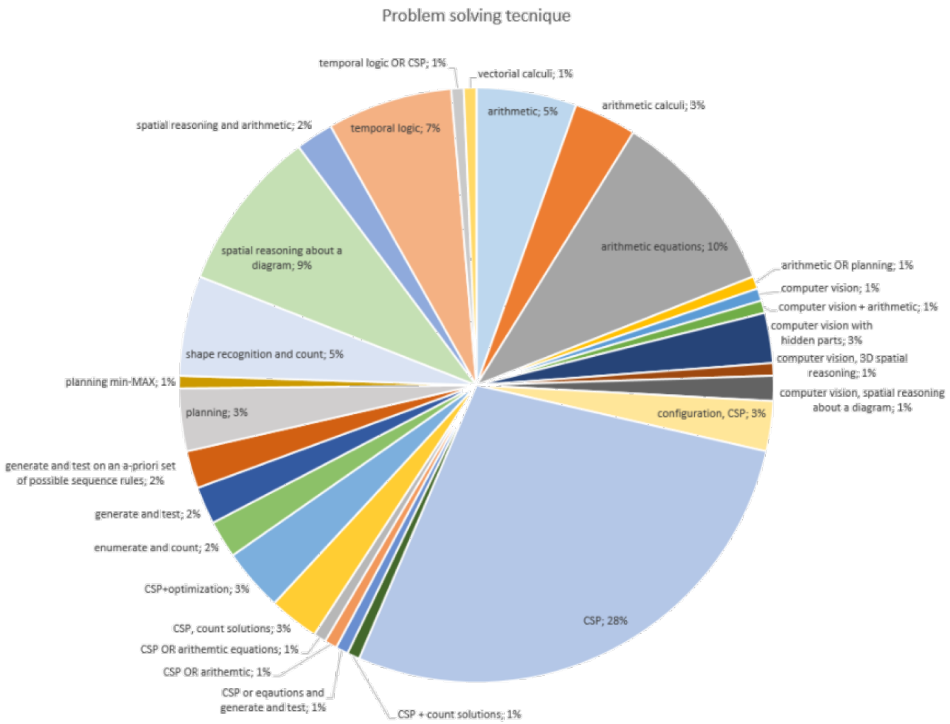
- Let us focus on problems for children aged 9-10 years (4^{th}-5^{th} grade of primary school)
- Why? no need of any specific notion, except default common reasoning…

Problems, problem types and AI: any classification possible?

- Focusing on the Bocconi database, category "CE" (4th and 5th grade of primary school)
- Years 2001 – 2017, for a total of 147 puzzles

Clear distinction between:

- problem type
- modelling approach
- solution approach



Credit: Ing. Riccardo Buscaroli
(Master Thesis ongoing project)

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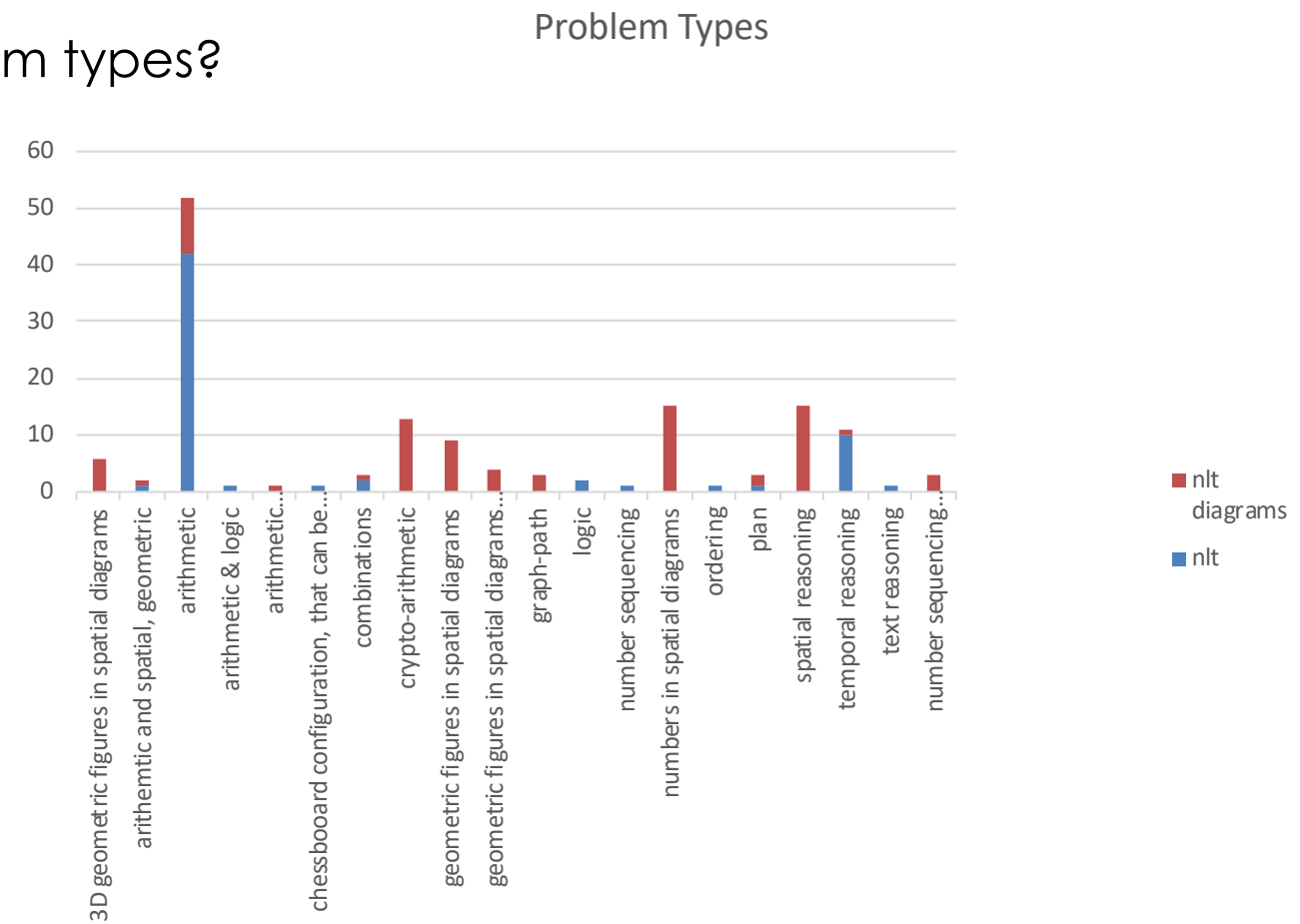
Problems, problem types and AI: any classification possible?

- Which problem types?



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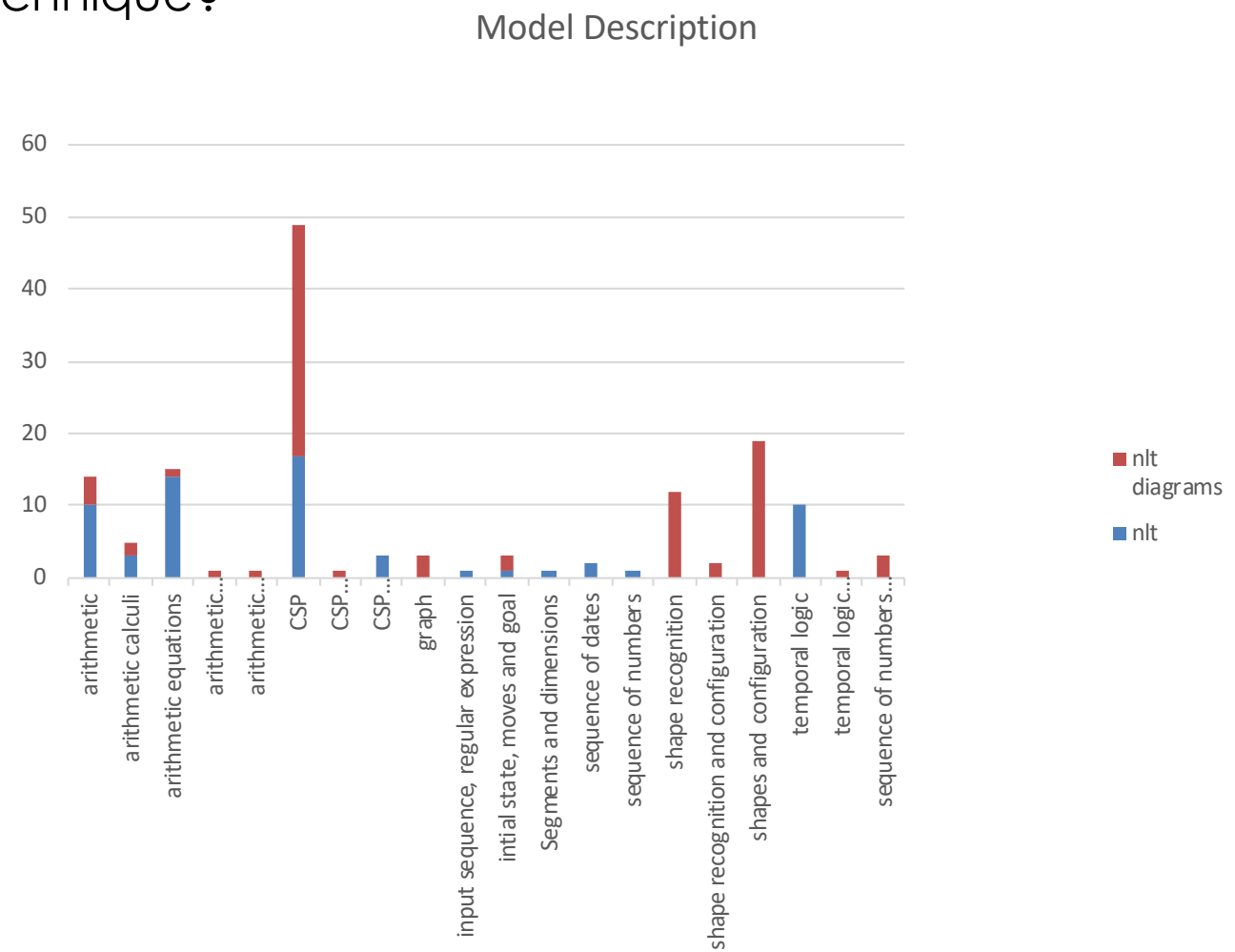
Problems, problem types and AI: any classification possible?

- Which modelling technique?



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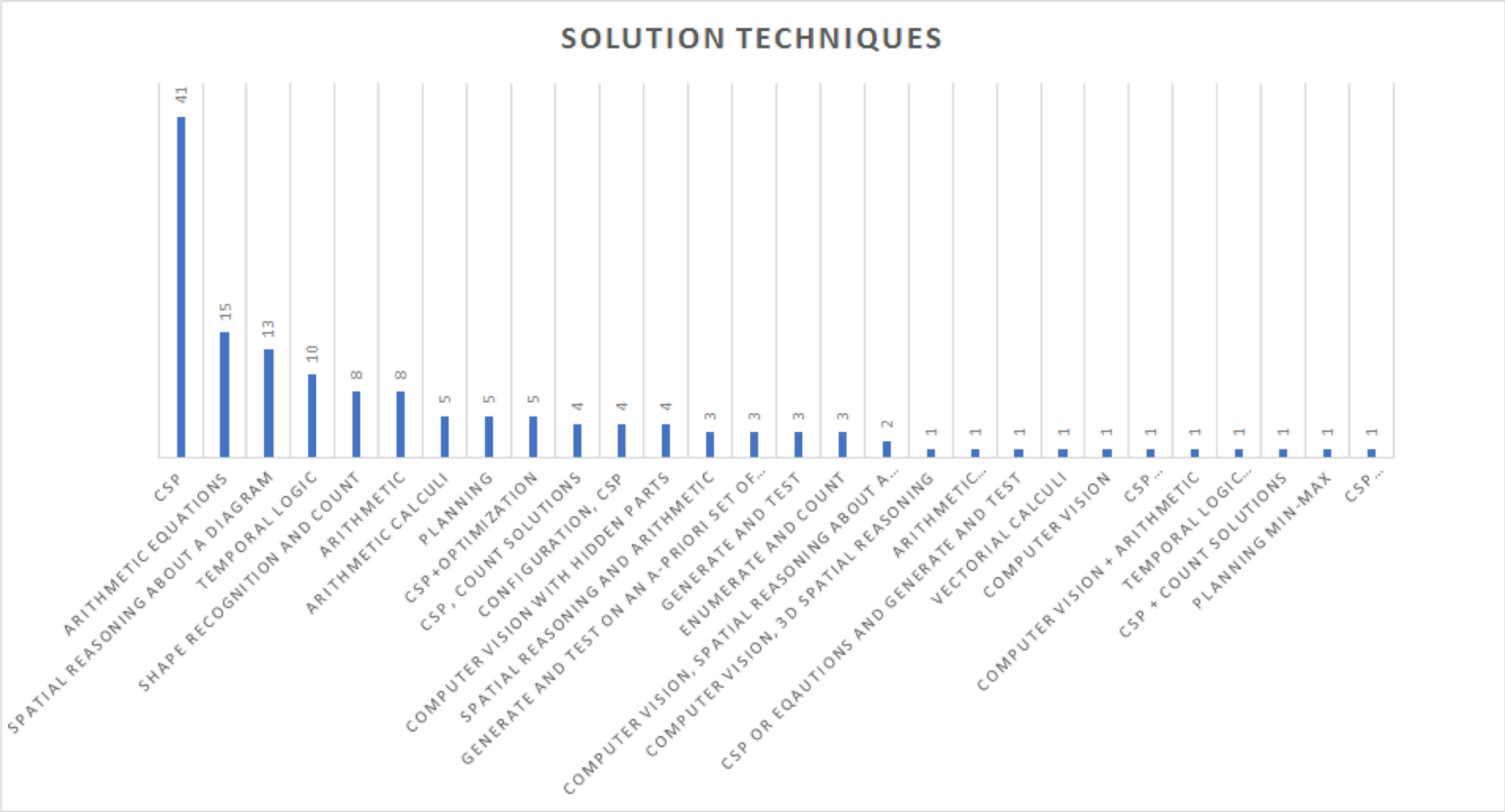
Problems, problem types and AI: any classification possible?

- Which solution technique?



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Problems, problem types and psychological/cognitive sciences: any classification possible?

Puzzles characteristics (from the cognitive viewpoint)

- E.g., is the diagram suggesting the solution? Is it suggesting the question? Is it misleading?

… but also, focus on which competences are required to solve them

- Still an ongoing work
- Starting point: the Cattell-Horn-Carroll Theory of Cognitive Abilities
    - not all the abilities are of interest: e.g., psychomotor abilities, or tactile abilities are currently ignored

Dealing with NLP and diagrams to provide a unified problem understanding – how?

Currently, we are not dealing with this issue…

However, it is worthy to mention the Visual Query Answering research field

How to mix information coming from the different source modalities?

- Is the text missing some information, that I will find in the diagram?
- Is the diagram missing some information, that I will look for in the text?
- Is the text/diagram suggesting what I should look for? I.e., my goal?

Goyal, Y., Khot, T., Summers-Stay, D., Batra, D., Parikh, D. Making the V in VQA Matter: Elevating the Role of Image Understanding in Visual Question Answering. CVPR 2017: 6325-6334

Johnson, J., Hariharan, B., van der Maaten, L., Fei-Fei, L., Zitnick, C. L., Girshick, R. B. CLEVR: A Diagnostic Dataset for Compositional Language and Elementary Visual Reasoning. CVPR 2017: 1988-1997

Which reasoning process?
How a child select a solution method rather than another?

A possible experimental research direction:

- let us create several problems, with increased difficulty levels
- let us observe children 9-10 years old and grab some insight on how they approach the problem

Problems: map coloring type; dimensions:

- number of variables
- number of domain values
- number of constraints
- constraint types (inequalities, disequalities, simple mathematical equations)

From the problem description…
…to the problem understanding…
…to a problem representation – how?

Frankly, no idea yet…

Several cognitive architectures exist; to cite few:

- ACT-R – Anderson J. R. (1983): *The architecture of cognition*. Harvard University Press
  http://act-r.psy.cmu.edu/
- SOAR – Newell A. (1990). *Unified theories of cognition*. Harvard University Press
  Laird J. E., Lebiere C., and Rosenbloom P. S. (2017). A Standard Model for the Mind: Toward a Common Computational Framework across Artificial Intelligence, Cognitive Science, Neuroscience, and Robotics., AI Magazine 38(4).
  https://soar.eecs.umich.edu/

Conclusions

- AI is experimenting a momentum, but a class of problems is still too difficult for current AI
- Mathematical puzzles are a subclass of those problems…
- … many research questions, that we are currently investigating

Several practical applications and consequences… for example:

- New models of interaction between humans and machines – machines that understand the goals and partner with humans
- Software agents able to aid children in the learning process
- From computation and AI thinking (humans adapt problems to the machine methods) to machines able to understand the problems

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